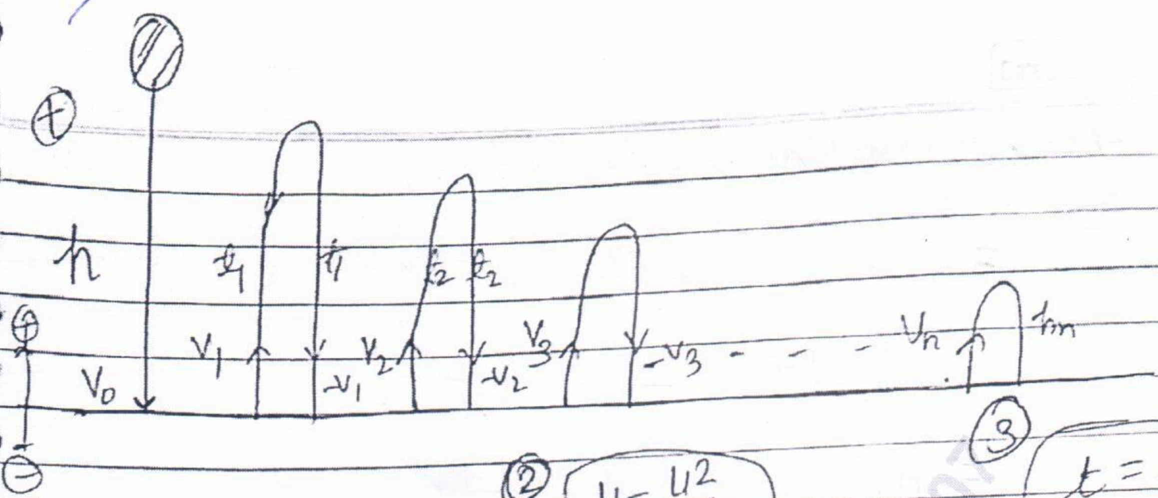

① $\boxed{V_1 = eV_0}$

$V_2 = eV_1 = e^2 V_0$

$V_3 = e^3 V_0$

$\boxed{V_n = e^n V_0}$

$\boxed{V_n = e^n \sqrt{2gh}}$

② $\boxed{H = \frac{u^2}{2g}}$

$h_n = \frac{V_n^2}{2g}$

$h_n = e^{2n} \frac{V_0^2}{2g}$

$\boxed{h_n = e^{2n} h}$

③ $\boxed{t = \frac{u}{g}}$

$t_n = \frac{V_n}{g}$

$t_n = e^n \frac{V_0}{g}$

$\boxed{t_n = e^n \sqrt{\frac{2h}{g}}}$

④ Total time $= t + 2t_1 + 2t_2 + 2t_3 + \cdots$

$t_n = e^n t$

$= t + 2\times et + 2\times e^2 t + 2\times e^3 t + \cdots$

$= t[1 + 2e + 2e^2 + 2e^3 + \cdots]$

$= t[1 + 2e(\underbrace{1 + e + e^2 + e^3 + \cdots}_{\text{G.P}})]$

G.P common ratio $= r$, $r = e$, $e<1$ $\therefore r<1$, $\boxed{S_\infty = \frac{a}{1-r}}$

$= t\left[1 + 2e\left[\frac{1}{1-e}\right]\right]$

$= t\left[\frac{1-e+2e}{1-e}\right]$

$\boxed{\text{Total} = t\left[\frac{1+e}{1-e}\right] = \sqrt{\frac{2h}{g}}\left[\frac{1+e}{1-e}\right]}$

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(5) Total distance $= h + 2h_1 + 2h_2 + 2h_3 + \dots$

$= h + 2\times e^2 h + 2\times e^4 h + 2 e^6 h + \dots \quad h_n = e^{2n} h$

$= h[1 + 2e^2 + 2e^4 + 2e^6 + \dots]$

$= h[1 + 2e^2 \underbrace{[1 + e^2 + e^4 + \dots]}_{G.P}] \quad C.R = r$

total distance $= h\left[\dfrac{1+e^2}{1-e^2}\right]$

(3) Perfectly Inelastic Collision:

→ $e = 0$

→ COLM ✓

→ COKE ✗

→ COME ✗

→ COTE ✓

→ Max^m loss in K.E

After collision bodies stick with each other

COLM,

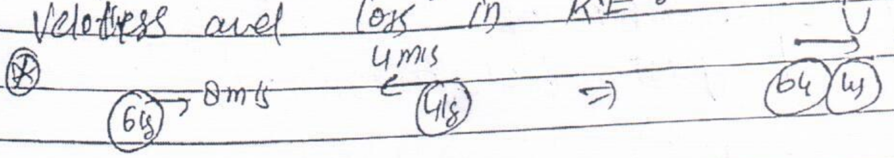
$m_1\vec{u_1} + m_2\vec{u_2} = \cancel{m_1\vec{v_1} + m_2\vec{v_2}}$
$= (m_1+m_2)\vec{V}$

$$\boxed{\vec{V} = \frac{m_1\vec{u_1} + m_2\vec{u_2}}{m_1+m_2}}$$

Loss in K.E

$$\Delta K = \frac{1}{2}\times\frac{m_1 m_2}{m_1+m_2}(\vec{u_1}-\vec{u_2})^2$$

Que. A ball of mass 6kg is moving with velocity 8m/s, collides with another body of mass 4kg moving in opp. dirn with speed 4m/s. after collision Both stick find their final velocities and loss in K.E.



$$8\times6 - 4\times4 = 10v$$

$$48 - 16 = 10V$$

$$\frac{32}{10} = V$$

$$V = 3.2\ m/s$$

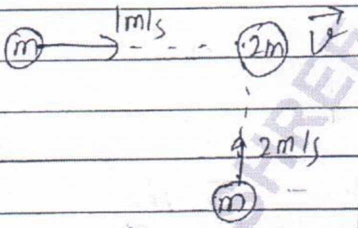
loss in K.E

$$\Delta K = \frac{1}{2}\frac{m_1m_2}{(m_1+m_2)}(\vec{u_1}-\vec{u_2})^2$$

$$= \frac{1}{2}\times\frac{6\times4}{10}\times(8-(-4))^2$$

$$= 172.8\ \text{Joule}$$

Que. Two identical particles are moving mutually ~~part~~ ⊥ dirth with speed 1m/s & 2m/s. they collides and sticks with each other find magnitude of velocity of their combination just after collision.



COLM

$$m_1\vec{u_1} + m_2\vec{u_2} = (m_1+m_2)\vec{V}$$

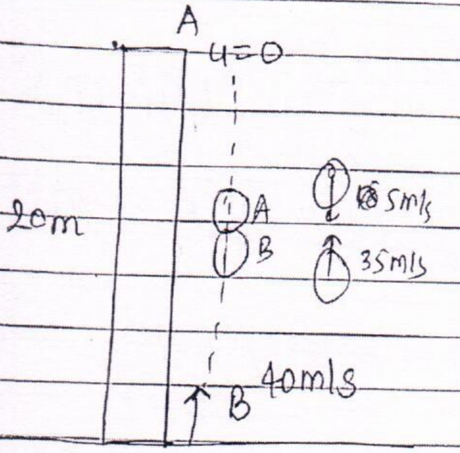
$$(m\times1)\hat{i} + m(2)\hat{j} = (2m)\vec{V}$$

$$\vec{V} = \frac{\hat{i}+2\hat{j}}{2}\ m/s$$

$$\vec{V} = \frac{1}{2}(\hat{i}+2\hat{j})$$

$$|\vec{V}| = \frac{1}{2}\sqrt{1^2+2^2} = \frac{\sqrt{5}}{2}\ m/s$$

Q. Particle A is droped from top of a tower of height 20m. Simultaneously another identical particle 'B' is projected upwards from foot of tower with speed 40 m/s. after collision they stick with each other find their combined speed just after collision.



$S_{rel} = u_{rel}t + 0$ $\quad \boxed{\vec{g}_{rel} = 0}^{*}$

$20 = (0+40)t$

$t = \frac{1}{2}\text{sec} \rightarrow$ collide after $\frac{1}{2}$ sec

$V_A = u + at$

$V_A = 0 + 10 \times \frac{1}{2}$

$= 5\ m/s$ downwards $= 5\ m/s \downarrow$

$V_B = u - gt$

$= 40 - 10 \times \frac{1}{2} = 35\ m/s$ upwards $= 35\ m/s \uparrow$

Velocity just before collision

COLM,

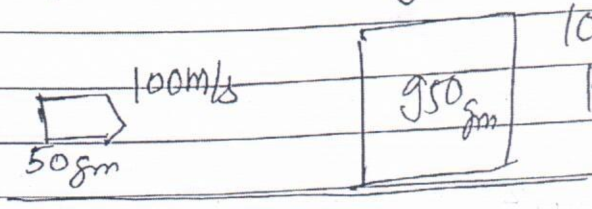
just before collision = just after collision

$$m_1\vec{u_1} + m_2\vec{u_2} = (m_1+m_2)\vec{V}$$

$$\vec{V} = \frac{m(-5) + m(35)}{2m} = \frac{30}{2} = 15\ m/s$$

$|\vec{V}| = 15\ m/s$ upwards

Q. A bullet is fired with speed 100 m/s towards a block of mass 950gm placed on a frictionless surface. Bullet embeds into the block find % loss in K.E if mass of bullet is 50gm & also find Combined velocity just after bullet is embeded.



loss in K.E

$$\frac{\Delta K}{K_i} = \frac{\cancel{\frac{1}{2}} \frac{m_1 m_2}{(m_1+m_2)} [u_1 - 0]^2}{\cancel{\frac{1}{2}} \cancel{m_1} u_1^2}$$

$$\frac{\Delta K}{K_i} \times 100\% = \frac{m_2}{m_1+m_2} \times 100\%$$

$$= \frac{950}{1000} \times 100 = 95\%$$

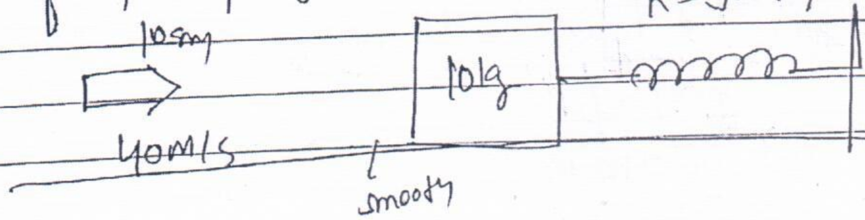
$$m_1\vec{u_1} + m_2\vec{u_2} = (m_1+m_2)\vec{V}$$

$$50 \times 100 + 950 \times 0 = (1000)\vec{V}$$

$$\frac{5000}{1000} = \vec{V}$$

$$\boxed{\vec{V} = 5 m/s}$$

Que A bullet of mass 10gm is fired with speed 40 m/s towards a block of mass 10kg acc to the fig. If bullet is embed in the block Find maxm Compression of the spring.



Scanned by CamScanner

COLM

block-bullet $\quad m_1 u_i + m_2 u_i = (m_1 + m_2) V$

$$V = \frac{10 \times 10^{-3} + 40 \times 0}{10 + 0.01} = \frac{0.4}{10.01} = 0.04\ m/s$$

$W = \Delta K$

$W_{sp} = K_f - K_i^{0}$

$\frac{1}{2} K x^2 = 0 + \frac{1}{2} \times 10 \times (0.04)^2$

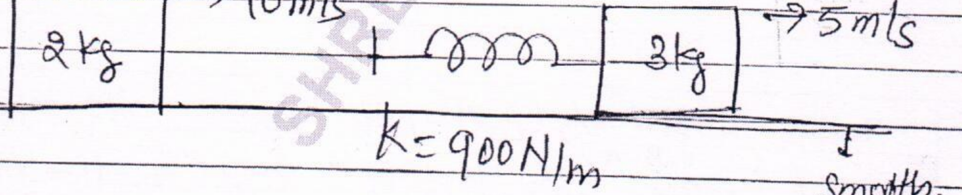
$900 x^2 = 10 \times 16 \times 10^{-4}$

$x^2 = \frac{16 \times 10^{-3}}{900}$ $\qquad x = \frac{4}{3} \times \frac{10^{-2}}{\sqrt{3}}\ m$

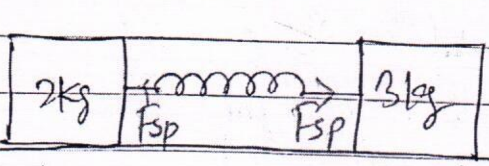
$x = \frac{4}{3\sqrt{10}}\ cm$

★

Que. Two blocks of mass 2kg & 3kg are moving on horizontal frictionless surface in same direction a/c to the figure. Find Max^m compression in the spring.



method-1



Speed down

Speed up

until both have same velocity.

When $\vec{V}$ are equal than compression is max^m

common velocity

COLM, $m_1\vec{u_1} + m_2\vec{u_2} = (m_1+m_2)\vec{V}$

$$\vec{V} = \frac{2\times10 + 3\times5}{5} = 7 m/s$$

$$W_{sp} = K_f - K_i$$

$$-\frac{1}{2}kx^2 = \frac{1}{2}\times5\times7^2 - \left[\frac{1}{2}\times2\times100 + \frac{1}{2}\times3\times25\right]$$

$$-900x^2 = 245 - 275$$

$$x^2 = \frac{1}{30}$$

$$x = \frac{1}{\sqrt{30}}\ m$$

Method-2

Short Trick

$$\boxed{\frac{1}{2}\frac{m_1m_2}{(m_1+m_2)}(\vec{u_1}-\vec{u_2})^2 = \frac{1}{2}kx^2}$$

loss in K.E in inelastic collision

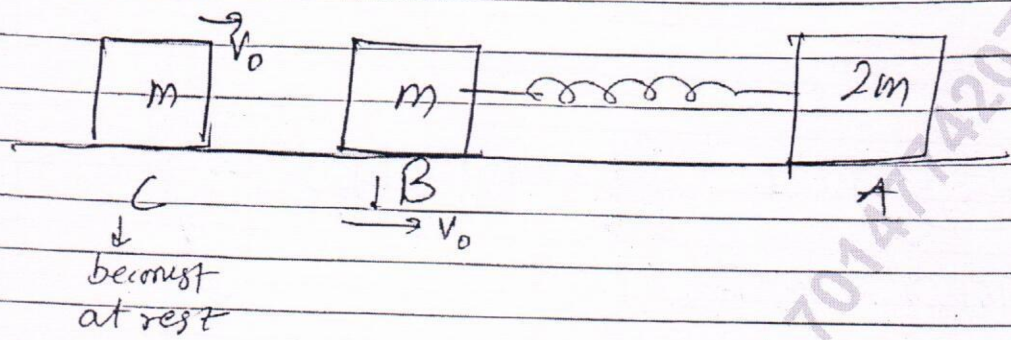
convert in to P.E

$$\frac{1}{2}\times\frac{2\times3}{5}\left[100-25\right] = \frac{1}{2}\times900\times x^2$$

$$\frac{2\times3}{5}\times\frac{75}{900} = x^2 \qquad x = \frac{1}{\sqrt{30}}\ \text{ans}$$

loss of KE in ~~elasti~~ perfectly ~~in~~ e in inelastic collision $= \Delta K = \frac{1}{2}\frac{m_1 m_2}{(m_1+m_2)}(u_1 - u_2)^2$

Que. B & A blocks at rest on frictionless surface & spring in natural length. Now block 'C' moving with velocity V_0 collides head on elastically with B block. If max^m compression in spring is 'x'. Find spring const.

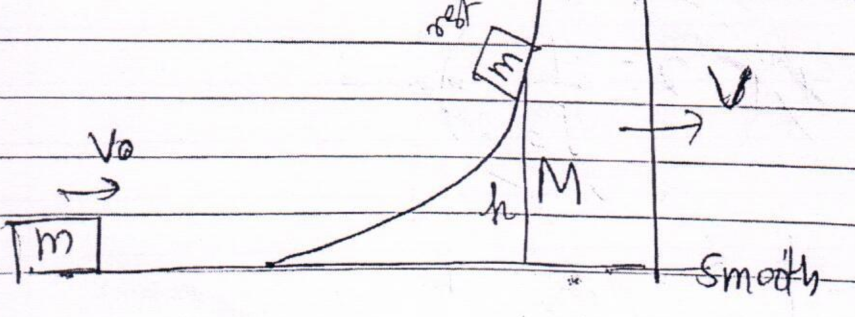


Short trick.
$$\frac{1}{2}\frac{m_1 m_2}{m_1+m_2}\left[\vec{u}_1 - \vec{u}_2\right]^2 = \frac{1}{2}kx^2$$

$$\frac{1}{2}\times\frac{m\times 2m}{3m}\left[V_0 - 0\right]^2 = \frac{1}{2}kx^2$$

$$\boxed{K = \frac{2mV_0^2}{3x^2}}$$

Q. An Inclined plane of mass 'M' is placed on frictionless horizontal surface. Now a block of 'm' is projected with speed V_0 towards the inclined plane. Find max^m height attained by the block on inclined.



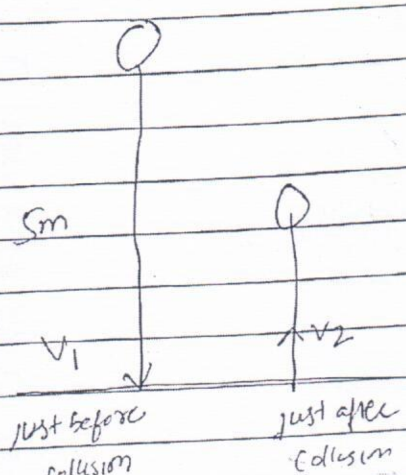
loss in K.E = gain in P.E of block
inelastic

$$\frac{1}{2}\frac{mM}{M+m}\left[V_0 - 0\right]^2 = mgh$$

$$\boxed{h = \frac{M V_0^2}{2(M+m)g}}$$

as blocks starts up to on Inclined then the Inclined also starts moving on floor when the block comes to rest momentarily Relative to Inclined then Both are moving with same velocity [perfectly Inelastic collision]

Q. A ball is droped from 5m on floor. after collison ball attain height 1m. By what facture speed of ball is lost in collision



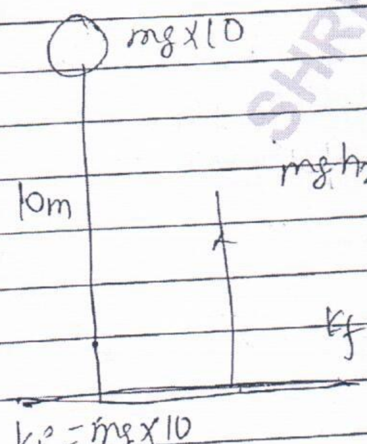
$$= \frac{v_f - v_i}{v_i} = \frac{\sqrt{2g\times1.8} - \sqrt{2g\times5}}{\sqrt{2g\times5}}$$

$$= \frac{\sqrt{\frac{18}{10}} - \sqrt{5}}{\sqrt{5}}$$

$$= \frac{\frac{3}{\sqrt{5}} - \sqrt{5}}{\sqrt{5}} = \frac{3-5}{5}$$

$$= -2/5$$

lost 2/5



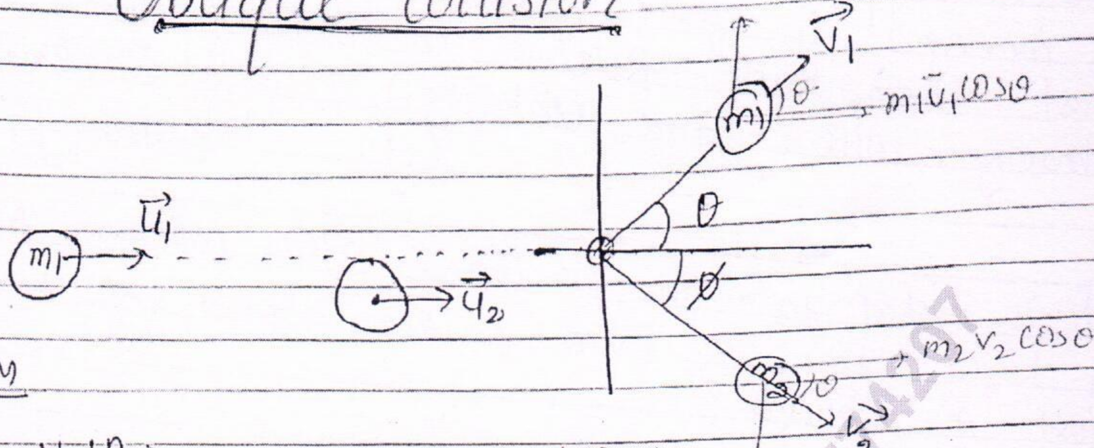
$mgh_2 = mg\times8$

~~$K_f = mg\times10\times\frac{80}{100} = mg\times8$~~

If 20% energy is lost

in what facture height

Oblique collision



COLM

x-dirn $\Rightarrow$ $m_1u_1 + m_2u_2 = m_1u_1\cos\theta + m_2u_2\cos\phi$ $\quad m_2v_2\sin\theta$

y-dirn $\Rightarrow$ $0 + 0 = m_1v_1\sin\theta - m_2v_2\sin\phi$

$m_1v_1\sin\theta = m_2v_2\sin\phi$ [cancel each other]

Elastic collision

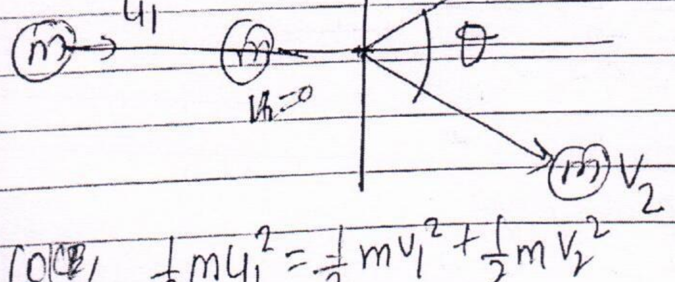
COKE,

$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$

→ Identical bodies
→ one body at rest
→ collision elastic

$\Rightarrow$ After collision velocities are Mutually perpendicular.

$$\boxed{[\theta + \phi] = 90^\circ}$$



COKE, $\frac{1}{2}mu_1^2 = \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2 \qquad u_1^2 = v_1^2 + v_2^2$ —— (1)

COLM

$$m\vec{u_1} + 0 = m\vec{v_1} + m\vec{v_2}$$

$$\vec{u_1} = \vec{v_1} + \vec{v_2}$$

$$\underline{\underline{\vec{u_1}}} = \sqrt{v_1^2 + v_2^2 + 2v_1v_2\cos\theta}$$

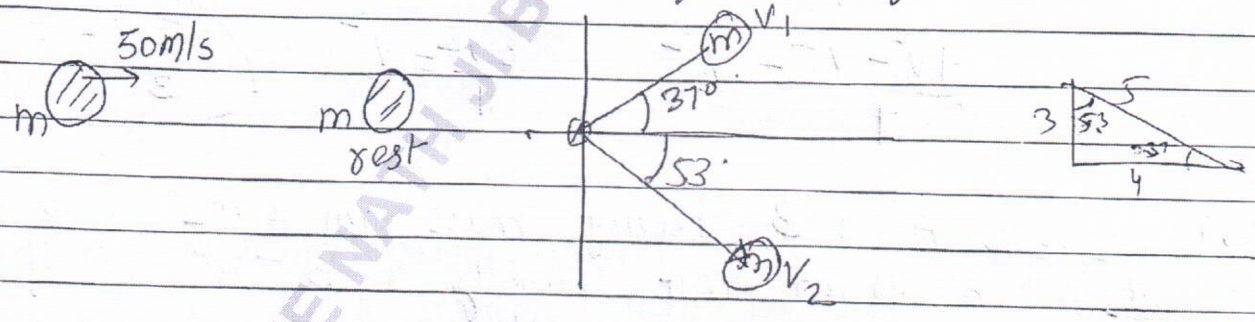
$$v_1^2 + v_2^2 = v_1^2 + v_2^2 + 2v_1v_2\cos\theta$$

$$2v_1v_2\cos\theta = 0$$

$$\cos\theta = 0$$

$$\theta = 90$$

Que. A ball moving with velocity 50 m/s collides with another identical stationary ball elastically. the incident ball deflects by 37°. Find speeds of ball after collision.



$x \rightarrow$

$$50m + 0 = mv_1\cos37° + mv_2\cos53$$

$$50 = v_1 \times \frac{4}{5} + v_2 \times \frac{3}{5}$$

$$250 = 4v_1 + 3v_2 \quad \text{—(I)}$$

$$\frac{1}{2}\cancel{m}(50)^2 = \frac{1}{2}\cancel{m}v_1^2 + \frac{1}{2}\cancel{m}v_2^2$$

$$250 = v_1^2 + v_2^2$$

y-dirt^n

~~$0 + 0 = mv_1\sin37° + mv_2\sin53$~~

$$0 + 0 = mv_1\sin37° + mv_2\sin53°$$

$$v_1 \times \frac{3}{5} = v_2 \times \frac{4}{5}$$

$$3v_1 = 4v_2 \quad \text{—(II)}$$

fro (I) & (II)

$$250 = 4\left[\frac{4v_2}{3}\right] + 3v_2$$

$$250 = \frac{16v_2 + 9v_2}{3}$$

$\frac{\cancel{250}}{10} \times 3 = \cancel{25} V_2$ $\qquad$ $3V_1 =$

$\boxed{V_2 = 30 \text{ m/s}}$

Q. A ball of mass m moving with velocity V collides with another ball of mass 'm' which is at rest. after collision the incident ball deflects through 'θ' with speed $V/3$. Find speed of another ball after collision. (elastically)

(m) $\quad$ (m) $\quad \Rightarrow$

V $\quad$ rest

COKE

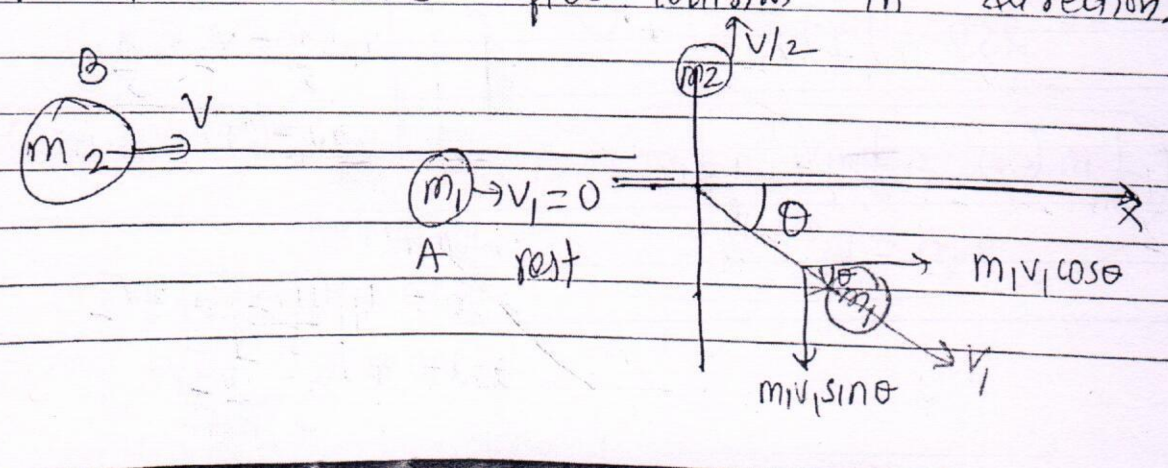
$$\frac{1}{\cancel{2}}\cancel{m}V^2 = \frac{1}{\cancel{2}}\cancel{m}\frac{V^2}{9} + \frac{1}{\cancel{2}}\cancel{m}V_2^2$$

$$V^2 = \frac{V^2}{9} + V_2^2$$

$$\frac{9V^2 - V^2}{9} = V_2^2 \qquad \frac{8V^2}{9} = V_2^2 \qquad \frac{2\sqrt{2}V}{3} = V_2$$

Q. Two sphere 'A' & 'B' having mass m_1 & m_2 respec. collides. 'A' is at rest initially & B is moving with velocity 'V' along x-axis. after collision 'B' has velocity $\frac{V}{2}$ in dir^n ⊥ to the original dir^n.

the mass 'A' moves after collision in direction:-



☆ Since, Before collision net Momentum is along x-dirth so, After collision net Momentum should be along x-direction.

along x-dirth COLM

$P_i = P_f$

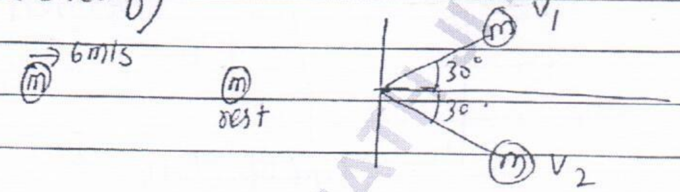
$m_2 V = m_1 V_1 \cos\theta$ — ①

$m_2 \frac{V}{2} = m_1 V_1 \sin\theta$ — ②

$\frac{2}{1} = \frac{1}{\tan\theta}$

$\tan\theta = \frac{1}{2}$ $\quad \boxed{\theta = \tan^{-1}\left(\frac{1}{2}\right)}$ Neet or $\boxed{\theta = -\tan^{-1}\left(\frac{1}{2}\right)}$

Q. A particle collides with ~~velocity~~ speed 6 m/s with identical stationary particle. after collision their velocities are at an angle 30° from the direction of incident particle. Find their speed after collision. (elastically)



x-dirth COLM

$\cancel{m(6) + 0 = \frac{mV_1}{2} + \frac{mV_2}{2}}$

$\cancel{12 = V_1 + V_2}$

$\cancel{12 = 2V_1}$

$\cancel{V_1 = 6\ m/s}$

$m(6) = mV_1 \cos 30 + mV_2 \cos 30$

$6 = \frac{V_1 \times \sqrt{3}}{2} + \frac{V_2 \times \sqrt{3}}{2}$

$\frac{12}{\sqrt{3}} = V_1 + V_2$ — ①

y-dirt COLM

$0 + 0 = mV_1 \sin 60 \mp mV_2 \sin 60$

$V_1 = V_2$ — ⑪

from ① & ⑪

$\frac{12}{\sqrt{3}} = 2V_1$

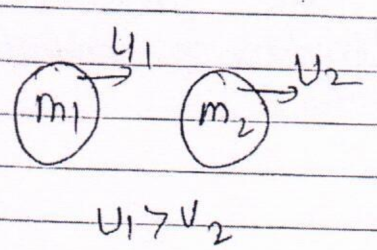
$V_2 = \frac{6}{\sqrt{3}} = \frac{3 \times 2}{\sqrt{3}}$

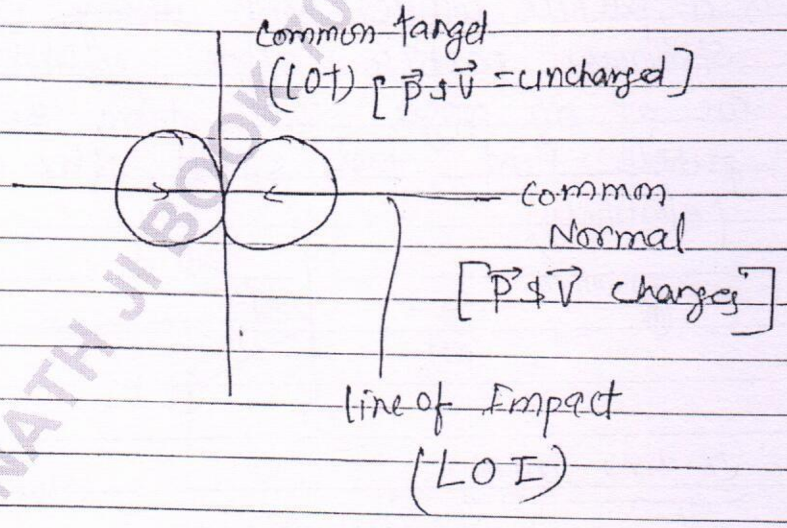
$\boxed{V_2 = 2\sqrt{3}}$ $\boxed{V_1 = 2\sqrt{3}}$ m/s

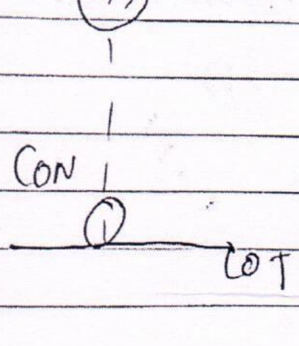
Note : During collision a pair of equal and opposite impulsive act along common normal direction hence linear momentum of individual particle do change along common Normal dirth.

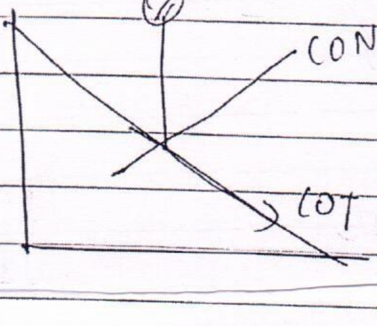
→ No component of impulsive force acts along common tangent dirn hence linear momentum & Velocity remains unchanged.

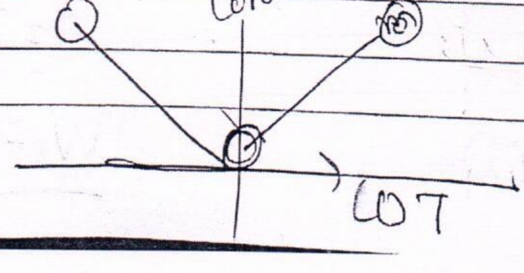
Definition of coefficient of restitution is applied along common Normal dirth.











CIRCULAR MOTION

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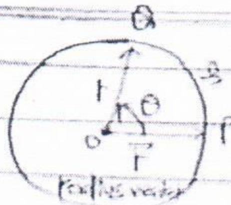


particle moves around a pt. by keeping const. separation.

$|\vec{r}|$ = const.

Radius vector - vector directed from centre of a circle towards the positⁿ and its magnitude remains constant.

1. Angular displacement - Angle through which radius vector turns.

$\theta = \frac{s}{r}$ radian. $\boxed{1 \text{ rev.} = 2\pi \text{ rad} = 360°}$

No. of rev. = N   → radian

$\boxed{\theta = 2\pi N \text{ rad.}}$ $[M^0L^0T^0]$

Axial vector -

Infinitesimal small.

$d\vec{\theta}_1 + d\vec{\theta}_2 = d\vec{\theta}_2 + d\vec{\theta}_1$

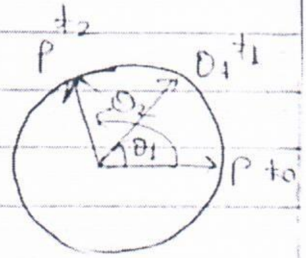
(Infinitesimally small angular displ. obey commutative law of vector additⁿ)

large.

$\vec{\theta}_1 + \vec{\theta}_2 \neq \vec{\theta}_2 + \vec{\theta}_1$

(large ang. displ. does not obey commutative law of vector additⁿ, so it is scalar.

2. Angular velocity -



Average angular velocity.

$\omega_{av.} = \frac{\text{net angular displacement.}}{\text{total time}} = \frac{\Delta\theta}{\Delta t} = \frac{\theta_2 - \theta_1}{t_2 - t_2}$

Instantaneous angular ~~displacement~~ vel.

$\vec{\omega}_{ins.} = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\vec{\theta}}{dt}$

⇒ ω = const. $\omega_{ins} = \omega_{av}$

Uniform circular motⁿ

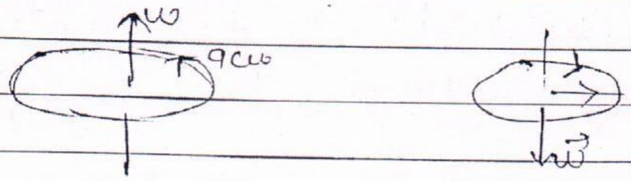
NO. of rev. | given ya asked → 4 hv θ
sec →

PAGE NO DATE

→ Axial vector -

→ directed along axis of rotation.

→ Right hand thumb rule.

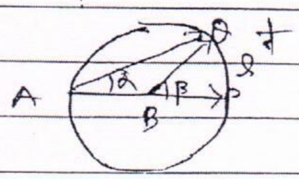


→ radian / sec. ?

→ (T^{-1}) ③

→ For a rigid body angular velocity of its pt. masses are equal for ~~about~~ ~~kiye~~ a given axis of radian.

* Angular velocity depends on the positn of observer.



$\alpha = \dfrac{s}{2r}$ $\quad$ $s = r$

$\beta = \dfrac{s}{r}$ $\quad$ $\dfrac{\alpha}{\beta} = \dfrac{1}{2}$

$\omega_A = \dfrac{\alpha}{t}$

$\omega_B = \dfrac{\beta}{t}$

$\dfrac{\omega_A}{\omega_B} = \dfrac{\alpha}{\beta}$

$\dfrac{\omega_A}{\omega_B} = \dfrac{1}{2}$

$\boxed{\omega_B = 2\omega_A}$

Frequency - No. of cycles per sec.

rps, rpm, cps, $\underbrace{Hz, s^{-1}}_{MKS}$ (S.I.)

$\boxed{\omega = 2\pi n}$

1 rps = 60 rpm.

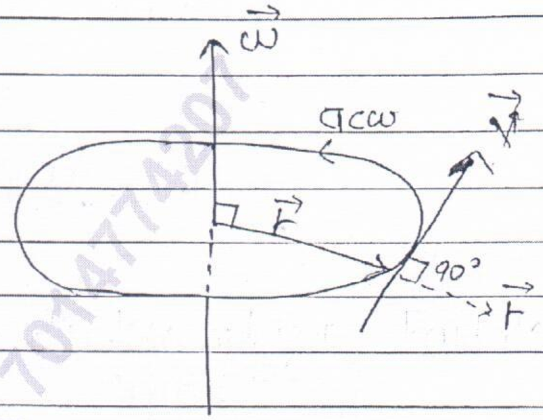
$\frac{1}{60}$ rps = 1 rpm

Time period -

time taken to complete one revolutⁿ.

$$\boxed{\omega = \frac{2\pi}{T}} \qquad n = \frac{1}{T}$$

Relatⁿ b/w 'v' & 'ω'



$$\theta = \frac{s}{r}$$

$$s = r\theta$$

$$\frac{ds}{dt} = r\frac{d\theta}{dt} \qquad \vec{\omega} \perp \vec{r} \perp \vec{v}$$

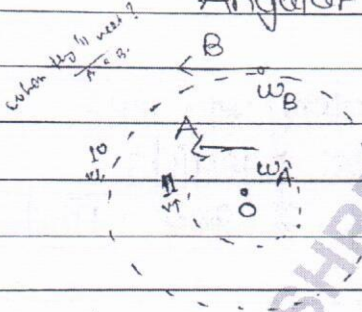
$$\boxed{v = r\omega} \qquad \left[\begin{array}{l} \vec{\omega}\cdot\vec{r} = 0 \\ \vec{r}\cdot\vec{v} = 0 \\ \vec{\omega}\cdot\vec{v} = 0 \end{array}\right]$$

$$\boxed{\vec{v} = \vec{\omega} \times \vec{r}}$$

Thumb $\vec{v}$ के dirⁿ. में

Relative Angular velocity →

Angular velocity of B w.r.to A



$$\boxed{\vec{\omega}_{rel.} = \vec{\omega}_{B/A} = \vec{\omega}_B - \vec{\omega}_A}$$

$$\omega_{rel.} = \omega_{B/A} = \omega_B - \omega_A$$

Time taken by 1 particle to complete 1 more or 1 less revolⁿ than other, or time interval after which they again meet.

$$T = \frac{2\pi}{\omega_{rel.}}$$

also (applicable in satellite) and observer on earth and satellite for reappearance.

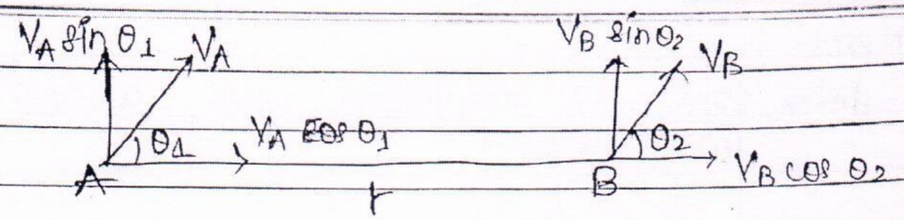
$$T = \frac{2\pi}{\omega_B - \omega_A}$$

$$T = \frac{2\pi}{\frac{2\pi}{T_B} - \frac{2\pi}{T_A}}$$

Time-pd. ↓

$$\boxed{T = \frac{T_A T_B}{T_B - T_A}}$$

#



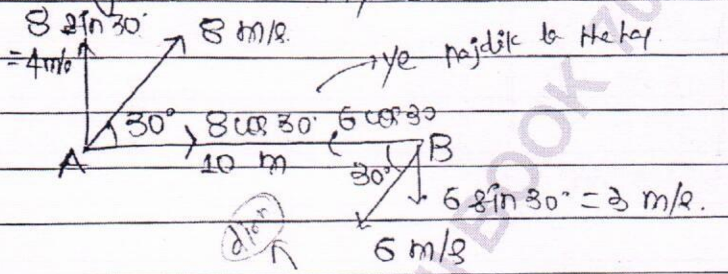
$$\omega_{rel.} = \frac{V_{rel.}(\perp \text{ to } r_{rel.})}{r_{rel.}}$$

$$\omega_{rel.} = \frac{V_B \sin\theta_2 - V_A \sin\theta_1}{r}$$

7/11/16

Q. Find angular velocity of B w.r.t. A

done

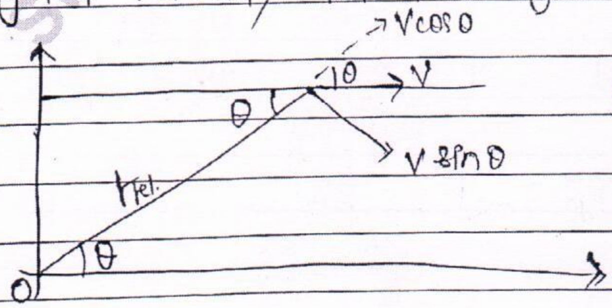


rotatn effect wo peeta hai jo r ke ⊥ hote hai

→ $\omega_{rel.} = \frac{4+3}{10} = 0.7 \text{ rad s}^{-1}$

Q. A particle is moving with speed 'V' along st. line, $y = b$. When positn vector of the particle relative to origin makes $\angle\theta$ from x-axis. Find its angular velocity rel. to origin.

→



$\sin\theta = \frac{b}{r_{rel.}}$

$r_{rel.} = \frac{b}{\sin\theta}$

$$\omega_{rel.} = \frac{V\sin\theta}{\frac{b}{\sin\theta}} = \frac{V\sin^2\theta}{b}$$

* particle st. line me chalte hue V kisi pt. k respect से ang. vel. /const. नहीं होती

ω = const.

$$\omega = \frac{\theta}{t} \Rightarrow \theta = \omega t.$$

$$\omega_{sec.} = \frac{2\pi}{60} \text{ rad/s.} \qquad \omega_{earth \text{ about own axis}} = \frac{2\pi}{24 \times 60 \times 60} \text{ rad/s.}$$

$$\omega_{min.} = \frac{2\pi}{60 \times 60} \text{ rad/s.}$$

$$\omega_{hr.} = \frac{2\pi}{12 \times 60 \times 60} \text{ rad/s.}$$

Angular Acceleration :- (α)

Rate of change of angular velocity.

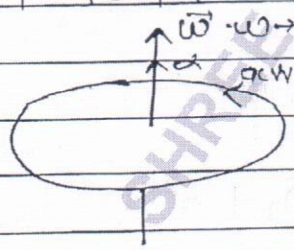
Average acc^n =

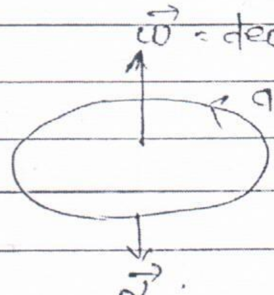
$$\alpha = \frac{\Delta\omega}{\Delta t} = \frac{\omega_2 - \omega_1}{t_2 - t_1}.$$

Instantaneous angular acc^n

$$\alpha_{in} = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}.$$

→ Axial vector





→ rad/s^2 $[M^0L^0T^{-2}]$.

$\boxed{\alpha = 0, \ \omega = \text{const. Uniform circular motion}}$

Resultant acc^n -

$$\vec{v} = \vec{\omega} \times \vec{r}.$$

$$\frac{d\vec{v}}{dt} = \frac{d\vec{\omega}}{dt} \times \vec{r} + \vec{\omega} \times \frac{d\vec{r}}{dt} \qquad \vec{a}_{net} = \vec{\alpha} \times \vec{r} + \vec{\omega} \times \vec{v}.$$

$$\boxed{\vec{a}_{net} = \vec{a}_T + \vec{a}_c}$$

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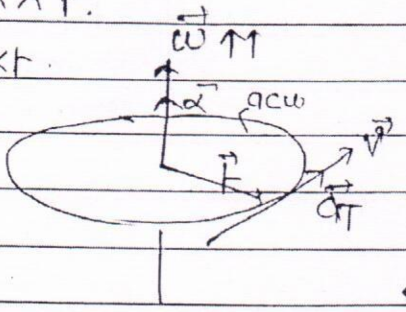
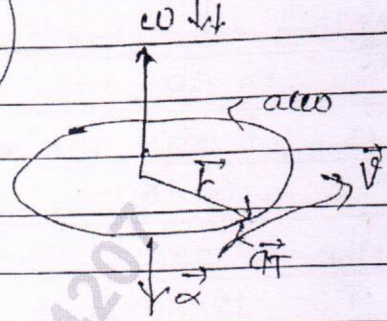
Tangential acc^n - $(\vec{a}_T)$ $\vec{a}_T = \frac{d|\vec{v}|}{dt}$ = rate of change of speed.

not necessary for $+$ int $\Rightarrow$ UCM.

$\vec{a}_T = \vec{\alpha} \times \vec{r}$.

$a_T = \alpha r$.

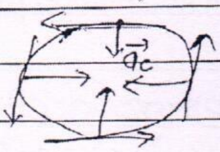




It only changes the speed. (not changes dirn).

Centripetal acc^n :-

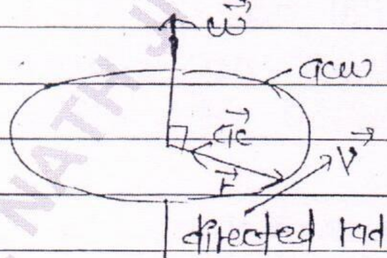
$\vec{a}_c = \vec{\omega} \times \vec{v}$.



→ Necessary for circular motion.

→ only changes direct^n of velocity not magnitude.

$$\boxed{\begin{aligned} a_c &= \omega v \\ a_c &= \frac{v^2}{r} \\ a_c &= \omega^2 r \end{aligned}}$$



directed radially inwards.

$\vec{a}_c = \vec{\omega} \times (\vec{\omega} \times \vec{r})$

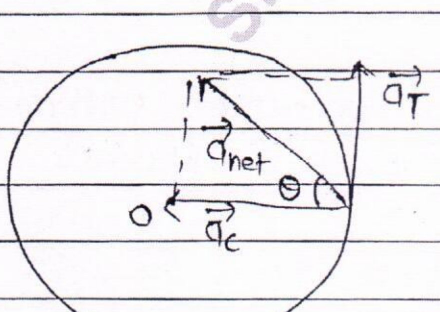
$= \hat{j} \times (\hat{j} \times \hat{i})$

$= \hat{j} \times (-\hat{k})$

$= -\hat{i}$.

→ only acc^n present in UCM.

$\vec{a}_{net} = \vec{a}_T + \vec{a}_c$.



$|\vec{a}_{net}| = \sqrt{\left(\frac{dv}{dt}\right)^2 + \left(\frac{v^2}{r}\right)^2}$

Q. A particle is moving on a circle of radius 2cm. With speed $v = 4t$ cms^{-1}. Find its acc^n at time $t = 1$ sec.

(net) is not specific

→
$$a_c = \frac{v^2}{r} = \frac{(4\times1)^2}{2} = \frac{16}{2} = 8 \text{ cm/s}^2.$$

$$a_T = \frac{dv}{dt} = 4 \text{ cm/s}^2$$

$$|\vec{a}_{net}| = \sqrt{8^2+4^2} = \sqrt{64+16} = 4\sqrt{4+1} = 4\sqrt{5} \text{ cm s}^{-2}.$$

Q. A particle is moving on a circle of radius 'R', its speed, distance relat^n is $v = \alpha\sqrt{s}$ where α is a constant. Find its centripetal acc^n & resultant acc^n at distance 's'.

→
$$a_c = \frac{\alpha^2 s}{R}.$$

$$a_T = v\frac{dv}{ds}$$

(?)
$$= \alpha s^{1/2} \times \alpha \times \frac{1}{2} s^{-1/2}$$

$$a_T = \frac{\alpha^2}{2}$$

$$a_{net} = \sqrt{\left(\frac{\alpha}{2}\right)^2 + \left(\frac{\alpha^2 s}{R}\right)^2}$$

$$a_{net} = \sqrt{\frac{\alpha^4}{4} + \frac{\alpha^4 s^2}{R^2}}$$

$$= \alpha^2 \sqrt{\frac{1}{4} + \frac{s^2}{R^2}}.$$

Q. A particle is moving on circle radius 'r', its centripetal acc^n is $a_c = k^2 r t^2$, where 'k' is const. & 't' is time. Find

(i) Speed.

(ii) tangential acc^n.

(iii) resultant acc^n (mag)

(iv) angle subtended by resultant acc^n from tangential acc^n.

→
$$a_c = k^2 r t^2$$

(i) $a_c = \frac{v^2}{r} \Rightarrow v = \sqrt{a_c \times r} = \sqrt{k^2 r t^2 r}$

$= krt.$

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(ii) $a_T = \frac{dv}{dt} \Rightarrow kr.$

(iii) $|\vec{a}_{net}| = \sqrt{k^2r^2 + k^4r^2t^4}$

$= kr\sqrt{1 + k^2t^4}.$

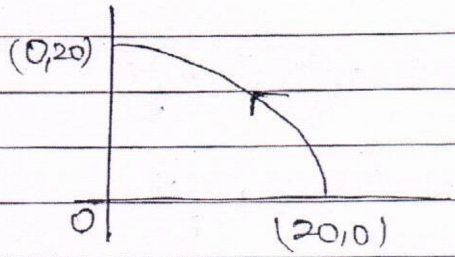
(iv) $\tan\alpha = \frac{\vec{a_c}}{\vec{a_T}} = \frac{k^2rt^2}{kr}$

$\tan\alpha = kt^2.$

$\alpha = \tan^{-1}(kt^2).$

JEE

Q. A particle 'p' starts moving on a circle of radius 20 mts in counter clockwise dir^n (acw) a/c to fig. It sweeps distance 's' a/c to, $s = t^3 + 5$ mts. Find its acc^n, at time 't' = 2 sec.



$\rightarrow v = 3t^2$

$a_c = \frac{v^2}{r} = \frac{(3\times(4))^2}{20} = \frac{144}{20}$ m/s².

$a_T = 6t.$

$a_T = 12$ m/s².

$|\vec{a}_{net}| = \sqrt{144 + \frac{144\times144}{20\times20}}.$

$= 12\sqrt{1 + \frac{144}{400}} \Rightarrow \frac{12}{20}\sqrt{544}$

$= 0.6 \times 23.32$

$= 13.994 \simeq 14$ m/s².

2010 JEE ADVL

Q. A particle is moving on a circle of radius 'R' in counter cw dir^n in xy plane with centre @ origin. with speed 'v'. It is in UCM. At an instant its posit^n vector is (R θ) where, θ = ∠ made by posit^n vector from x-axis. acc^n of particle at this moment —

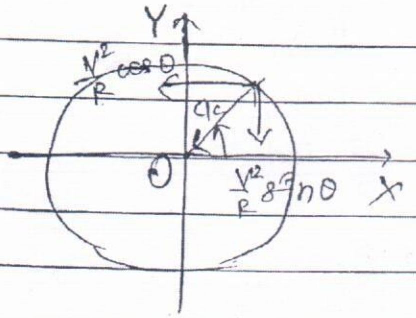
(i) $\frac{V^2}{R}\hat{i} + \frac{V^2}{R}\hat{j}$. m/s²

(ii) $-\frac{V^2}{R}\sin\theta\,\hat{i} - \frac{V^2}{R}\cos\theta\,\hat{j}$ m/s

(iii) $-\frac{V^2}{R}\sin\theta\,\hat{i} + \frac{V^2}{R}\cos\theta\,\hat{j}$.

(iv) $-\frac{V^2}{R}\cos\theta\,\hat{i} - \frac{V^2}{R}\sin\theta\,\hat{j}$

→



$\vec{a}_c = -\frac{V^2}{R}\cos\theta\,\hat{i} - \frac{V^2}{R}\sin\theta\,\hat{j}$.

9/11/17

$\alpha = \text{const.}$

$\omega = \omega_0 + \alpha t$

$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$.

$\omega^2 = \omega_0^2 + 2\alpha\theta$.

$\theta_{nth} = \omega_0 + \frac{\alpha}{2}(2n-1)$.

$$\left[\begin{array}{l} \theta = \left(\frac{\omega+\omega_0}{2}\right)t \\ \\ \alpha = \frac{\omega-\omega_0}{t}. \end{array}\right.$$

Q. A fan blade is rotating 100 rev/sec, Now it is switched off. After 5 min, blades stops. consider uniform angular retardatn. Find –

(i) total no. of revolutn made by blade before coming to rest after switching off.

(ii) Angular retardatn.

(iii) AV. angular velocity after switching off.

$\boxed{\text{revolut}^n \approx \theta}$

" per sec → frequency

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(i) $\theta = \left[\frac{0 + 2\pi \times 100}{2}\right] \times 300$

$\theta = 2\pi \times 15000$ rad.

$\theta = 2\pi N = 2\pi \times 15000$

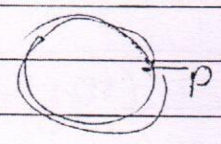
$\Rightarrow N = 15000.$

(ii) $\alpha = \frac{0 - 2\pi \times 100}{300} = -\frac{2\pi}{3}$ rad/s².

retardat^n $= \frac{2\pi}{3}$ rad/s².

(iii) $\omega_{av.} = \frac{2\pi \times \cancel{15000}^{50}}{\cancel{300}} = 100\pi$ rad/s.

Q. A particle starts circular mot^n with radius $20/\pi$ mt. under constant a_T. After complit^n of 2nd revolut^n, its speed become 80 m/s. find a_T?



→ $\omega^2 = \omega_0^2 + 2\alpha\theta$

$\frac{v^2}{r^2} = 0 + 2\frac{a_T}{r}\theta$

$v^2 = 2a_T r\theta$

~~$v^2 = 2a_T r$~~ $v^2 = 2a_T r \times 2\pi N.$

$v^2 = 2a_T \times 2\pi r \times N.$

$(80)^2 = 4a_T \times \cancel{\pi} \times \frac{20}{\cancel{\pi}} \times 2$

$\frac{80 \times 80}{80 \times 2} = a_T \Rightarrow a_T = a_T = 80$

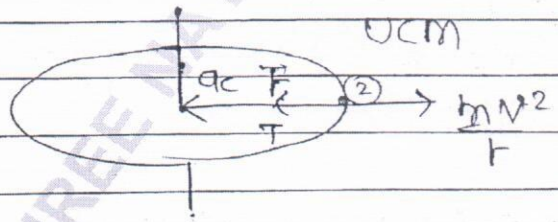
centripetal Force ($\vec{F_c}$)

→ It is compulsory for circular motion.
→ It is a not new kind of practical force.
→ It is demand of circular motn to continously change the dirn of velocity and it is provided by other forces like gravity, coulomb force, tension etc.

- $F_c = \frac{mv^2}{r}$
- $F_c = m\omega^2 r$
- $F_c = m\omega v$

centrifugal force -

→ pseudo force, because circular motn is accelerated frame,
→ so, it is non-inertial frame.
→ It is for observer too. (jo particle pe baitha hai).



Q. Two identical particles are tied with a string at a dist. 2 m & 3 m from one end. Now string is revolved in H.z plane with const. 'ω' by this end from which distance is measured. Find ratio of tension.

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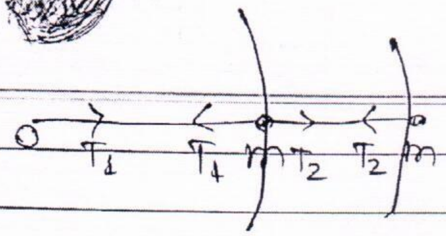
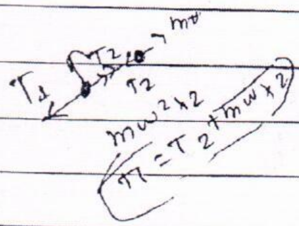


$T_1 - T_2 = m\omega^2 \times 2$ $\quad T_2 = m\omega^2 \times 3$

$T_1 = m\omega^2 \times 2 + m\omega^2 \times 3$

$T_1 = m\omega^2 \times 5$

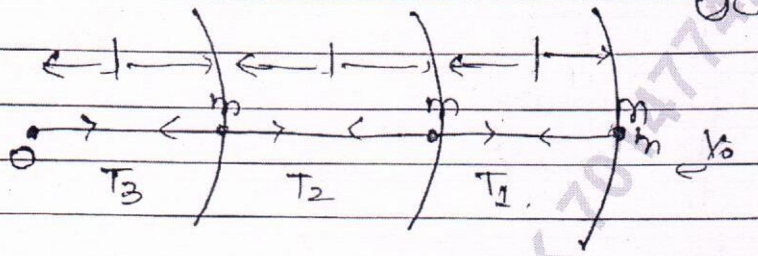
$T_2 = m\omega^2 \times 3$



$$\boxed{\frac{T_1}{T_2} = \frac{5}{3}}$$

Q. Find ratio of tension -

UCM.



→ $T_1 = m\omega^2 \times 3l$ —— (1)

$T_2 - T_1 = m\omega^2 \times 2l$

$T_2 = m\omega^2 \times 2l + m\omega^2 \times 3l$

$T_2 = m\omega^2 \times 5l$ — (2)

$T_3 - T_2 = m\omega^2 \times l$

$T_3 = m\omega^2 \times l + m\omega^2 \times 5l$

$T_3 = m\omega^2 \times 6l$ — (3)

$T_1 : T_2 : T_3 = 3 : 5 : 6.$

Q. A particle is moving on a circle of radius 'r'. Its K.E. is changing with distance a/c to relatⁿ -
$K = As^2$, where, A is const.
Find force on the particle.

→ $\frac{1}{2} mv^2 = As^2$.

$v = \sqrt{\frac{2A}{m}}\, s.$

$$a_c = \frac{v^2}{r} = \frac{2A}{m}\frac{s^2}{r} \text{ —— (1)}$$

$$a_T = \frac{2As}{m} \text{ —— (2)}$$

$$a_T = v\frac{dv}{ds}$$

$$= \sqrt{\frac{2A}{m}} \cancel{\times s \times \frac{2A}{m}} \times 1$$

$$= \sqrt{\frac{2A}{m}}s \times \sqrt{\frac{2A}{m}} \times 1$$

$$a = \sqrt{\left(\frac{2A}{m}\frac{s^2}{r}\right)^2 + \left(\frac{2As}{m}\right)^2}$$

$$a = \frac{2A}{m}s\sqrt{\frac{s^2}{r^2}+1}$$

$$F = m \times \frac{2As}{m}\sqrt{\frac{s^2}{r^2}+1}$$

JEE 2016

Q. A particle is moving on a circle of rad. 'r'. s.t. a_c is $a_c = k^2rt^2$, where 'k' is const & 't' is time. Find power delivered to the particle.

→ $a_c = k^2rt^2$

$$\frac{v^2}{r} = k^2rt^2$$

$$v = krt$$

$a_T = kr$

Power $= F_T v$

$= mkr \times krt$

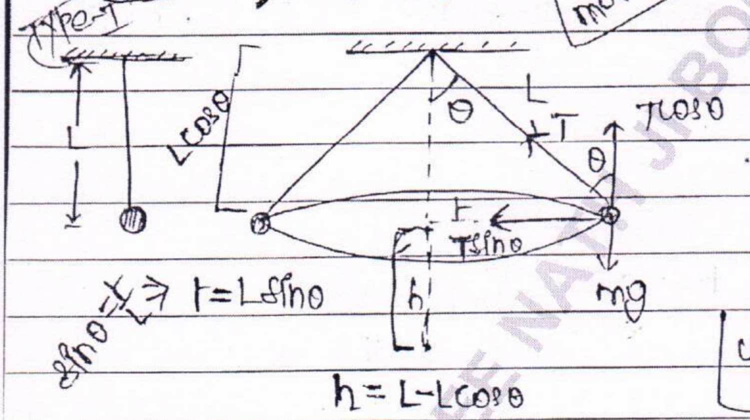
$= mk^2r^2t$

UCM :-	Non-UCM :-
$\to \vec{a}_T = 0, \vec{\alpha} = 0, \frac{dv}{dt} = 0.$	$\to \vec{a}_T \neq 0, \vec{\alpha} \neq 0, \frac{dv}{dt} \neq 0$
$\to \vec{\omega}$ = const. [both in mag. & dirn]	$\to \vec{\omega}$ = variable
$\to \vec{v}$ = variable only mag. const. $\lvert\vec{v}\rvert$ = const	$\to \vec{v}$ = variable [both in mag. & dirn)
$\to \vec{a}_{net} = \vec{a}_c$ variable (dirn change) but magnitude constant.	$\to \vec{a}_{net} = \vec{a}_T + \vec{a}_c$ $\lvert\vec{a}_{net}\rvert = \sqrt{a_T^2 + a_c^2}$
$\to \vec{F}_T = 0$	$\to \vec{F}_T \neq 0.$
$\to \vec{F}_c \neq 0$ $\vec{F}_{net} = \vec{F}_c$	$\to \vec{F}_c \neq 0,$ $\vec{F}_{net} = \vec{F}_T + \vec{F}_c$
→ No work done.	→ work done only by $\vec{F}_T$.
→ No power delivered.	→ Power delivered only by $\vec{F}_T$.

moving away → due to property of inertia of dirn.

Conical Pendulum :-

Type-1



$$T \sin\theta = m\omega^2 r$$

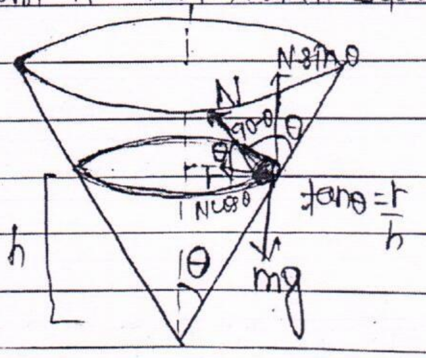
$$T \sin\theta = m\omega^2 L \sin\theta$$

$$T \cos\theta = mg$$

$$\frac{1}{\cos\theta} = \frac{\omega^2 L}{g}$$

$$\boxed{\omega = \sqrt{\frac{g}{L\cos\theta}}} \quad \boxed{T = 2\pi\sqrt{\frac{L\cos\theta}{g}}}$$

Q. A bead is projected with speed 'v' on a surface of a funnel a/c to the fig. Its circular plane is @ a ht. 9.8 cm from its vertex. All surfaces are frictionless find speed.



$$N\cos\theta = \frac{mv^2}{r} \qquad N \sin\theta = mg$$

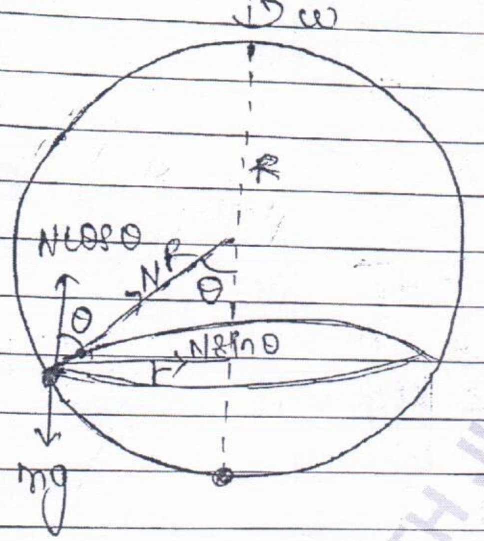
$$\tan\theta = \frac{gr}{v^2}$$

$$v = \sqrt{\frac{rg}{\tan\theta}}$$

circular tangent का $\perp^r$ radius

$$v = \sqrt{\frac{kg}{\frac{r}{h}}} = \sqrt{gh} = \sqrt{9.8 \times 9.8 \times 10^{-2}} = 0.98\ m/s.$$

Q. A ring of radius 'R' is held in vertical plane with small bead. Now, ring is rotated about vertical axis with angular velocity 'ω', find the ∠ made by the line joining bead at centre of ring from vertical in dynamic eqb^m. All surfaces are frict^{n}less.



$r = R \sin\theta$

$N \sin\theta = m\omega^2 r$

$N \sin\theta = m\omega^2 \times R \sin\theta$

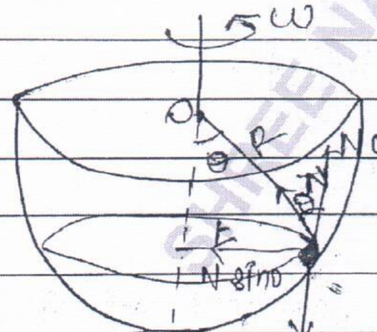
$N \cos\theta = mg$

$\frac{1}{\cos\theta} = \frac{\omega^2 R}{g}$

$\cos\theta = \frac{g}{\omega^2 R}$

$\theta = \cos^{-1}\left(\frac{g}{\omega^2 R}\right)$

Q.



Hemispherical bowl is rotating around symmetrical axis with angular vel. 'ω'. The line joining the particle & centre of complete sphere makes ∠θ from Vt. Find 'ω' of bowl.

$\sin\theta = r/R$

$r = R\sin\theta$

$N \sin\theta = m\omega^2 r$

$N \sin\theta = m\omega^2 \times R \sin\theta$

$N\cos\theta = mg$

$\frac{1}{\cos\theta} = \frac{\omega^2 R}{g} \Rightarrow \boxed{\omega = \sqrt{\frac{g}{R\cos\theta}}}$

Type-II.

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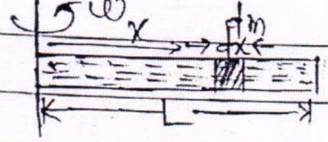
Q. A tube is filled with an incompressible fluid and rotated in Hz plane about 1 end. Find force on the other end.



$$dm = \frac{M}{L}dx.$$

$$dF = dm\,\omega^2 x$$

$$\int dF = \int_0^L \frac{M}{L}dx\,\omega^2 x.$$

$$F = \frac{M\omega^2}{L}\int_0^L x\,dx = \frac{M\omega^2}{L}\left[\frac{x^2}{2}\right]_0^L = \frac{m}{L}\frac{\omega^2 L^2}{2}$$

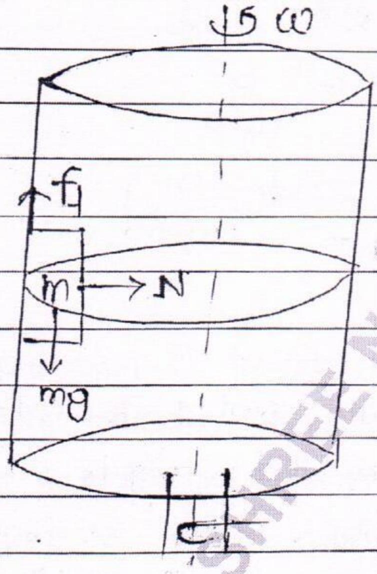
$$\boxed{F = \frac{m\omega^2 L}{2}}$$

OR

$$dm = \frac{M}{L}dx$$

$$\frac{M\omega^2 L}{2}.$$

Death/Well Rotor



$$mg \le f_1$$

$$mg \le \mu N.$$

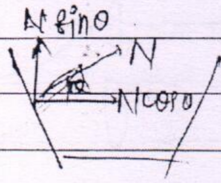
$$\cancel{m}g \le \mu \cancel{m}\omega^2 r$$

$$\frac{g}{\mu r} \le \omega^2$$

$$\boxed{\omega \ge \sqrt{\frac{g}{\mu r}}}$$

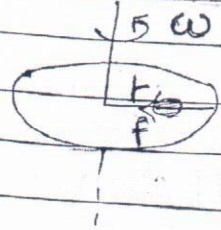
$$\boxed{\omega_{min} = \sqrt{\frac{g}{r\mu}}}$$

Here 'N' force provides centripetal force.



Q. A coin is placed on a gramophone record at a distance 'r' from axis of rotation. Angular velocity of record is 'ω'. For what condition, the coin will not slide coefficient of static friction 'μ'.

(1) $r \le \frac{\omega^2 g}{\mu}$ (2) $r \ge \frac{\omega^2 g}{\mu}$ (3) $r \le \frac{\mu g}{\omega^2}$ (4) NOT



$f_1 \geq m\omega^2 r$

$\mu mg \geq m\omega^2 r$

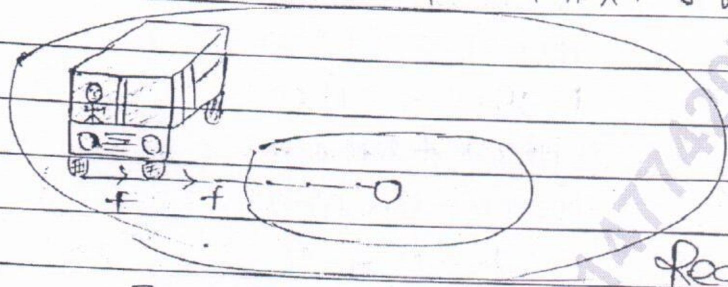
$\frac{\mu g}{\omega^2} \geq r$ ⇒ $\boxed{r \leq \frac{\mu g}{\omega^2}}$

Maxm value of ω so that coin doesnot slide

$\omega^2 \leq \frac{\mu g}{r}$

$\omega_{max} = \sqrt{\frac{\mu g}{r}}$

Car on circular flat road - (Maxm safe speed



Reqd centripetal force provided by static frictn.

$f_1 \geq \frac{mv^2}{r}$

$\mu mg \geq \frac{mv^2}{r}$

$\mu rg \geq v^2$

$v^2 \leq \mu rg$

13.11.17

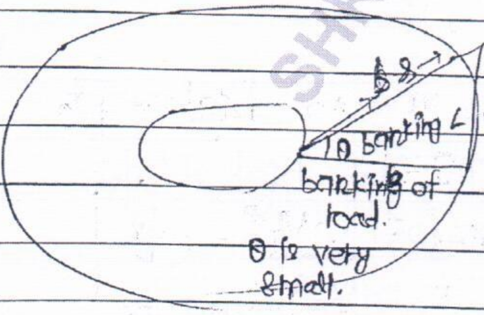
$\boxed{V_{max} = \sqrt{\mu rg}}$

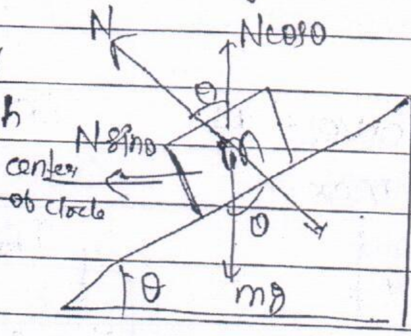
$v > V_{max}$ car skid radialy outwards.

Banking of Road -

Outer edge of the road is slightly raised w.r.t. inner.

friction not involved





$N \sin\theta = \frac{mv^2}{r}$

$N \cos\theta = mg$

$\tan\theta = \frac{v^2}{rg}$

$\boxed{v = \sqrt{rg\tan\theta}}$

$\sin\theta \simeq \theta \simeq \tan\theta$

$\sin\theta = \frac{h}{b} = \tan\theta = \frac{h}{s}$

$\boxed{\tan\theta = \frac{h}{s}}$

same in case of aeroplane.

+ W/PE

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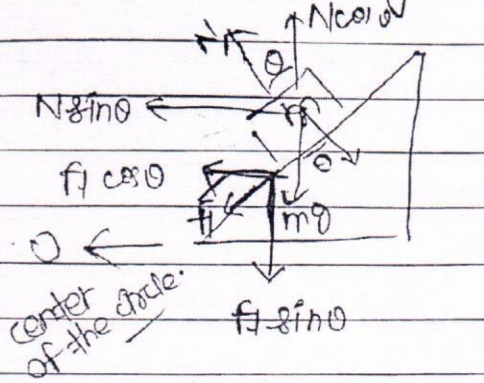
Now, frictⁿ is also given.



$$N \sin\theta + fl\cos\theta = \frac{mV_{max}^2}{r} \quad \text{—(1)}$$

$$N\cos\theta = mg + fl\sin\theta$$

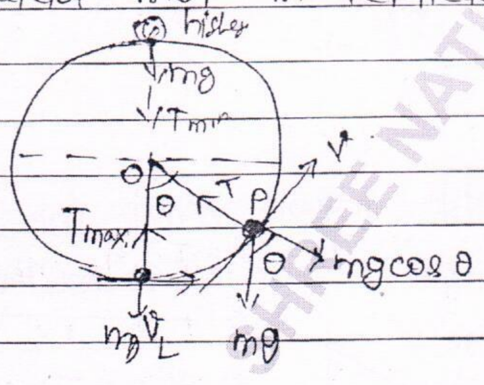
$$N\cos\theta - fl\sin\theta = mg \quad \text{—(2)}$$

$$\frac{N\sin\theta + \mu N\cos\theta}{N\cos\theta - \mu N\sin\theta} = \frac{V_{max}^2}{rg}$$

$$\frac{\tan\theta + \mu}{1 - \mu\tan\theta} = \frac{V^2_{max}}{rg}$$

$$\boxed{V_{max} = \sqrt{rg\left[\frac{\tan\theta + \mu}{1 - \mu\tan\theta}\right]}}$$

Circular motⁿ in verticle plane - NON-UCM



$$T - mg\cos\theta = \frac{mV^2}{r}$$

$$\boxed{T = \frac{mV^2}{r} + mg\cos\theta}$$

Case -I : Lowest pt.

$\theta = 0°$, T = max.

$$\boxed{T_{max} = \frac{mV_L^2}{r} + mg}$$

Case II : At highest pt.

$\theta = 180°$

$$\boxed{T_{min} = \frac{mV_T^2}{r} - mg}$$

Case III - mid point

$\theta = 90°$

$$\boxed{T_{mid} = \frac{mV^2_{mid}}{r}}$$

$\frac{mv_T^2}{r} - mg \geq 0$

$\frac{v_T^2}{r} \geq g \Rightarrow \boxed{v_T \geq \sqrt{rg}}$

For just completion of circle, or looping the loop →
At top, $T_T \geq 0$. [Not bfq the highest point] $\frac{mv_T^2}{r} - mg \geq 0$.

For just complet^n of circle, $\boxed{v_T \geq \sqrt{rg}}$
Critical speed at top, $\boxed{v_T = \sqrt{rg}}$

For just complet^n of circle, critical speed at lowest point,

Loss in P.E. = Gain in K.E.

COME

$mg \times 2r - 0 = \frac{1}{2}mv_L^2 - \frac{1}{2}m(\sqrt{rg})^2$

$2rg + \frac{1}{2}rg = \frac{1}{2}v_L^2$

** $\boxed{v_L = \sqrt{5rg}}$ critical speed at lowest pt. to just complete the loop.

mid-point $v_m = \sqrt{3rg}$

For just complet^n, T :-

$\boxed{T_{min} = 0}$

$T_{max} = \frac{m \times 5rg}{r} + mg$

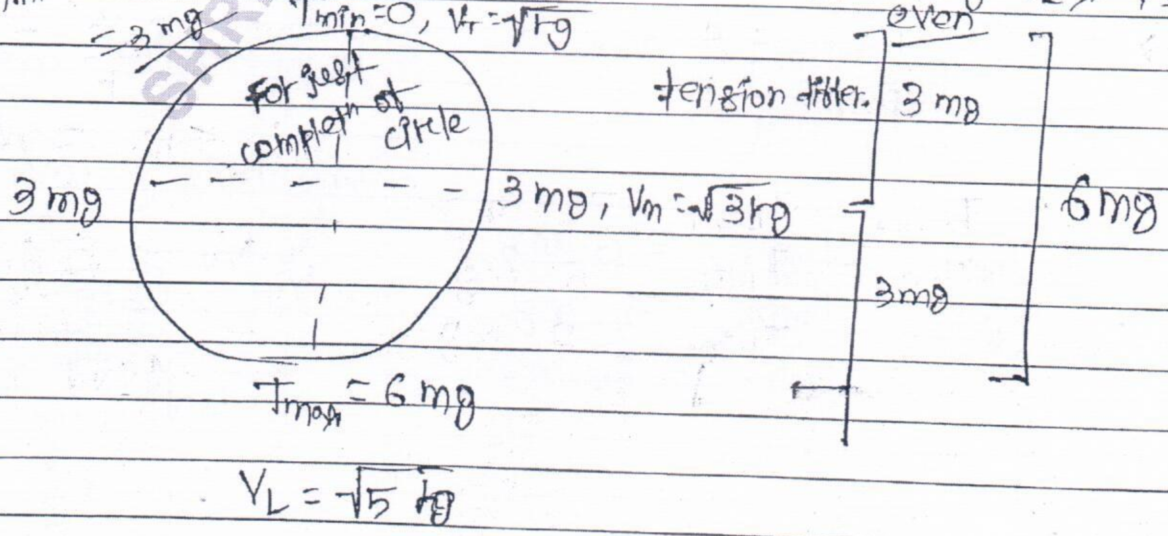
$T_{max} = 6mg$

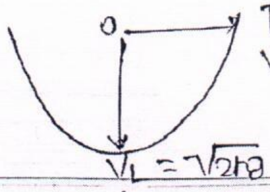
$\boxed{T_{max} - T_{min} = 6mg}$

Always

including $v_L > \sqrt{5rg}$
even

$T_{mid} = \frac{m(\sqrt{3rg})^2}{r} = 3mg$



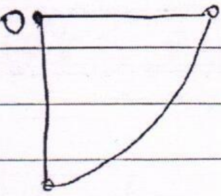


COME :
gain in P.E. = Loss in KE

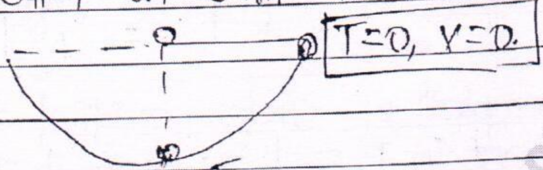
$mgr = \frac{1}{2} m V_L^2$

$\boxed{V_L = \sqrt{2rg}}$

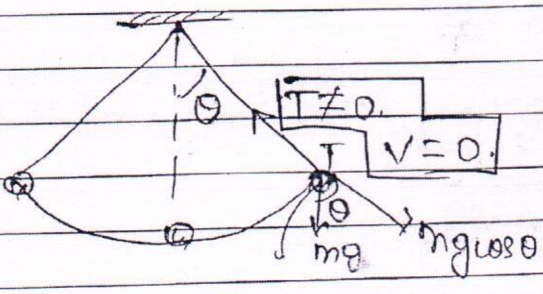
condⁿ for not completⁿ of circle -



(a) $V_L = \sqrt{2rg}$
oscillatⁿ on semi-circle.

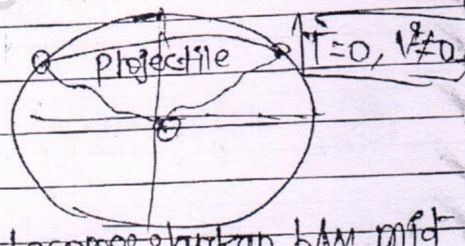


(b) $V_L < \sqrt{2rg}$
oscillatⁿ on arc.



$T = mg\cos\theta$

(c) $\sqrt{2rg} < V_L < \sqrt{5rg}$
move around centre
leave circle b/w mid
& highest point



~~string becomes slacken b/w mid pt. to top most pt. But speed non-zero so particles keep going~~ on around in center but leave the circle.

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Q. The ratio of maxᵐ to minᵐ tension in a string in circular motⁿ in vt. plane is 4:1. Find speed of the particle at lowest and highest point. radius 10 mt.

→ $\frac{T_{max}}{T_{min}} = \frac{4}{1}$

$T_{max} - T_{min} = 6\,mg$

$4T_{min} - T_{min} = 6\,mg$

$T_{min} = 2\,mg$

$T_{min} = \frac{mV_T^2}{r} - mg$

$2mg + mg = \frac{mV_T^2}{r}$

$V_T = \sqrt{3rg}$

$= \sqrt{3 \times 10 \times 10}$

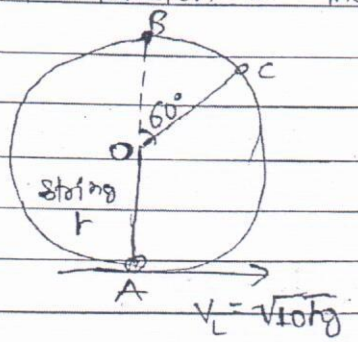
$= 10\sqrt{3}$ m/s.

$$T_{max.} = \frac{mV_l^2}{r} + mg$$

$$8\,mg - mg = \frac{mv_l^2}{r}$$

$$V_L = \sqrt{7rg} = \underline{40\sqrt{7}\ m/s}$$

Q. Find tension in the string at pt. A, B and C.



(A)

$$T_{max.} = \frac{m \times 10\,rg}{r} + mg$$

$$= 11\ mg.$$

(B) $T_{max.} - T_{min} = 6\,mg$

$11\,mg - 6\,mg = T_{min}.$

$T_{min.} = 5\ mg.$

(C) $T + mg\cos 60° = \frac{mv^2}{r}$

$$T = \frac{mv^2}{r} - \frac{mg}{2}$$

$$= \frac{m \times 7rg}{r} - \frac{mg}{2}$$

$$= \frac{13}{2}\ mg = 6.5\ mg.$$

COME,

Loss in K.E. = gain in P.E.

$K_i - K_f = P_f - P_i$

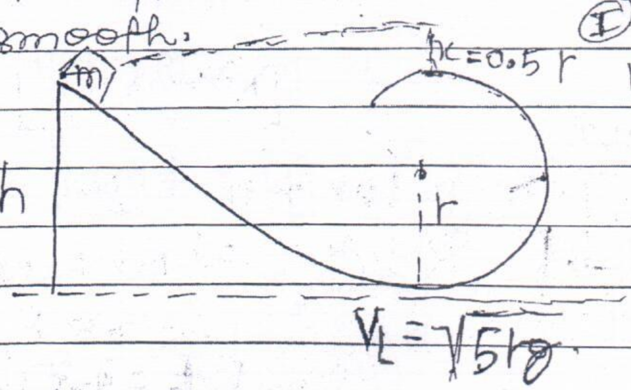
$$\frac{1}{2} m \times 10\,rg - \frac{1}{2} mv^2 = mg\,(r + r\cos 60°)$$

$$10\,rg - 3\,rg = v^2.$$

$$v^2 = 7\,rg.$$

2019

Q. For just completion of circle. Find value of 'h'. All surfaces are smooth.



(I) COME,

Loss in PE = gain in KE.

$$mgh = \frac{1}{2} mv^2$$

$$v^2 = 2gh.$$

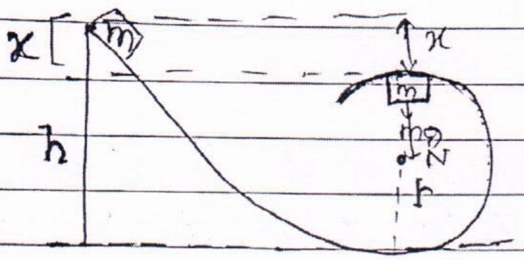
(II)

$2gh = 5rg$

$h = \frac{5}{2} r$

$\boxed{h = 2.5\,r}$

Q. Find the value of 'h' so that @ the highest pt. of circle, normal force on the block is mg.



$$N + mg = \frac{mv^2}{r}$$

$$\boxed{v^2 = 2rg}$$

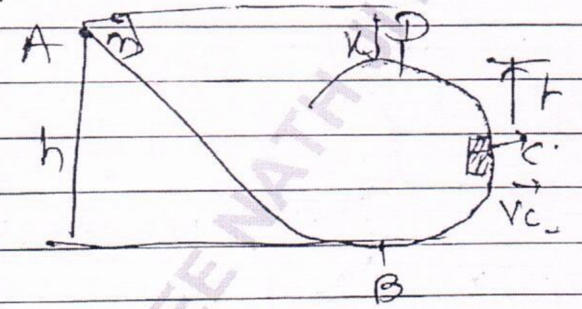
COME,

Loss in P.E. = Gain KE.

$$mgx = \frac{1}{2} m \times 2rg$$

$$x = r. \quad \Rightarrow \boxed{h = 3r}.$$

Q. Find speed of block at D, Normal force at D, ht. 'h',



$v_C = \sqrt{7rg}$

COME,

C D

Loss in KE = Gain in PE.

$K_i - K_f = P_f - P_i$

$$\frac{1}{2} m \times 7rg - \frac{1}{2} m v_D^2 = mgr$$

$$7rg - 2rg = v_D^2$$

$$\boxed{v_D = \sqrt{5rg}}$$

$$N + mg = \frac{m v_D^2}{r}$$

$$N = \frac{m}{r} \times 5rg - mg.$$

$$\boxed{N = 4mg}$$

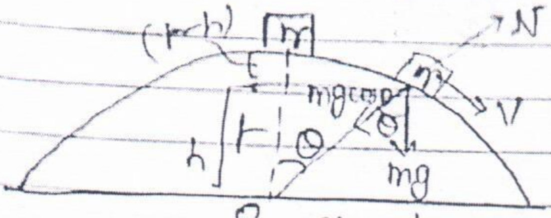
Loss in PE = gain in KE

$$mgx = \frac{1}{2} m \times 5rg$$

$$x = 2.5r$$

$$\boxed{h = 4.5r}$$

Q. In the given fig. the block just start sliding. At what ht. from ground, it will leave the surface. All surface frictless.



$\cos\theta = \frac{h}{r}$

$mg\cos\theta - N = \frac{mv^2}{r}$

Leaves surface.

$N = 0$.

$v^2 = rg\cos\theta$ —— (1)

C o ME,

Loss in P.E. = gain in KE.

$P_i - P_f = K_f - K_i$

$mg(r-h) = \frac{1}{2}mv^2$

$2g(r-h) = rg\cos\theta$

$2(r-h) = r\cos\theta = r \times \frac{h}{r}$

$2r - 2h = h$, $\Rightarrow \boxed{h = \frac{2}{3}r}$

$\cos\theta = \frac{2}{3}\frac{r}{r}$

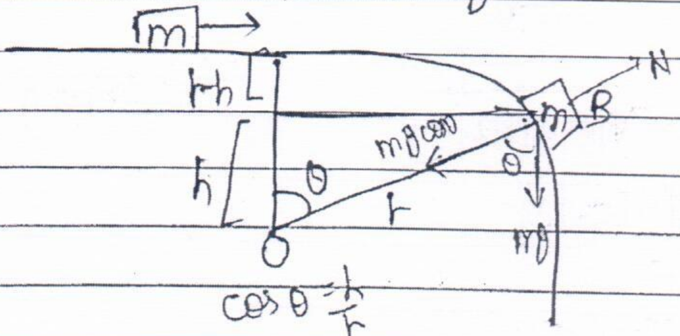
$\boxed{\theta = \cos^{-1}(2/3)}$

$h' = \frac{2}{3}r + r$

$\boxed{\theta = \cos^{-1}(2/3)}$

Q. At pt. 'B', the block leaves the surface. Find 'θ'.

$V_0 = 0.5\sqrt{rg}$



$mg\cos\theta - N = \frac{mv^2}{r}$

$v^2 = rg\cos\theta$ —— (1)

$\cos\theta = \frac{h}{r}$

$h = r\cos\theta$

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COME,

Loss in ~~KE~~ P.E = gain in KE.

$P_i - P_f = K_f - K_i$.

$\cancel{m}g(r-h) = \frac{1}{2}\cancel{m}v^2 - \frac{1}{2}\cancel{m} \times 0.25 \times rg$.

$2rg - 2gh + 0.25\,rg = v^2$.

$2rg - 2g \times r\cos\theta + 0.25\,rg = v^2$.

$2.25\,rg - 2rg\cos\theta = v^2$.

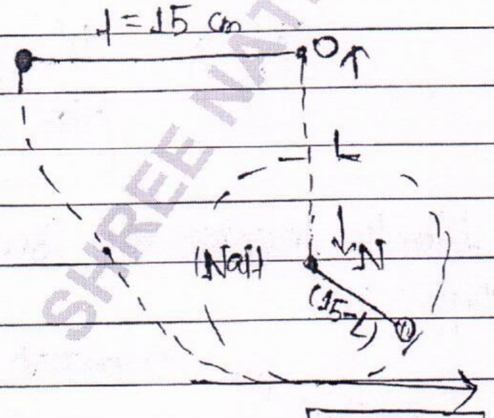
$v^2 = 2.25\,\cancel{rg} - 2\cancel{rg}\cos\theta = \cancel{rg}\cos\theta$

$2.25 = 3\cos\theta$

$\cos\theta = \frac{2.25}{3.00} = \frac{9}{12} = \frac{3}{4}$

$\theta = \cos^{-1}\left(\frac{3}{4}\right)$.

Q. Find min^m value of 'L' so that the bob just complete circle around nail.



$\sqrt{5(15-L)g}$

COME,

Loss in PE = gain in KE,

$mg \times r = \frac{1}{2} m v_L^2$.

$v_L^2 = 2rg$.

$v_L = \sqrt{2rg}$.

$2 \times \overset{3}{\cancel{15}} \times \cancel{g} = \cancel{5}(15-L)\cancel{g}$

$6 = 15 - L$

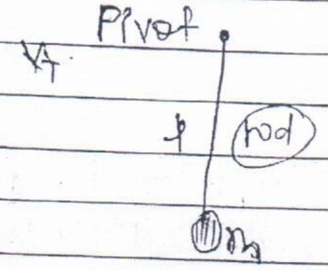
$L = 15 - 6$

$= 9$ cm.

Scanned by CamScanner

Q. A particle of mass 'm' is tied with 1 end of a uniform thin rod of length 'l'. The other end is pivoted. The particle is at lowest posn a/c to the figure. Find min^n speed given, so that the particle just complete the circle.

→



At, top, speed = 0, string — top — tension = 0

$v_T = 0$

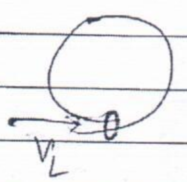
COME,

Loss in KE = Gain PE

$\frac{1}{2} m v_L^2 = mg \times 2r$

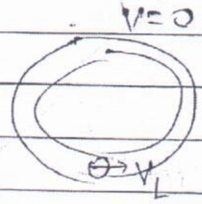
$$\boxed{V_L = \sqrt{4rg}}$$

Similar cases –



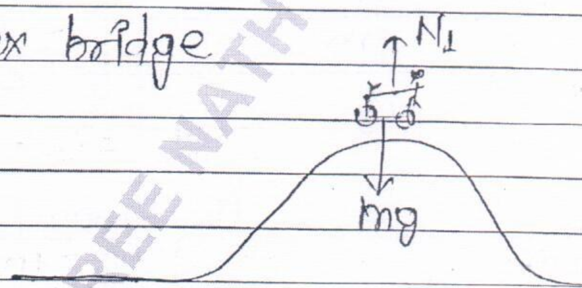
Ring-bead

$$\boxed{V_L = \sqrt{4gr}}$$



$$\boxed{V_L = \sqrt{4gr}}$$

convex bridge



$mg - N_1 = \frac{mv^2}{r}$

$$\boxed{N = mg - \frac{mv^2}{r}}$$

$\frac{18}{5} \times \frac{18}{5} \cdot \frac{15}{9}$ 210

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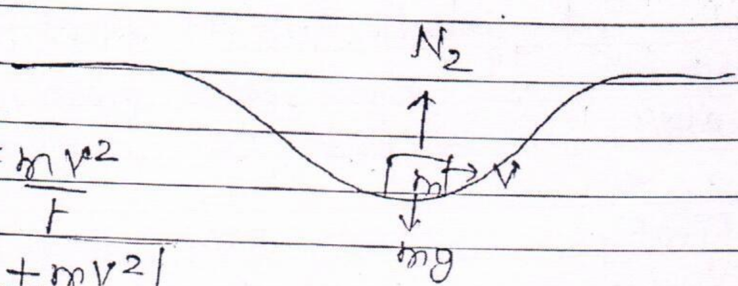
concave bridge

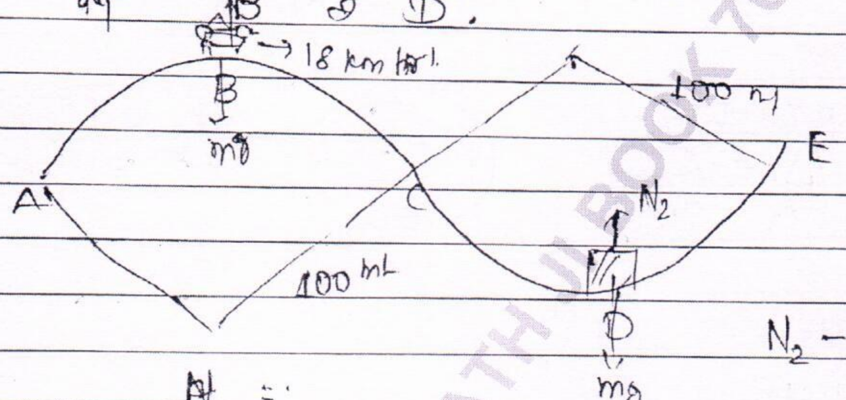
X zhan tak जलाती से कितना jyada lagega & kon N' jyata hai.



$N_2 - mg = \frac{mv^2}{r}$

$\boxed{N_2 = mg + \frac{mv^2}{r}}$

Q. combined mass of cycle & rider is 100 kg. His speed throughout the path is 18 km/hr. Find Normal at B & D.



At :

$mg - N_1 = \frac{mv^2}{r}$

$N_1 = 1000 - \frac{100 \times 25}{100}$

$N_1 = 975$ N.

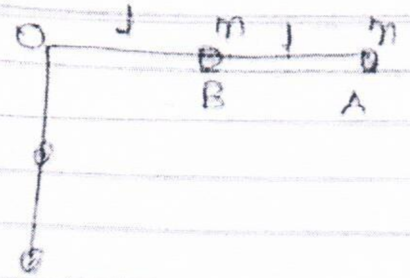
$N_2 - mg = \frac{mv^2}{r}$

$N_2 = 1000 + \frac{100 \times 25}{100}$

$= 1025$ N.

Q. 2 identical particle of mass 'm' are tied with thin rod kept in hz. plane & released when rod becomes Vt. find speed of B.

Pull to rigid body no relative pt will be there



$$mgl + mg(2l) = \frac{1}{2} mV_A^2 + \frac{1}{2} mV_B^2$$

$V = r\omega$

$\frac{V_A}{V_B} = \frac{r_A}{r_B} = \frac{2l}{l}$

$V_A = 2V_B$

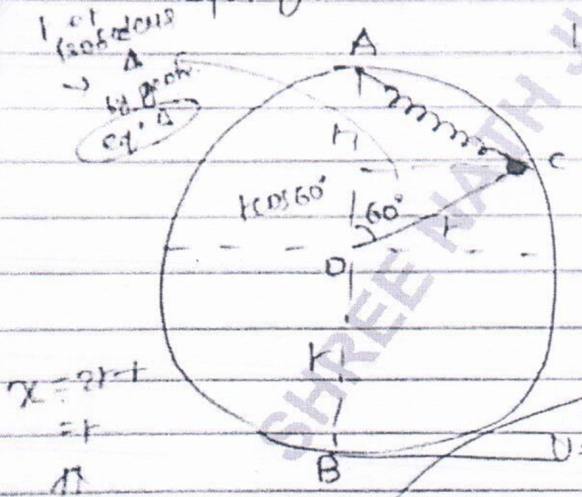
$$6gl = V_A^2 + V_B^2$$

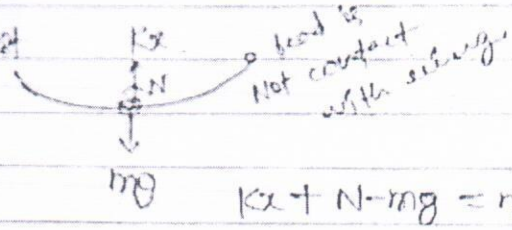
$$6gl = 4V_B^2 + V_B^2.$$

$$V_B^2 = \frac{6gl}{5}$$

$$V_B = \sqrt{\frac{6}{5}gl}.$$

Q. Radius of the ring is 20 cm & natural length of spring is also 20 cm. mass of the bead is 5 kg. bead is released from pt. 'C' its looses its contact with ring at lowest pt. 'B'. find spring const. of spring.





$$kx + N - mg = \frac{mV_B^2}{r}$$

$$kx - mg = \frac{mV_B^2}{r}.$$

COME -

C B

$$K_1 + P_1 = K_2 + P_2$$

$$0 + mg\left(r + \frac{r}{2}\right) + \frac{1}{2}k(0)^2 = \frac{1}{2}mV_B^2 + 0 + \frac{1}{2}kx^2.$$

$$mgr\left(\frac{3}{2}\right) = \frac{1}{2}mV_B^2 + \frac{1}{2}kx^2$$

$$3mg - \frac{kx^2}{r} = \frac{mV_B^2}{r}$$

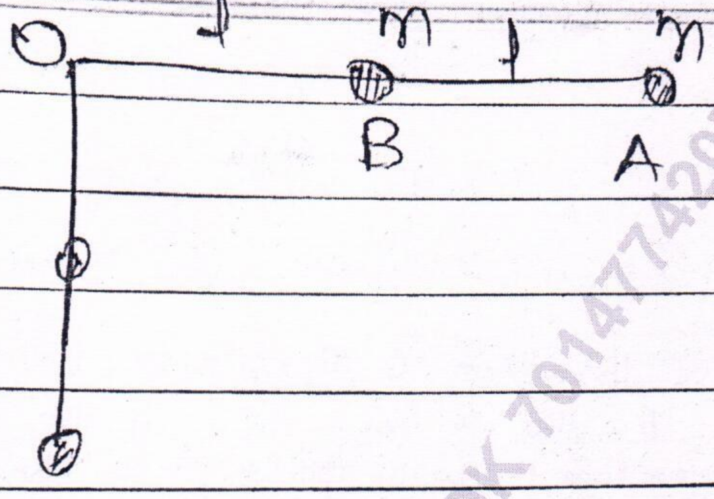
$$kx - mg = 3mg - \frac{kx^2}{r}$$

$$kr + kr = 4mg$$

$$k = \frac{2mg}{r} = \frac{2 \times 5 \times 10}{0.2}$$

$= 500$ N/m.

This is rigid body
no matter pt
must be figure



$mgl +$

$6gl$:

$6gl =$

V_B^2 :

V_B

IGM

Q. Radius of the ring is 20 c
spring is also 20 cm . r
bead is released from pt.
with ring at lowest pt.

AIIMS-18 Ex-II

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$\alpha = 2\ rad/s^2$

$5s$

$100\ rad$

$t=0 \quad \omega_1 \quad t \quad 100\ rad \quad 5s \quad \omega_2$

$\cancel{100}^{20} = \frac{\omega_1 + \omega_2}{2} \times \cancel{5}$

$2 = \frac{\omega_2 - \omega_1}{5}$

$\omega_1 + \omega_2 = 40 \quad -(1)$

$-\omega_1 + \omega_2 = 10$

$\omega_1 = 15\ rad/s$

$15 = 0 + 2 \times t$

$t = 7.5\ s$

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AIIMS-14 Ex-II

$\alpha = 2\ rad/s^2$

5 s

100 rad.

ω_1 100 rad ω_2

t = 0 t 5 s

$$\cancel{100}^{20} = \frac{\omega_1 + \omega_2}{2} \times \cancel{5}$$

$$2 = \frac{\omega_2 - \omega_1}{5}$$

$$\omega_1 + \omega_2 = 40 \quad -(1)$$

$$-\omega_1 + \omega_2 = 10$$

$$\omega_1 = 15\ rad/s$$

$$15 = 0 + 2 \times t$$

$$t = 7.5\ s.$$

Rotational Motion

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Rigid body :-

It is a system of large pt. masses at/each pair have infinitesimally small separatⁿ & this separatⁿ never changes in any circumstance. means, deformatⁿ & compression not possible.

→ For a given axis of rotatⁿ, each particle of rigid body have same angular displacement in same interval of time, so 'ω' of all particles is same but 'v' is different because, 'r' is different.

Moment of Inertia :- (I)

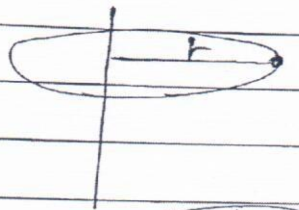
M.I. of a body is a measure to resist change in rotational state of motⁿ.

I. M.O.I. of 'n' particle system -

→ dim. → $M^1L^2T^0$

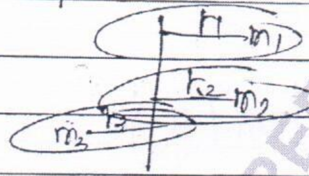
→ unit = $kg\,m^2$.

→ Tensor quantity.



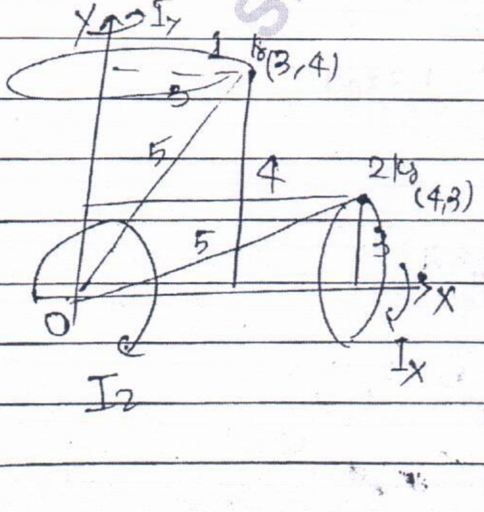
$\boxed{I = mr^2}$

↳ ⊥ʳ distance.



$\boxed{I = m_1r_1^2 + m_2r_2^2 + \dots + m_nr_n^2}$

Q. find M.O.I. of system about x-axis, y-axis and z-axis.



$I_X = 1 \times 4^2 + 2 \times 3^2 = 16 + 18 = 34\ kg\,m^2$.

$I_Y = 1 \times 3^2 + 2 \times 4^2 = 9 + 32 = 41\ kg\,m^2$

$I_Z = 1 \times 5^2 + 2 \times 5^2 = 25 + 50 = 75\ kg\,m^2$.

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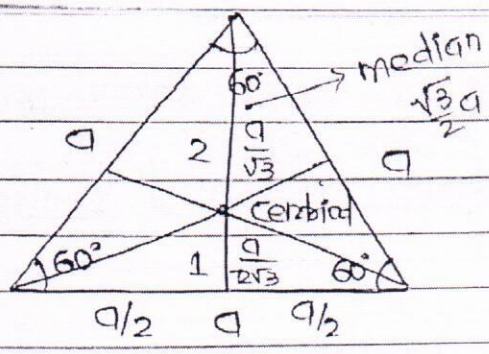
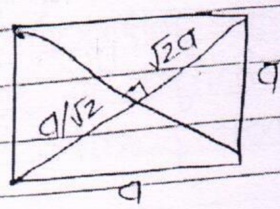
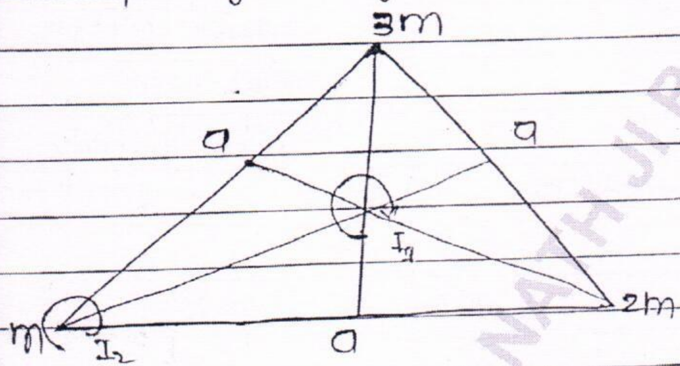
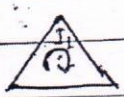




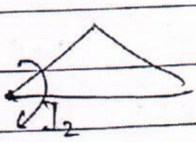
Q. Find M.O.I. of the system about an axis -
(i) passing through centriod & ⊥r to its plane.
(ii) passing through 'm' & ⊥r to its plane.
(iii) passing through the side joining 'm' & '2m'.
(iv) passing through 'm' & ∥ through a side
(v) passing through median. (3m)



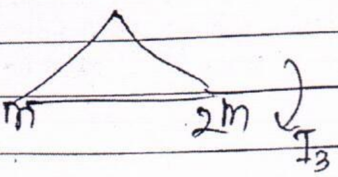
(i) 

$$I = m\times\left(\frac{a}{\sqrt3}\right)^2 + 2m\left(\frac{a}{\sqrt3}\right)^2 + 3m\left(\frac{a}{\sqrt3}\right)^2$$

$$= \frac{a^2}{3}\times 6m = 2ma^2.$$

(ii) 

$$I = 0 + 2ma^2 + 3ma^2$$
$$= 5ma^2.$$

(iii) 

$$I = 3m\left(\frac{\sqrt3}{2}a\right)^2$$

$$= \frac{9}{4}ma^2.$$

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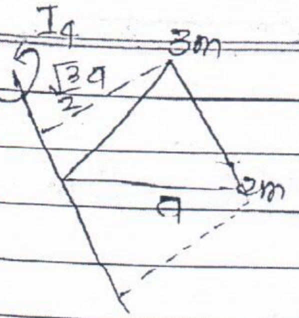
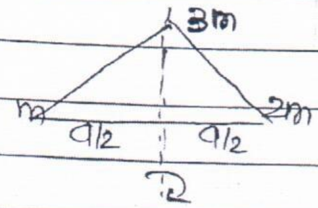
(iv)



$$I_4 = 0 + 2m\left(\frac{\sqrt{3}a}{2}\right)^2 + 3m\left(\frac{\sqrt{3}a}{2}a\right)^2$$

$$= \frac{15}{4} ma^2.$$

(V)

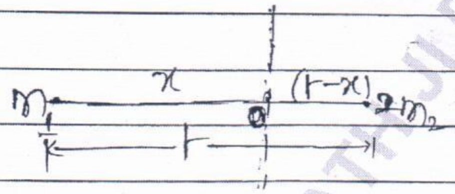


$$I_5 = \frac{ma^2}{4} + \frac{2ma^2}{4}$$

$$= \frac{3ma^2}{4}.$$

Q. Two particles of mass m_1 and m_2 are separated by distance 'r'. Find m.o.I. of the system about an axis passing through their COM and $\perp^r$ to line joining them.

→



$$m_1 x = m_2(r-x)$$

$$x = \frac{m_2 r}{m_1+m_2}.$$

$$x = \frac{m_2 r}{m_1+m_2} \qquad \frac{m_1 r}{m_1+m_2}.$$

$$I = m_1 r_1^2 + m_2 r_2^2$$

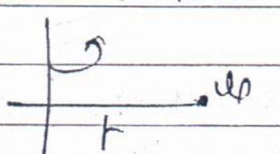
$$= \frac{m_1 \times m_2^2 r^2}{(m_1+m_2)^2} + \frac{m_2 \times m_1^2 r^2}{(m_1+m_2)^2}$$

$$= \frac{m_1 m_2 r^2}{(m_1+m_2)^{\cancel{2}}} \cancel{(m_1+m_2)}$$

$$\boxed{I = \frac{m_1 m_2}{m_1+m_2} r^2}$$

yaad rakho

$$\mu = \frac{m_1 m_2}{m_1+m_2} \quad \text{reduced mass.}$$



$$I = \mu r^2.$$

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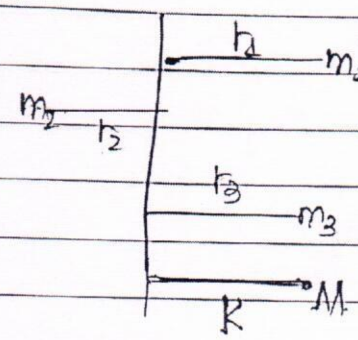
Radius of gyration (K) :-



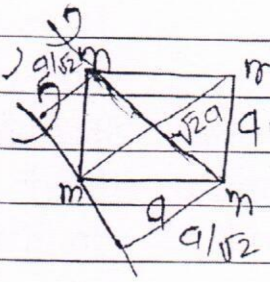
$I = m_1r_1^2 + m_2r_2^2 + \dots m_nr_n^2$.

$M = m_1 + m_2 + \dots$

$$\boxed{I = MK^2}$$

$$\boxed{K = \sqrt{\frac{I}{M}}}$$

Q.



$I = 0 + m\left(\frac{a}{\sqrt{2}}\right)^2 \times 2 + m(\sqrt{2}a)^2$.

$I = 3ma^2$.

$K = \sqrt{\frac{3ma^2}{4m}}$

$K = \frac{\sqrt{3}}{2}a$.

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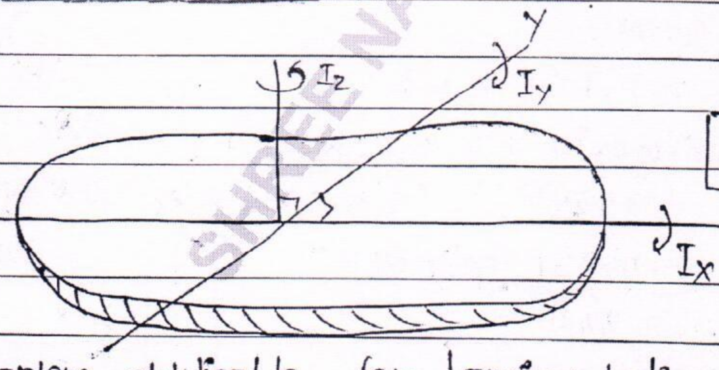
⊥ axis Theorem :-



$$\boxed{I_z = I_x + I_y}$$

→ This theorem applicable for laminar bodies or 2-D str. like ring, disc, lamina etc.

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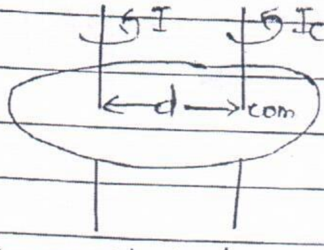
|| axis theorem –

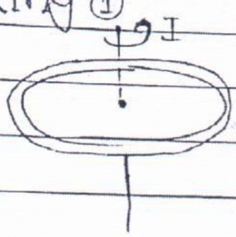
→ applicable for all bodies, all axis even axis of rotatⁿ is outside the body.



$$\boxed{I = I_c + Md^2}$$

M.O.I. of Rigid body / continous mass distribution :–

1. Ring ①

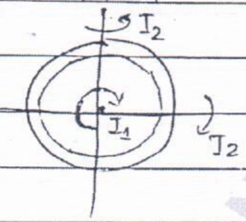


1. Axis passing through centre & ⊥ʳ to its plane.

$$\boxed{I_1 = MR^2}$$

$$\boxed{K_1 = R}$$

2. diametrical axis

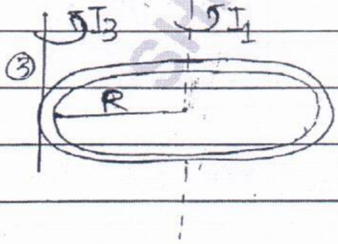


$$I_2 = I_x + I_y$$

$$MR^2 = I_2 + I_2$$

$$\boxed{I_2 = \frac{MR^2}{2}} \qquad \boxed{K_2 = \frac{R}{\sqrt{2}}}$$

3. tangential & ⊥ʳ to its plane –

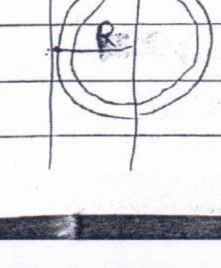


$$I_3 = I_1 + MR^2$$

$$I_3 = MR^2 + MR^2$$

$$\boxed{I_3 = 2MR^2} \qquad \boxed{K_3 = \sqrt{2}R}$$

4. tangential & || to its plane –

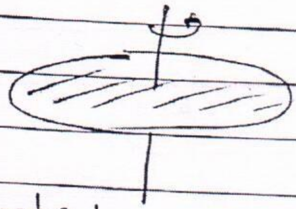


$$I_4 = \frac{MR^2}{2} + MR^2 \qquad \boxed{K_4 = \sqrt{\frac{3}{2}}R}$$

$$\boxed{I_4 = \frac{3}{\cancel{R}} MR^2} \qquad \boxed{I_4 = \frac{3}{2} MR^2}$$

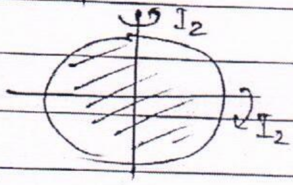
2. Disc :-

1. axis passing through center & ⊥ to its plane.



$\boxed{I_1 = \frac{MR^2}{2}}$ $\boxed{K_1 = \frac{R}{\sqrt{2}}}$

2. diametrical

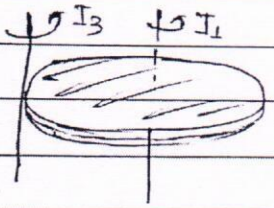


$I_2 = I_x + I_y$

$\frac{MR^2}{2} = 2I_2$

$\boxed{I_2 = \frac{MR^2}{4}}$ $\boxed{K_2 = \frac{R}{2}}$

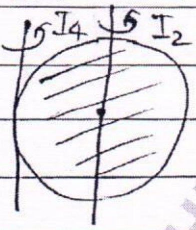
3. tangential and ⊥r to its plane.



$I_3 = \frac{mR^2}{2} + MR^2$

$\boxed{I_3 = \frac{3}{2} MR^2}$ $\boxed{K_3 = \sqrt{\frac{3}{2}} R}$

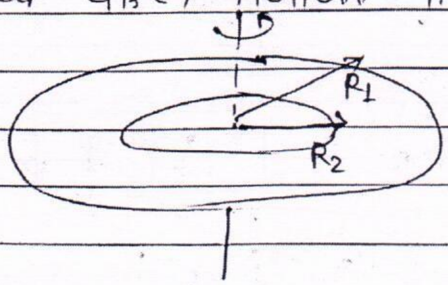
4. tangential & ∥ to its plane.



$I_4 = \frac{mR^2}{4} + mR^2$

$\boxed{I_4 = \frac{5}{4} mR^2}$ $\boxed{K_4 = \frac{\sqrt{5}}{2} R}$

Annular disc / Hollow disc.

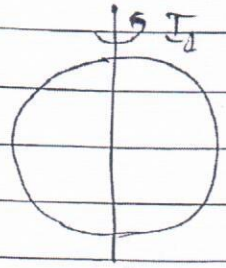


$\boxed{I = \frac{M}{2} (R_1^2 + R_2^2)}$

$\boxed{K = \frac{R_1 + R_2}{\sqrt{2}}}$

3. Solid sphere :-

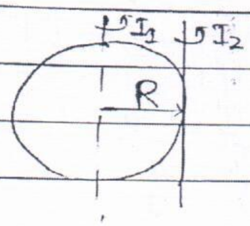
1.



$$\boxed{I_1 = \frac{2}{5} MR^2}$$

$$\boxed{K_1 = \sqrt{\frac{2}{5}} R}$$

2.

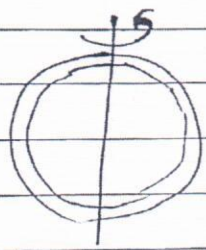


$$I_2 = \frac{2}{5} MR^2 + MR^2$$

$$\boxed{I_2 = \frac{7}{5} MR^2} \qquad \boxed{K_2 = \sqrt{\frac{7}{5}} R}$$

4. Hollow sphere or spherical shell.

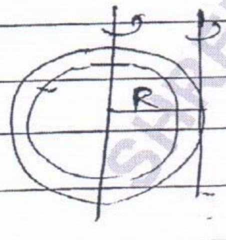
1.



$$\boxed{I_1 = \frac{2}{3} MR^2}$$

$$\boxed{K = \sqrt{\frac{2}{3}} R}$$

2.

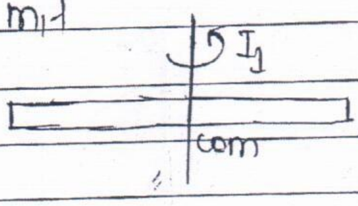


$$I_2 = \frac{2}{3} mR^2 + mR^2$$

$$\boxed{I_2 = \frac{5}{3} MR^2} \qquad \boxed{K = \sqrt{\frac{5}{3}} R}$$

5. Uniform Thin Rod -

1. m, l



$$\boxed{I_1 = \frac{ml^2}{12}}$$

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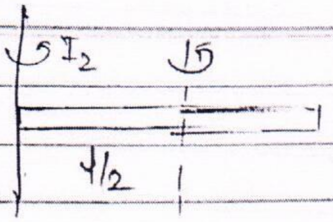
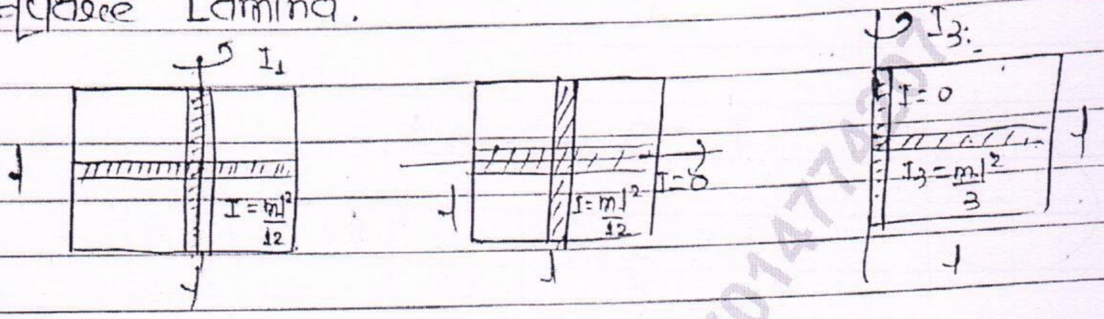
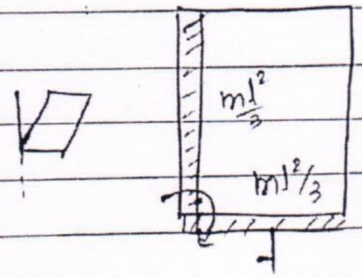
2. 

$$I_2 = \frac{ml^2}{12} + m\left(\frac{l}{2}\right)^2$$

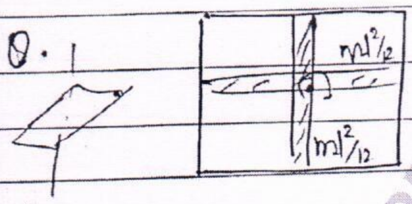
$$\boxed{I_2 = \frac{ml^2}{3}}$$

Square Lamina.



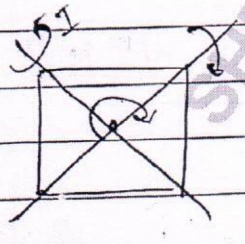
Q. 

$$I = \frac{2ml^2}{3}$$

Q. 1 

Passing through centre & ⊥ to plane

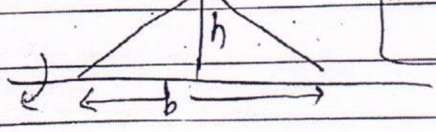
$$I = \frac{ml^2}{12} + \frac{ml^2}{12} = \frac{ml^2}{6}.$$

Q. 

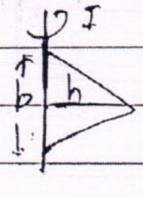
$$\frac{ml^2}{6} = I + I$$

$$I = \frac{ml^2}{12}$$

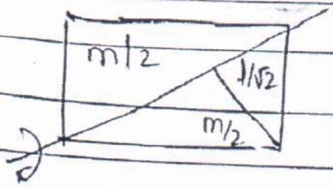
Triangular lamina.



$$\boxed{I = \frac{mh^2}{6}}$$



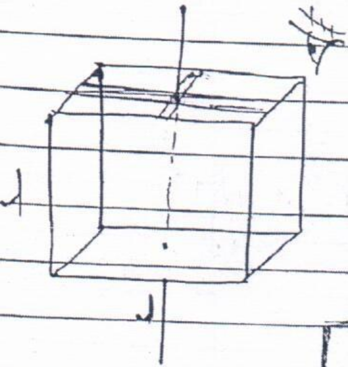
Q.



$$= \left[\frac{M}{2} \times \frac{1}{6}\left(\frac{l}{\sqrt{2}}\right)^2\right] \times 2.$$

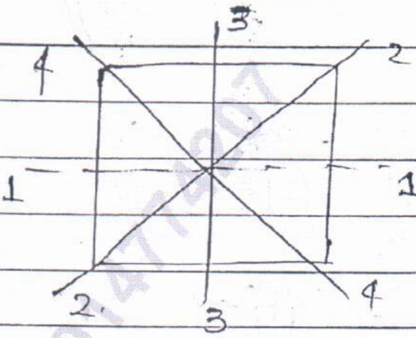
$$= \frac{ml^2}{12}.$$

#



$$\frac{ml^2}{6}.$$

#



Square lamina

$$\boxed{I_{11} = I_{22} = I_{33} = I_{44} = \frac{ml^2}{12}}$$

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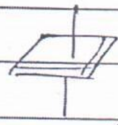
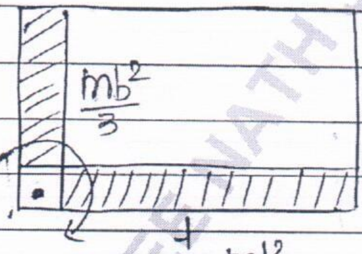
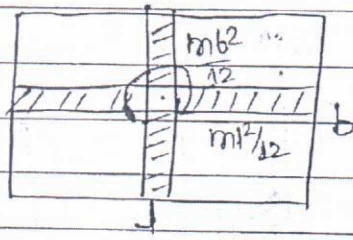
Rectangular Lamina -

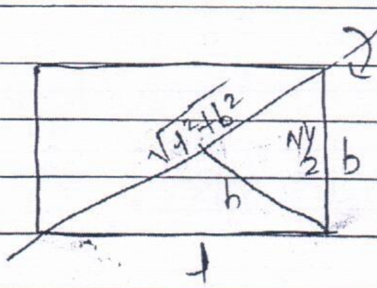






$$I = \frac{ml^2}{3} + \frac{mb^2}{3}.$$

$$I = \frac{ml^2}{12} + \frac{mb^2}{12}.$$



$$\frac{l \times b}{2} = \frac{1}{2} \times \sqrt{l^2+b^2} \times h$$

$$h = \frac{l \times b}{\sqrt{l^2+b^2}} \qquad I = \frac{mh^2}{6}$$

$$= \left[\frac{M}{2} \times \frac{l^2 \times b^2}{6(l^2+b^2)}\right] \times 2$$

$$\boxed{I = \frac{m}{6}\,\frac{l^2 \times b^2}{l^2+b^2}}$$

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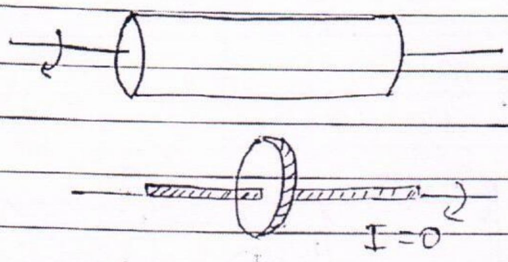
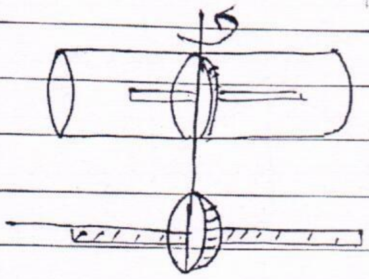
Solid cylinder →

1.



$$I = \frac{MR^2}{2} \; ; \; K = \frac{R}{\sqrt{2}}.$$

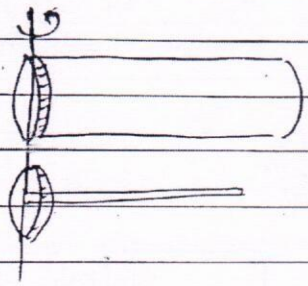
2.



d/cm ⊥c.

$$I = \frac{mR^2}{4} + \frac{ml^2}{12},$$

3.



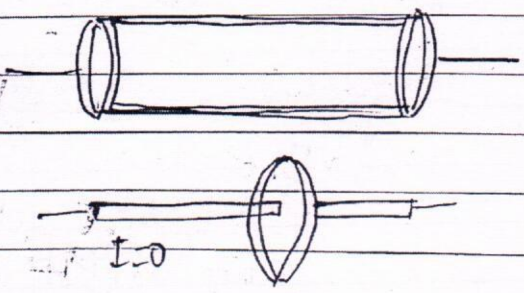
d/cm one end.

$$I = \frac{mR^2}{4} + \frac{ml^2}{3},$$

4.

$$\boxed{I = \frac{3MR^2}{2}}$$

5. Hollow sphere -



$$I = MR^2.$$

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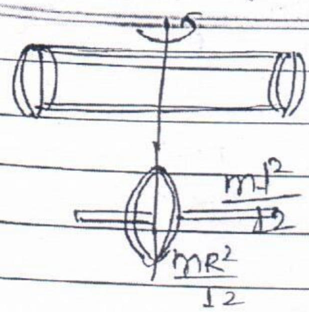
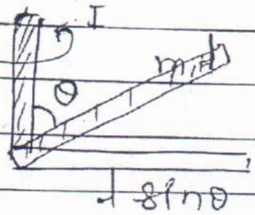
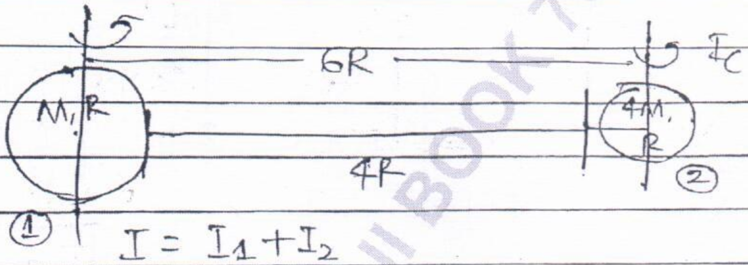


$$I = \frac{ml^2}{12} + \frac{mR^2}{12},$$



$$I = \frac{ml^2 \sin^2\theta}{3}.$$

Q.



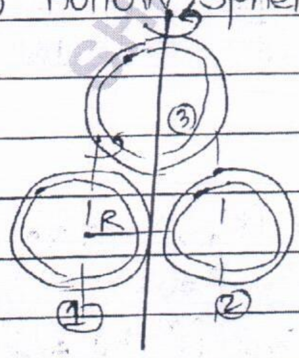
$$I = I_1 + I_2$$

$$= \frac{2}{5}MR^2 + \left[\frac{8}{5}MR^2 + 144R^2M\right]$$

$$= MR^2\left[\frac{2}{5} + \frac{8}{5} + 144\right]. \quad = \frac{10 + 720}{5} mR^2.$$

$$= \frac{\cancel{730}^{146}}{\cancel{5}} mR^2, = 146mR^2.$$

Q. Each hollow sphere of m, r.



$$= 2\left[\frac{2}{3}mR^2 + mR^2\right] + \frac{2}{3}mR^2$$

$$= mR^2\left[\frac{4}{3} + 2 + \frac{2}{3}\right].$$

$$= \frac{4 + 6 + 2}{3} mR^2 = 4mR^2.$$

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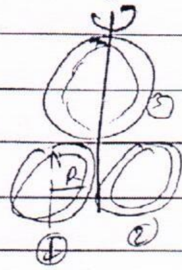
Q In the above questn, all 3 are ring, find M.O.I. of their given axis.

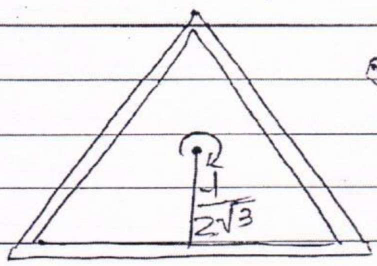


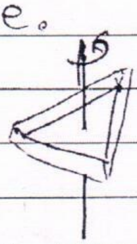
$$= 2\left[\frac{mR^2}{2} + mR^2\right] + \frac{mR^2}{2}$$

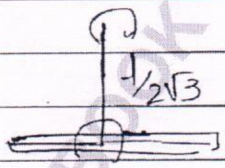
$$= mR^2\left[1 + 2 + \frac{1}{2}\right]$$

$$= \frac{7}{2} mR^2$$

Q, Find M.O.I. about an axis passing through centroid & $\perp^r$ to plane. mass m length 'l'





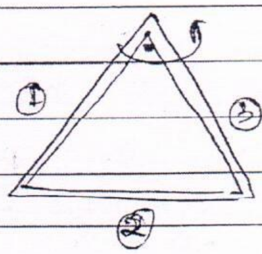


$$I = \left[\frac{ml^2}{12} + m\left(\frac{l}{2\sqrt{3}}\right)^2\right] \times 3$$

$$I = ml^2\left[\frac{1}{12} + \frac{1}{12}\right] \times 3$$

$$= \frac{ml^2}{2}$$

Q,

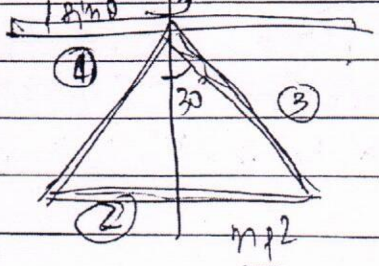


$$I = \frac{ml^2}{3} \times 2 + \left[\frac{ml^2}{12} + m\left(\frac{\sqrt{3}}{2} l\right)^2\right]$$

$$= ml^2\left[\frac{2}{3} + \frac{1}{12} + \frac{3}{4}\right]$$

$$= \frac{8+1+9}{12} = \frac{18}{12} ml^2 = \frac{3}{2} ml^2$$

Q, through meth.



$l\sin 30°$

$\frac{ml^2}{12}$

$$= \left[\frac{ml^2 \sin^2 30}{3}\right] \times 2 + \frac{ml^2}{12}$$

$$= ml^2\left[\frac{1}{3} \times \frac{1}{4} \times 2 + \frac{1}{12}\right]$$

$$= ml^2\left[\frac{1}{6} + \frac{1}{12}\right] = \frac{ml^2}{4}$$

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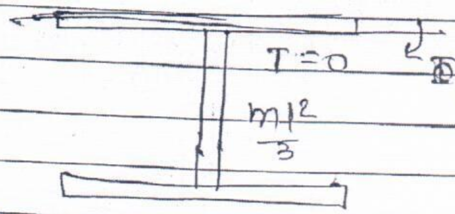
Q. All 3 rod are 666 apart.



$I = 0$

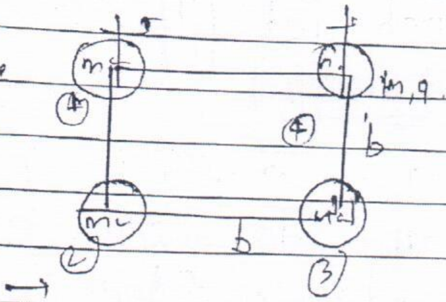
$\frac{ml^2}{3}$

$= 0 + ml^2$.

$I = 0 + \frac{ml^2}{3} + ml^2$

$= \frac{4}{3} ml^2$,

Q.



solid sphere.

$I = \left[\frac{2}{5} Ma^2\right] \times 2 + 2\left[\frac{2}{5} ma^2 + mb^2\right]$

$= \frac{4}{5} mR^2 + \frac{4}{5} Ma^2 + 2mb^2$.

$= \frac{8ma^2}{5} + 2mb^2$.

Q. Two disk of same ~~p~~ mass & thick ~~doent~~ ness have densities σ_1 & σ_2 o/c. Find ratio of their m.o.I about an axis passing through com & ⊥r to plane.

→

$m_1 = m_2$

$\pi R_1^2 t\, \sigma_1 = \pi R_2^2 t\, \sigma_2$.

$\left(\frac{R_1}{R_2}\right)^2 = \frac{\sigma_2}{\sigma_1}$.

$I = \frac{mR^2}{2} \Rightarrow \frac{I_1}{I_2} = \left(\frac{R_1}{R_2}\right)^2 = \frac{\sigma_2}{\sigma_1}$.

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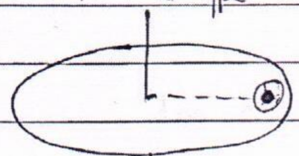
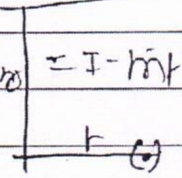
Deattachment →

$\boxed{I_{remaining} = I_{complete} - I_{removed}}$

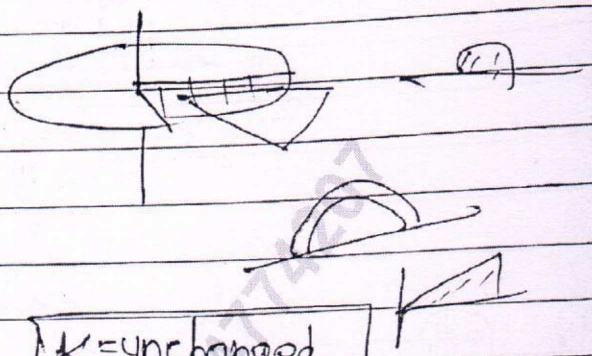
1. Asymmetrical ~~attachment~~ deattachment 2. Symmetrical deattachment



$I_{remaining} = I - mr^2$



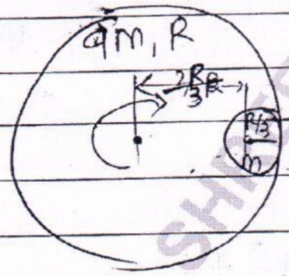
$K = \boxed{\sqrt{\dfrac{I - mr^2}{m - m}}}$



$\boxed{K = \text{unchanged}}$

Q. From a uniform disk of mass 9m & radius 'R' A disk of radius R/3 is removed such that at one point their's edges meet. Find m.o.I of the remaining part about an axis passing through center complete disc & ⊥r to its plane.

→



$m' = \sigma \times \dfrac{\pi R^2}{9}$

$9m = \sigma \times \pi R^2$

$m' = m.$

$= \dfrac{m(R/3)^2}{2} + m\left(\dfrac{2R}{3}\right)^2$

$= mR^2\left[\dfrac{1}{18} + \dfrac{4}{9}\right]$

$= mR^2\left[\dfrac{1+8}{18}\right]$

$= \dfrac{mR^2}{2}.$

$I_{remaining} = I_{comp} - I_{remov}$

$= \dfrac{9mR^2}{2} - \dfrac{mR^2}{2}.$

$= 4mR^2.$

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Symmetrical detachment ⇒ Radius of gyration unchanged

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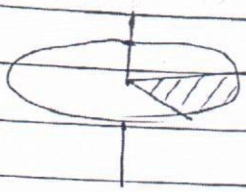
Q. From a disc of mass M, & radius R, 1/4th part is cut-out. Find M.O.I. of the remaining part about an axis passing through com and $\perp^r$ to plane.



$I = \dfrac{MR^2}{2}$

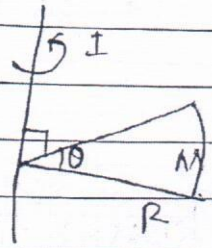
$K = \dfrac{R}{\sqrt{2}}$

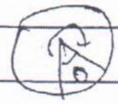
$I_{remaining} = \dfrac{3M}{4} K^2$

$= \dfrac{3M}{4} \times \dfrac{R^2}{2}$

$= \dfrac{3}{8} MR^2$

Eg.





$K = \dfrac{R}{\sqrt{2}}$

$I = MK^2$

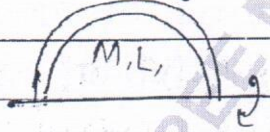
$= \dfrac{MR^2}{2}$

$I = \dfrac{mR^2}{4} \Rightarrow K = R/2$

$I = MK^2 = \dfrac{MR^2}{4}$

Q. A wire of length 'L' & mass 'M' is bent in semi-circle a/c to fig. Find M.O.I. about the given axis.

→



$I = \dfrac{MR^2}{2}$, $k = \dfrac{R}{\sqrt{2}}$

$I = mK^2$

$= \dfrac{mR^2}{2}$

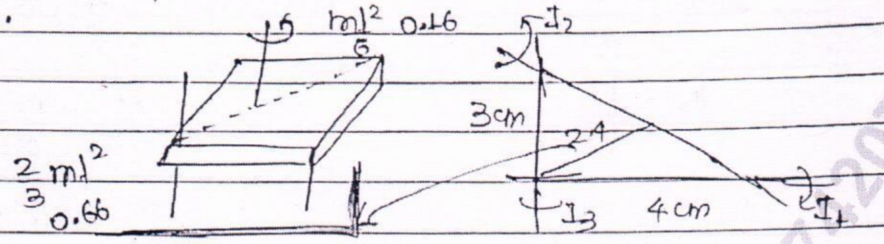
$\pi R = L$

$R = \dfrac{L}{\pi}$

$I = \dfrac{ML^2}{2\pi^2}$

* M.O.I. depend on -
  mass.
* M.O.I. depends on Axis of rotatⁿ,
* M.O.I. depends on mass-distributⁿ from axis of rotatⁿ.

Eg.



$h = \frac{12}{\sqrt{3^2+4^2}} = \frac{12}{5} = 2.4$

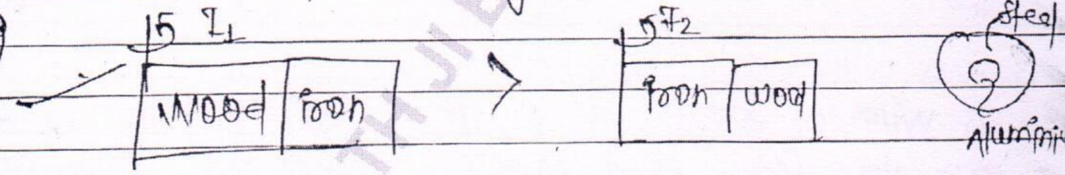
$I_2 = \frac{m}{6}(2.4)^2$ $\quad I_3 = \frac{m}{6}(4)^2$

$\boxed{I_3 > I_1 > I_2}$

$I_1 = \frac{m}{6}(3)^2$

* Larger mass, or larger density at larger distance from axis of rotatⁿ, larger is the M.O.I.

eg



* If ice on poles of earth melts then M.O.I. of earth ↑es because the water shifts away from Axis of rotatⁿ & contributes in M.O.I.

* If raw egg & boiled egg are spinning with same freq. & left, then M.O.I. of raw egg'll be more becoz, the fluid shifts away from axis of rotatⁿ. so stopping time of raw egg 'll be more. $I_{raw} > I_{boiled}$

stopping $t_{raw} > t_{boiled}$.

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$$\frac{1}{2}kx^2 = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2$$

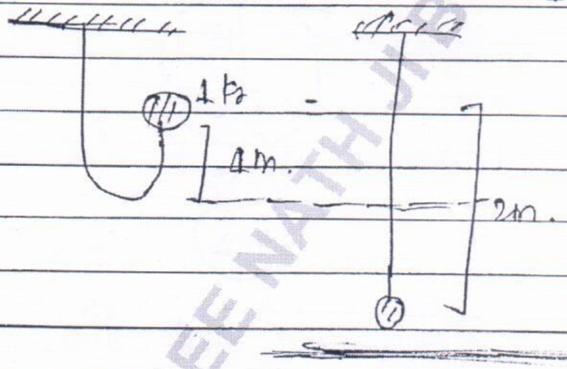
$$\cancel{\frac{1}{2}}kx^2 = \cancel{\frac{1}{2}}m_1 v_1^2 + \cancel{\frac{1}{2}}m_2\left[\frac{m_1 v_1}{m_2}\right]^2$$

$$kx^2 = m_1 v_1^2 + \cancel{m_2}\frac{m_1^2 v_1^2}{m_2^{\cancel{2}}}$$

$$= m_1 v_1^2\left[1 + \frac{m_1}{m_2}\right]$$

$$v_1 = \sqrt{\frac{m_2 kx^2}{m_1(m_2+m_1)}}$$

Q. If the sphere is released a/c to fig. find impulse given by string to the sphere, when the string becomes just taut.



$I = \Delta P$

$= m\sqrt{2gh}$

$= \sqrt{2\times10\times2}$

$= \sqrt{40}$.

Impulse,

J or $I = m(\vec{v_f} - \vec{v_i})$

$= m\sqrt{2gh}$

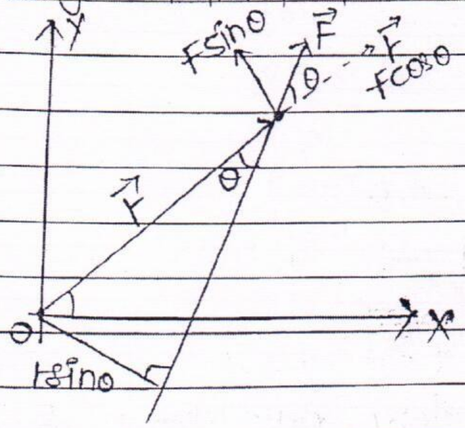
$= 1\sqrt{2\times10\times2}$

$= 2\sqrt{10}$ N-s

TORQUE :- $(\vec{\tau})$

→ Moment of force.

→ To change rotational state of motn torque is reqd.



$\tau = r \times$ [component of force ⊥ to $\vec{r}$]

$= r \times F\sin\theta$.

$\tau = F \times$ [⊥ distance of line of actn of force from axis of rotn]

$\boxed{\tau = Fr\sin\theta}$

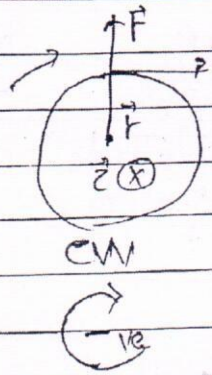
$\boxed{\vec{\tau} = \vec{r} \times \vec{F}}$

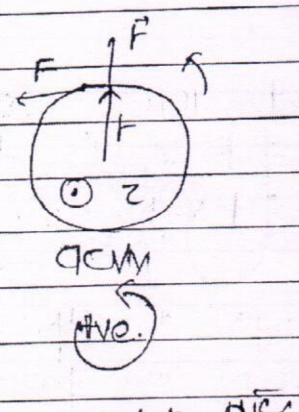
unit: N-m

dimension → $M^1L^2T^{-2}$.

→ Axial vector.

Sign convention.





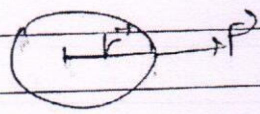
$\tau = -ve$ अंदर

$\tau = +ve$. बाहर

1. $\theta = 0°$ $\sin 0 = 0$

$\tau = 0$.

→ If line of actn of force passes through axis of rotatn then torque is zero.



2. $\theta = 90°$ $\sin 90 = +1$

$\boxed{\tau_{max.} = Fr}$

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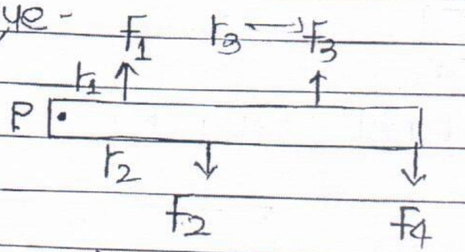
Net torque -



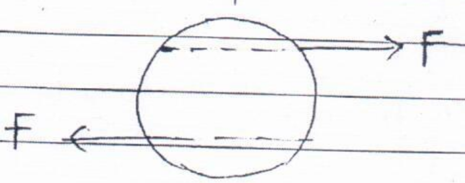
$\vec{\tau}_{net} = \vec{\tau}_1 + \vec{\tau}_2 + \dots + \vec{\tau}_n$

$\tau_{net} = +F_1r_1 - F_2r_2 + F_3r_3 - F_4r_4.$

$\boxed{\vec{\tau}_{net} = 0}$ rotational equilibrium.

Force couple -

eg. नल , I shape instrument



Translatory equilibrium but not in rotational equilibrium.

moment of force couple -

$\tau = F \times$ [⊥ distance b/w line of act^n of forces).

$= F \times \underbrace{r \sin\theta}_{\text{Lever arm}}$.

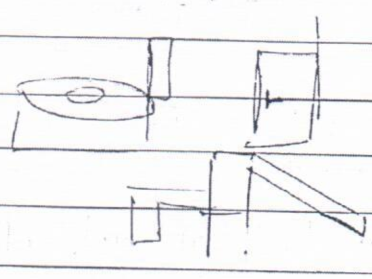
→ By ↑ lever arm, a desired torque can be produced with the help of lesser force.

→ Two equal & opp. forces & their line of act^n passes through 2 different pts.

23.11.17

Torque equat^n-

$\boxed{\vec{\tau} = I\vec{\alpha}}$



$\tau = Fr\sin\theta$.

Liver arm

$\tau = $ const

$\downarrow F \propto \dfrac{1}{(r\sin\theta)\uparrow\uparrow}$

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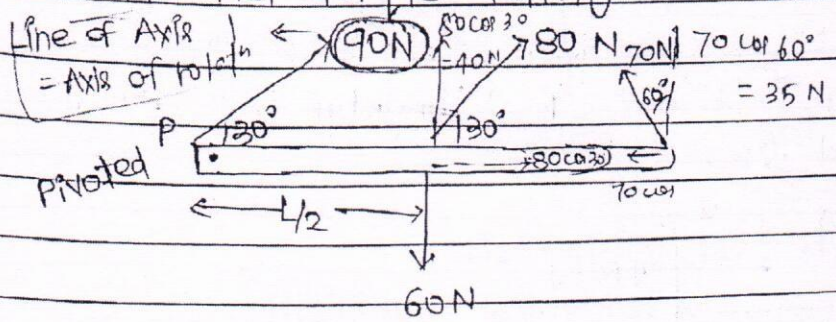
Q. Find net torque acting on the rod.



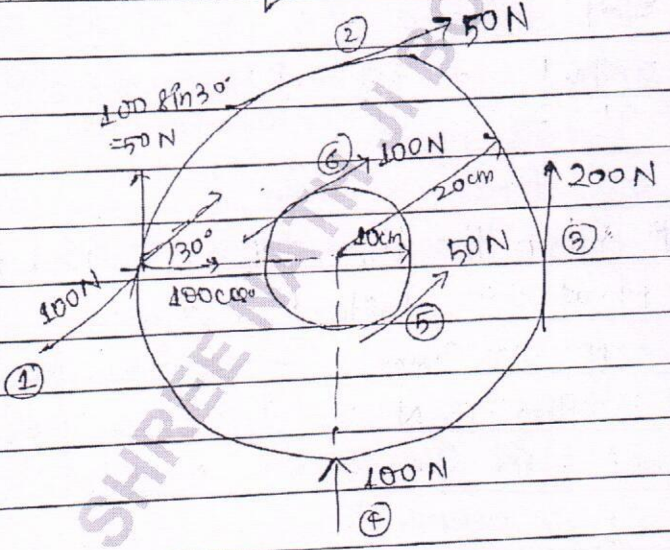
→ $\tau_P = -60 \times \frac{L}{2} + 40 \frac{L}{2} + 35 L$

$= -30L + 20L + 35L$

$= +25\ L$

$= +25 \times 0.3 = 7.5$ N-m acw Outwards.

Q. Find net torque on the wheel.



$\tau = -50 \times 0.2 - 50 \times 0.2 + 200 \times 0.2 + 0 + 50 \times 0.1 - 100 \times 0.1$

$= -10 - 10 + 40 + 5 - 10$

$= +15$ N-m

body rotate acw, τ outwards

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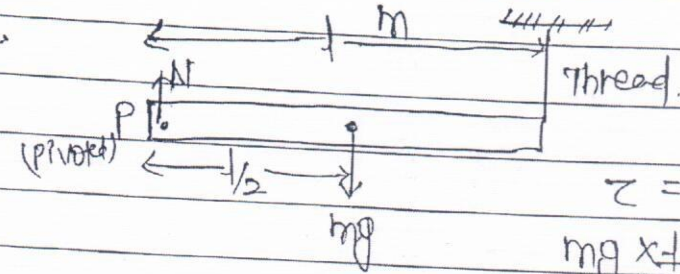
Q. A uniform rod of mass 'm' & length 'l' is kept horizontal as shown in fig. one end is pivoted to pt. P & is free to rotate. Now thread is cut. Find angular acc^n of rod & acc^n of com at this instant of time.



$$\tau = I\alpha$$

$$mg \times \frac{l}{2} = \frac{ml^2}{3} \times \alpha$$

$$\boxed{\alpha = \frac{3g}{2l}}$$

acc^n of com

$$a_{com} = \alpha r$$

$$= \frac{3g}{2l} \times \frac{l}{2} = \frac{3g}{4} \text{ m/s}^2$$

Q. In the prev. questⁿ when the thread is cut, find normal force on pivoted pt. at this instant of time.

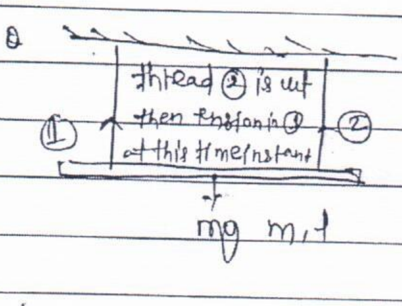
$$mg - N = m\,a_{com}$$

$$m(g - a_{com}) = N$$

$$N = m\left(g - \frac{3g}{4}\right)$$

$$N = \frac{mg}{4}$$

Q



$$mg - T = m\,a_{com}$$

$$T = \frac{mg}{4}$$

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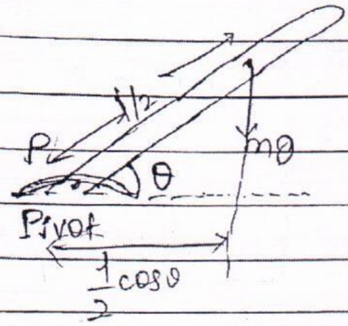
JEE-17

Q. The rod is left from given posit^n. Find acc^n of other point at this instant of time.



$$\tau = I\alpha$$

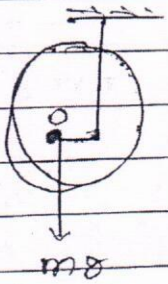
$$mg \times \frac{l}{2}\cos\theta = \frac{ml^2}{3} \times \alpha$$

$$\alpha = \frac{3g\cos\theta}{2l}$$

$$a_\theta = \alpha r$$

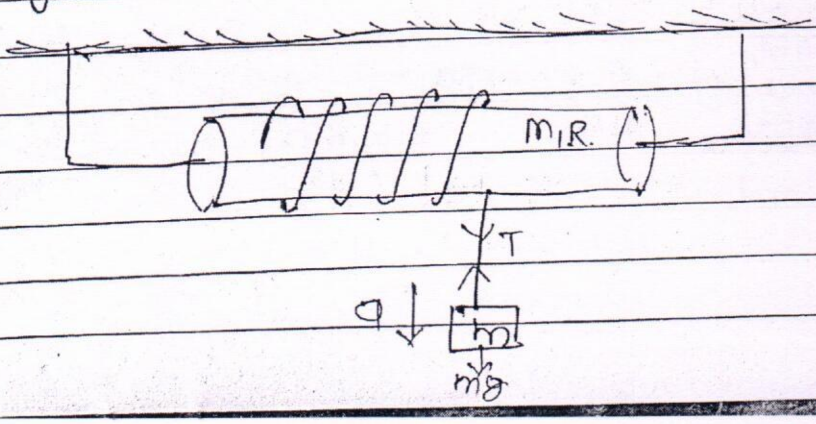
$$= \frac{3g\cos\theta}{2l} \times l$$

$$a_\theta = \frac{3}{2} g\cos\theta$$



$$mg\frac{r}{2} = \left[\frac{mr^2}{2} + \frac{mr^2}{4}\right]\alpha$$

Q. A ~~thread~~ string is wrapped over a solid cylinder of mass M, and radius 'R' & it is suspended such that it is free to rotate about geometrical axis on a hz. plane a/c to fig. A block of mass 'm' is tied with the end a/c to fig. & left. Find acc^n of block & tension in the string.



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$mg - T = ma$ —(1)

$mg - \frac{ma}{2} = ma.$

$mg = a\left[m + \frac{M}{2}\right]$

$$\boxed{a = \frac{2mg}{M+2m}}$$

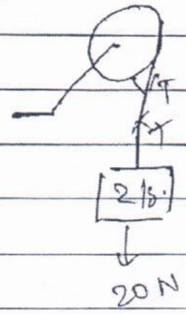
$\tau = I\alpha.$

$T \times \cancel{R} = \frac{MR^{\cancel{2}}}{2} \times \frac{a}{\cancel{R}}$

$T = \frac{Ma}{2}$ —②

$$\boxed{T = \frac{mMg}{M+2m}}$$

Q. A disc of radius 20 cm & M.O.I. 0.32 kg m² is free to rotate about geometrical axis. A string is wrapped over it with the hanging hand. block of mass 2 kg is tied & left. Find acc^n of block & tension in string.



$20 - T = 2a.$

$20 - 8a = 2a$

$a = 2 m/s^2.$

$\tau = I\alpha$

$T \times 0.2 = 0.32 \times \frac{a}{0.2}$

$T = \frac{0.32\,a}{0.04}$

$T = 8a$ —①

$T = 16$ N.

Q. Hollow cylinder, $m = 3$ kg

NEET

$r = 40$ cm

$F = 30$ N.

$\alpha = ?$

→ $\tau = I\alpha$

$\cancel{30}^{10} \times 0.4 = \cancel{3} \times (0.4)^2 \alpha$

$\alpha = \frac{4}{0.16} \times \cancel{100}^{25}$

$\alpha = 25$ rad / s².

Vaibhav

MPF
Com coll.
cfr.
Rotn MoI
Vec kin.

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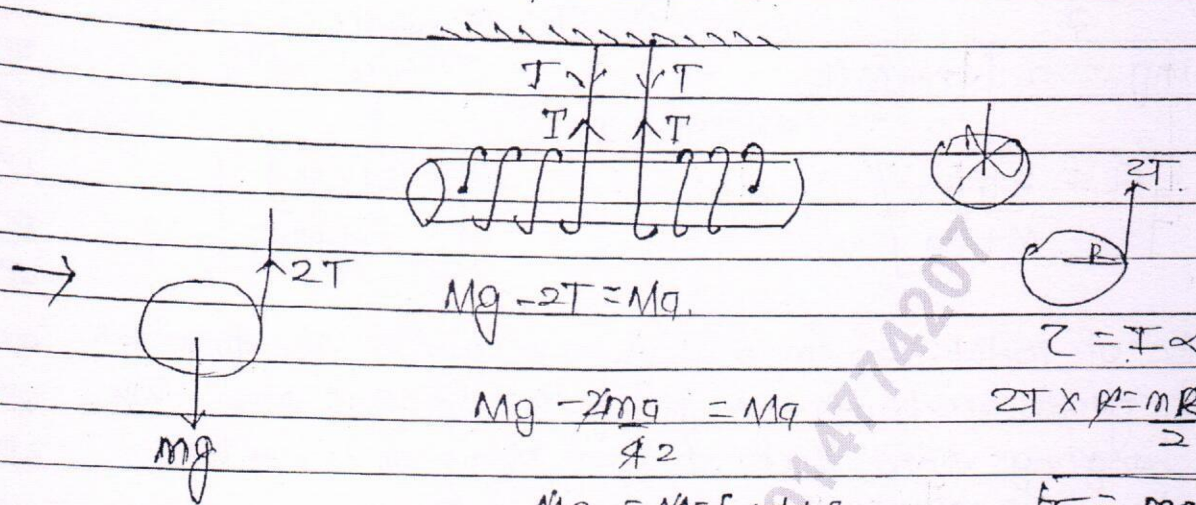
Q. A solid cylinder of mass 'M' & rad. 'R' is released. a/c to fig. Find acc^n of the cylinder.



$$Mg - 2T = Ma$$

$$Mg - \frac{\cancel{2}ma}{\cancel{4}\,2} = Ma$$

$$\cancel{M}g = \cancel{M}a\left[1 + \frac{1}{2}\right]$$

$$\boxed{a = \frac{2}{3}g}$$

$$\tau = I\alpha$$

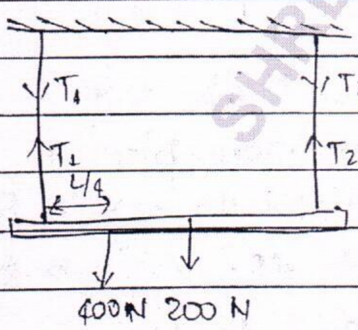
$$2T \times \cancel{R} = \frac{mR^{\cancel{2}}}{2} \times \frac{a}{\cancel{R}}$$

$$\boxed{T = \frac{ma}{4}}$$

24/11.

Rotn part Eqb^m

Q. A uniform ~~mass~~ rod of mass 20 kg is suspended by 2 string. Length of the rod is 'L'. A block of mass 40 kg is suspended from the rod at distance L/4 from end. Find tension in the strings.



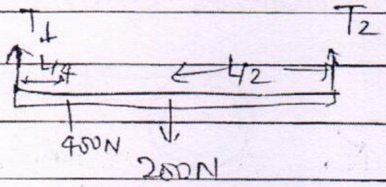
Translatory Eqb^m

$\Sigma F_y = 0$

$T_1 + T_2 = 600$ —(1)

$T_2 = 600 - 400$

$= 200$ N



$$-T_1 L + 400 \times \frac{3L}{4} + 200 \times \frac{L}{2} = 0$$

$300 + 100 = T_1$.

$T_1 = 400$ N

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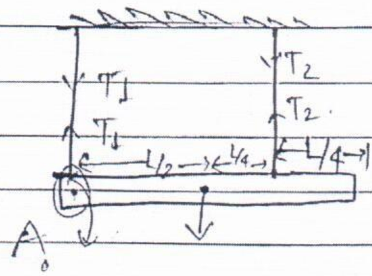
Q. Wt. of the rod is 'W' & length is 'L'. Find tension in the string.



Translatory eqb^n.

$T_1 + T_2 = W$ —(1)

Rotatory eqb^n

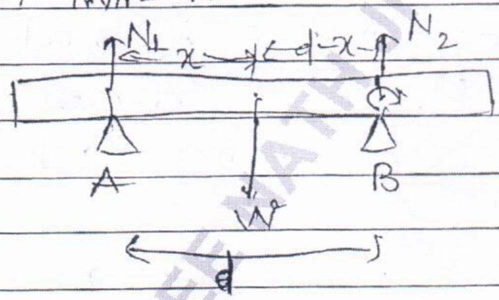
$\tau_A = 0$.

$-\frac{WL}{2} + T_2 \times \frac{3L}{4} = 0$

$\frac{3}{4_2} T_2 = \frac{W}{2}$ $\Rightarrow T_2 = \frac{2}{3} W$.

$T_1 = W - \frac{2W}{3} = \frac{W}{3}$.

Q. A uniform rod of wt. 'W' is placed horizontally over two knife edges A & B separated b/w knifes d. centre of gravity of rod is at distance x from A. Find normal force on knife A.



$-N_1 d + W(d-x) = 0$

$N_1 = \frac{W(d-x)}{d}$

$= W\left[1 - \frac{x}{d}\right]$

Q. A metre scale is balanced over knife edge. now 2 coins each of mass 5 gm are placed one over other at 25 cm mark to again balance knife edge is ~~kept~~ placed at 45 cm mark. find wt. of the metre scale.

$\Rightarrow$

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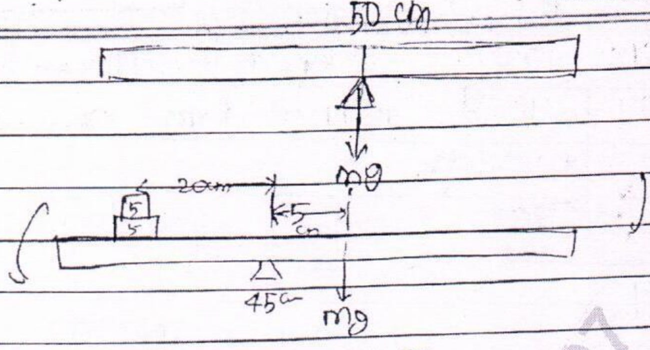
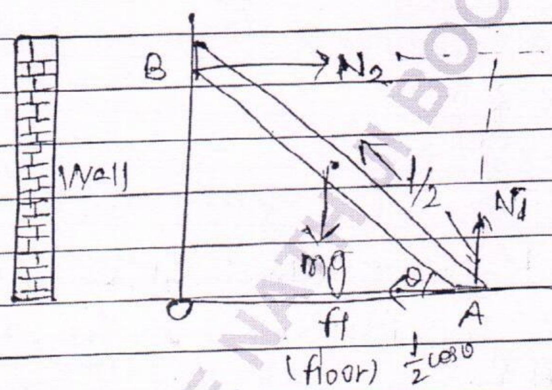


$$2mg \times 20 = mg \times 5$$

$$m = \frac{2 \times 5 \times 20}{5} = 40 \text{ gm}$$

Q. A rod of mass 'm' & length 'l' is placed against wall a/c to fig. wall is smooth while floor is rough. Find normal force due to wall, floor and frict^n force and also find resultant force on end 'A'.



Translatory eqb^n –

$\Sigma F_x = 0$.

$N_2 = f$ —(1).

$\Sigma F_y = 0$.

$N_1 = mg$ —(2)

Rotational Eqb^n,

$$mg \times \frac{l}{2}\cos\theta - N_2 l \sin\theta = 0$$

$$\frac{mg\cos\theta}{2} = N_2 \sin\theta$$

$$\boxed{N_2 = \frac{mg\cot\theta}{2}} \qquad \boxed{N_2 = f = \frac{mg\cot\theta}{2}}$$

$$f_{net}A = \sqrt{N_1^2 + f^2}$$

$$= \sqrt{(mg)^2 + \left(\frac{mg\cot\theta}{2}\right)^2}$$

$$\boxed{f_{net} = mg\sqrt{\frac{4+\cot^2\theta}{4}}}$$

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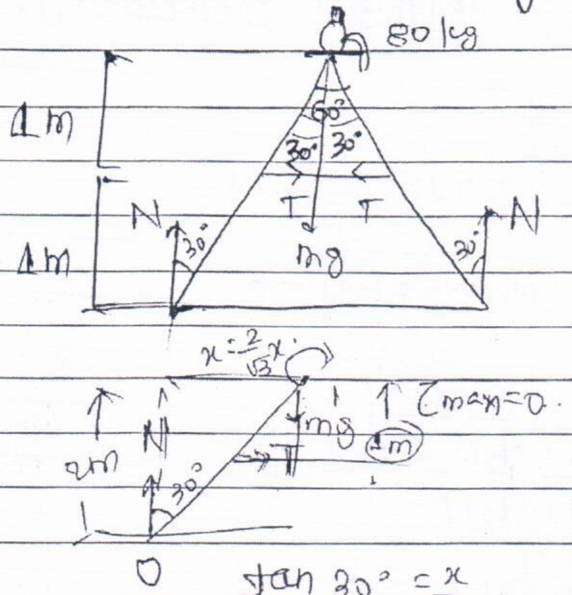
Q. A man of mass 80 kg is sitting on ladder as shown in fig.. Find normal force on each leg and tension in the string.



Translatory eqn –

$\Sigma F_y = 0$

$2N = mg$

$N = \frac{mg}{2} = \frac{800}{2} = 400\,N$

$\tan 30° = \frac{x}{2}$

$\frac{1}{\sqrt{3}} = \frac{x}{2} \Rightarrow x = \frac{2}{\sqrt{3}}\,m$

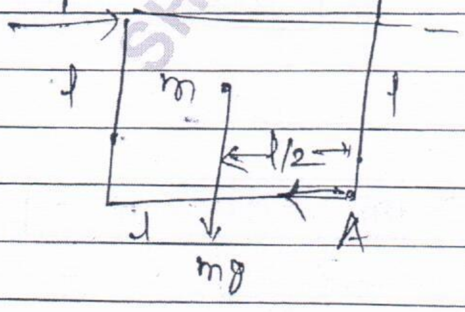
$+T \times 1 - N \times \frac{2}{\sqrt{3}}\,m = 0$

$T = \frac{400 \times 2}{\sqrt{3}}$

$= \frac{800}{\sqrt{3}}\,N$

wt. floor चार देगा N ड़ाली आ जाएगी

Q. A Force 'F' is applied on a cubical block a/c to fig. Find min^m value of force 'F' so that the cube start toppling. Fric^n is sufficiently large. so there is no sliding



$F \leq f l$

$\tau_F \geq \tau_{mg}$

$\tau_F = \tau_{mg}$

$F_{min} \times l = mg\frac{l}{2}$

$\boxed{F_{min} = \frac{mg}{2}}$

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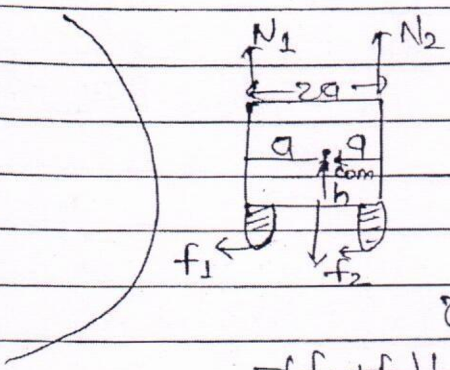
Q. A car is moving on circular track of radius 'r' its com is @ a ht. 'h' from road. separation b/w both wheels '2a'. Find the max^m speed without overturning.



Translatory eq.

$\Sigma F_x = 0, \; f_1 + f_2 = \frac{mv^2}{r}$ — ①

$\Sigma F_y = 0 \quad N_1 + N_2 = mg$ — ②

$\tau_{com} = 0$

$-(f_1+f_2)h - N_1 a + N_2 a = 0$

$a(N_2 - N_1) = (f_1+f_2)h$

$N_2 - N_1 = \frac{mv^2}{ra}h$

$N_1 + N_2 = mg$

$2N_2 = mg + \frac{mv^2}{ra}h$

$2N_1 = mg - \frac{mv^2}{ra}h$

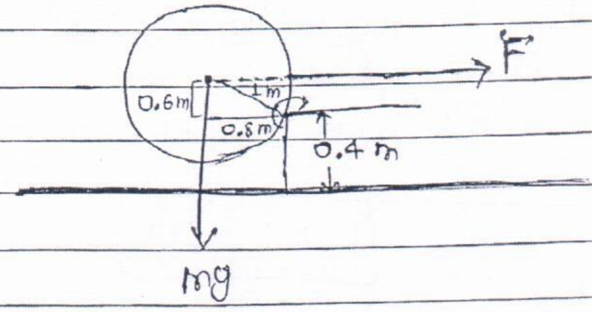
Overturning, $N_1 = 0$.

Inner wheel will leave the road.

$mg = \frac{mv^2}{ra}h$

$\boxed{v_{max} = \sqrt{\frac{gra}{h}}}$

Q. Find min^m force 'F' required to pull a wheel of mass 10 kg and radius 1 mt. a/c to fig.



$\rightarrow$

$\tau_F \geq \tau_{mg}$

$\tau_{F_{min.}} = \tau_{mg}.$

$F_{min} \times 0.6 = mg \times 0.8$

$F_{min} = \frac{100 \times 0.8}{0.6} \frac{4}{3}$

$= \frac{400}{3} = 133.33$ N.

Rotational Work, Energy, Power :-

$\vec{\tau} = const.$ $\tau = variable.$

$\omega = \vec{\tau}.\vec{\theta}.$ $\omega = \int_{\theta_1}^{\theta_2} \vec{\tau}.d\vec{\theta}$

$$\boxed{K.E._{rot.} = \frac{1}{2} I\omega^2}$$

Work Energy theorem,

$$\boxed{\omega_{all} = \frac{1}{2} I (\omega_f^2 - \omega_i^2)}$$

$$\boxed{Power = \vec{\tau}.\vec{\omega}.}$$

Q. A rod of mass 'm' & length 'l' is pivoted at pt. 'P' & free to rotate abt pt. 'P'. Rod is left free to fall from 'A' when rod is at $posit^n$ 'A' & 'C'. Find its angular speed & also find speed of other end.

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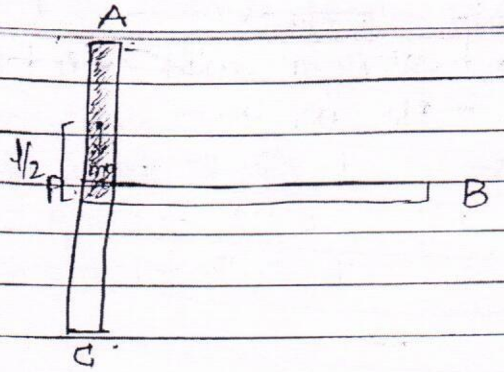


B/W 'A' & 'B'

Loss in P.E. = gain in K.E.rot.

$$mg\frac{l}{2} = \frac{1}{2} I \omega_B^2$$

$$\cancel{m}g\cancel{l} = \frac{\cancel{m}l^{\cancel{2}}}{3}\omega_B^2$$

$$\boxed{\omega_B = \sqrt{\frac{3g}{l}}}$$

$V_{end} = \omega r = \sqrt{\frac{3g}{l}} \times l$

$V_{end} = \sqrt{3gl}$.

B/W A & C.

Loss in P.E = Gain in KErot.

$$mgl = \frac{1}{2} I\omega_c^2$$

$$\cancel{m}gl = \frac{1}{2} \times \frac{\cancel{m}l^2}{3}\omega_c^2$$

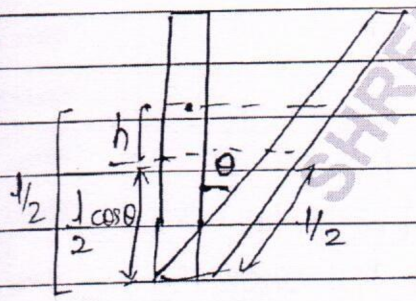
$$\boxed{\omega_c = \sqrt{\frac{6g}{l}}}$$

$V_{end} = \omega \times l$

$= \sqrt{\frac{6g}{l}} \times l$

$= \sqrt{6gl}$.

Q. Rod is held vt. on one end on floor, now it is left to fall freely. find speed of other end when it is fallen through 'θ'



Loss in PE = Gain in KErot.

$$mg\left(\frac{l}{2} - \frac{l}{2}\cos\theta\right) = \frac{1}{2} I\omega^2$$

$$\cancel{m}g\frac{\cancel{l}}{2}(1-\cos\theta) = \frac{1}{2} \times \frac{\cancel{m}l^2}{3}\omega^2$$

$$\frac{3g}{l} \times 2\sin^2\theta/2 = \omega^2.$$

$$\boxed{\omega = \sqrt{\frac{6g}{l}}\sin\theta/2}$$

हर जगह Practice करना. यह Quest^n है Exam में आएगा.

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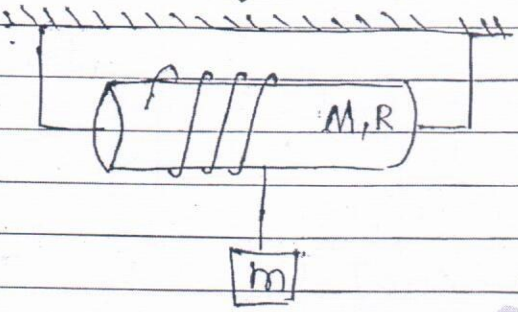
Q. A light string is wrapped over a solid cylinder which is mounted on hz. axle on free end of string, a block of mass 'm' is tied and left to fall freely. When the block falls through dist. 'h'. Find angular speed of cylinder.

→



COME,

Loss in PE of block = Gain in KE of block + Gain in KErot. of cyl.

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\frac{v^2}{R^2}$$

$$2mgh = v^2\left[m + \frac{I}{R^2}\right]$$

$$v^2 = \frac{2mgh}{mR^2+I}R^2$$

$$v = \sqrt{\frac{2mgh}{mR^2+I}}\,R$$

$$\omega = \frac{v}{R}$$

$$\omega = \sqrt{\frac{2mgh}{mR^2+I}}$$

VAIBHAV
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Q. A wheel is rotating about geometrical axis with speed 20 rad/s. now by applying const. torque it is brought to rest in 4 sec. Find work done by torque in 1st two sec., $I = 0.2 \ kg \ m^2$.

→ $\omega_0 = 20 \ rad./s$

$\tau = const.$

$t = 4 \ s$ rest.

$\alpha = \frac{\omega_f - \omega_0}{t}$

$= \frac{0-20}{4} = -5 \ rad/s^2$

$\omega_f = 20 - 5 \times 2$

$= 10 \ rad/s.$

$W = \frac{1}{2} I [\omega_f^2 - \omega_0^2]$

$= \frac{1}{2} \times 0.2 [100 - 400]$

$= 0.1 \times (-300) = -30 \ J.$

Loss in KE_{rot}. $\therefore W_\tau = 30 \ J.$

Q. A wheel of MOI. $10 \ kgm^2$ is rotating about principle axis @ of 10 rpm. Find work done by torque so that angular speed becomes 5 times of initial.

→ $W = \frac{1}{2} I (\omega_f^2 - \omega_0^2)$

$= \frac{1}{2} \times 10 [(5\omega_0)^2 - (\omega_0)^2]$

$= 5 \ [25 - 1] \ \omega_0^2$

$= 5 \times 24 \times [\frac{2\pi \ 10}{60}]^2$

$\pi^2 = 10$

$= 5 \times \cancel{24}^{2} \times 4 \times 10 \times \frac{1}{\cancel{36}_3}$

$= \frac{400}{3} = 133.33 \ J.$

1 rps = 60 rpm

$\frac{1}{60}$ rps = 1 rpm

$2\pi \times 10 \times \frac{1}{60}$ rps

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Q. Power of an electric motor is 1 hp at 6000 rpm. Find torque acting on it.

→ Power = $\tau . \omega$

$\cancel{746} = \cancel{6000 \times 2\pi \times \tau} / 60$ $\quad \tau = \frac{P}{\omega}$

$\cancel{\tau = \frac{746}{2\pi \times 10}}$

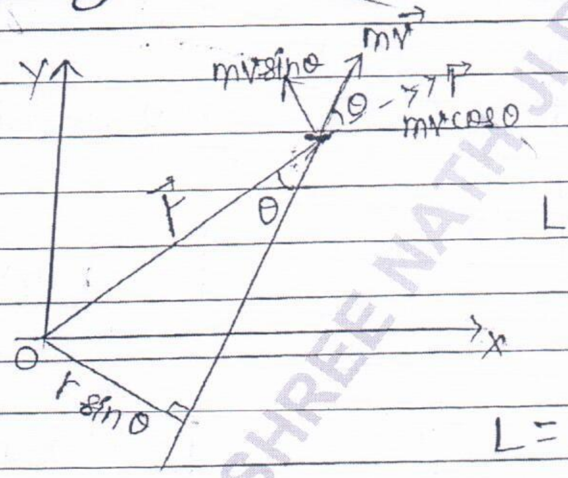
$= \frac{746}{2\pi \times 6000 \times \frac{1}{60}}$ (373)

$= \frac{373}{100\pi}$ N-m

$= \frac{373}{3.14}$ N-m $= 1.187$ N-m

Angular Momentum [$\vec{L}$]

moment of linear momentum is c/d angular momentum.



$L = r \times$ [component of linear momentum $\perp$ to $\vec{r}$]

$= r \times mv\sin\theta$.

$L = mv \times$ [$\perp$ distance of line of action of momentum from axis of rotation]

$L = mv \times r\sin\theta$.

$\boxed{L = mvr\sin\theta}$ $\quad L = pr\sin\theta$

$\boxed{\vec{L} = m(\vec{r} \times \vec{v})}$ $\quad \boxed{\vec{L} = \vec{r} \times \vec{p}}$

This formula is only for pt. mass particle not for rigid body.

Axial vector

unit = kgm^2/s $[M^1L^2T^{-1}]$ → planck const. का भी यही है

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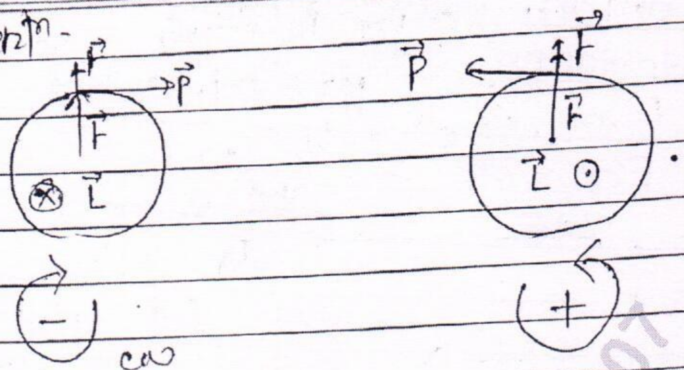
Sign. conventⁿ.



cw acw.

$L = -ve$ $L = +ve$.

In circular motⁿ,

$\theta = 90°$

$\boxed{L = mvr}$

$\theta = 0°$ $\sin 0 = 0$

$\boxed{L = 0}$

If line of actⁿ of linear momentum passes through axis of rotatⁿ then angular momentum zero.

For rigid body.

$\boxed{\vec{L} = I\vec{\omega}}$

$\boxed{\vec{\tau} = \frac{d\vec{L}}{dt}}$

$\boxed{K.E._{rot.} = \frac{L^2}{2I}}$

28/11

Q. A particle of mass 0.1 kg is moving with velocity $5\hat{i}$ m/s. When its positⁿ vector w.r.t. origin is $6\hat{i}+10\hat{j}$ mt. Find its angular momentum relative to origin.

→ $\vec{L} = m(\vec{r} \times \vec{v})$

$= 0.1\ [(6\hat{i}+10\hat{j}) \times 5\hat{i}]$

$= 0.1\ [50(-\hat{k})]$

$= 5\ (-\hat{k})\ \text{kg m}^2/\text{s}$

Vaibhav kumar

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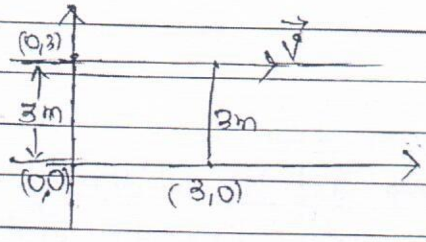
Q. A particle of mass 2 kg is moving with speed 3 m/s along +x-dirⁿ on st. line y = 3 mt. Find its angular momentum relative to (0,0), (3,0) & (0,3).

→



(1) (0,0)

$L_1 = mvr$

$= 2 \times 3 \times 3 = 18 \ kg \, m^2/s$

along −ve z-axis (cw)

(2) (3,0)

$L_2 = mvr$

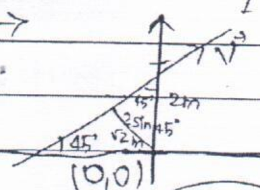
$= 18 \ kg \, m^2/s$.

(3) (0,3) $L_3 = 0$

$\theta = 0°$.

Q. A particle of mass 2 kg is moving with speed 2 m/s along st. line y = x+2. Find its angular momentum w.r.t. origin.

→



$y = x+2$

$y = mx + c$

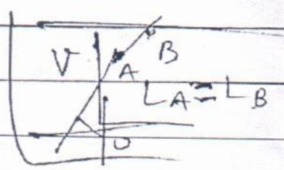
$m = 1 \Rightarrow \tan\theta$

$\theta = 45°$

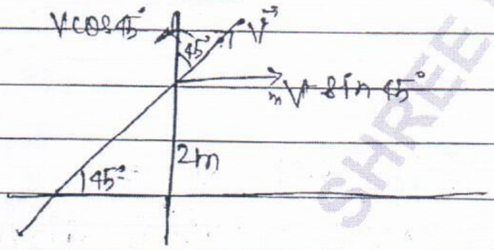
$L = mv\underline{r\sin\theta}$

$= 2 \times 2 \times \sqrt{2}$

$= 4\sqrt{2} \ kg \, m^2/s$.



Method-2



$= 2 \times mv\sin 45°$

$= 2 \times 2 \times 2 \times \frac{1}{\sqrt{2}} = 4\sqrt{2} \ kg \, m^2/s$.

Torque −ve

Q. A particle of mass 'm' is projected with speed V_0 @ an ∠ 45° from ground. Find its angular momentum related to pt. of projectⁿ where it is at highest pt. of its trajectory.

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$I = \frac{3}{2} \times 2 \times 0.16$.

$= 0.48 \ kg \ m^2$.

$L = I\omega$

$= 0.48 \times \left(2 \times \pi \times \frac{120}{60}\right)$

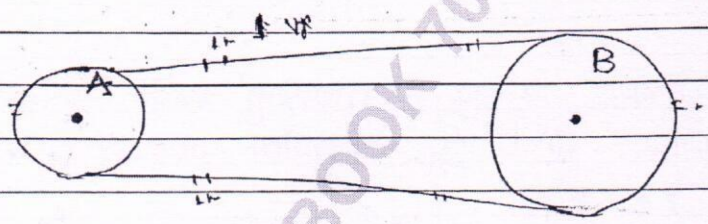
$= 1.92 \pi \ kg \ m^2/s$.

Q. Two wheels 'A' & 'B' are mounted on horizontal axle a/c to figure. They are connected with a belt which does not slip. Radius of B is of 3 times that of A. Find ratio of M.O.I. if —

(I) Both have same 'L'.

(II) " " " 'K.E.'.

speed / V of each point remains same.



$L = I\omega$

$v = \omega r$.

$\frac{\omega_1}{\omega_2} = \frac{r_2}{r_1}$

(I) $I_1\omega_1 = I_2\omega_2$.

$\frac{I_1}{I_2} = \frac{\omega_2}{\omega_1} = \frac{r_1}{r_2}$ ⇒ $\frac{I_1}{I_2} = \frac{r}{3r}$

(A) $\frac{I_1}{I_2} = \frac{1}{3}$

(II) $\frac{1}{2} I_1\omega_1^2 = \frac{1}{2} I_2\omega_2^2$.

$\frac{I_1}{I_2} = \left(\frac{\omega_2}{\omega_1}\right)^2$

$= \left(\frac{r_1}{r_2}\right)^2 = \left(\frac{r}{3r}\right)^2 = \frac{1}{9}$.

Q. Radius of wheel A & B are 50 cm & 80 cm. Wheel 'A' starts rotating with angular acc^n 0.8 rad/s². After some time wheel 'B' acquires angular speed 10 rad/sec. Find –

(I) accⁿ of pt. of belt and angular accⁿ of wheel 'B'.

(II) time after which wheel 'B' acquires angular speed 10 rad/sec.

→

A ($r_A = 50$ cm) B ($r_B = 80$ cm)

(I)

$a = \alpha_A r_A$
$= 0.8 \times 0.5$
$= 0.4\ m/s^2$.

$a = \alpha r$
$\alpha_A r_A = \alpha_B r_B$.
$0.8 \times 0.5 = \alpha_B \times 0.8$

$\Rightarrow \alpha_B = 0.5\ rad/s^2$.

(II)

$\omega_f = \omega_0 + \alpha t$.

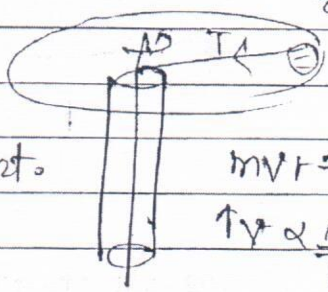
$t = \frac{\omega_f - \omega_0}{\alpha} = \frac{10-0}{0.5} = 20\ s$.

CONSERVATION OF ANGULAR MOMENTUM - [COAM]

COAM,

If external torque is zero act on a system then, angular momentum of system remains constant.

Eg. $\tau_T = 0$.



$mvr = const.$

$\uparrow v \propto \frac{1}{r \downarrow}$

$\vec{\tau}_{ext.} = 0$.

$\frac{d\vec{L}}{dt} = 0$.

$d\vec{L} = 0$.

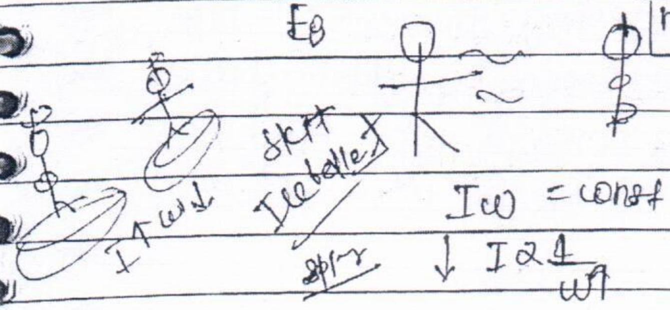
$\boxed{\vec{L} = const.}$

$\boxed{\vec{L}_1 + \vec{L}_2 + \vec{L}_3 + --- + \vec{L}_n = const.}$

$\boxed{I_1\omega_1 = I_2\omega_2}$

$\boxed{m_1v_1r_1 = m_2v_2r_2}$

Eg



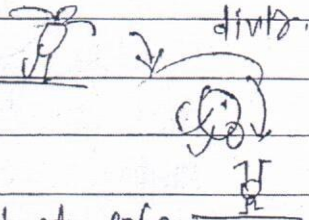
skat Ice ballet

$I\uparrow \omega\downarrow$

$I\omega = const$

spin $\downarrow I \propto \frac{1}{\omega\uparrow}$

Eg.



diving.

$I\uparrow \omega\downarrow$

Eg Ice melt at pole

$\omega = \frac{2\pi}{T}$ $\Rightarrow \uparrow I \propto \uparrow T$.

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Q. If earth suddenly shrinks upto 3/4th of its radius without any change in mass, find new duration of the day.

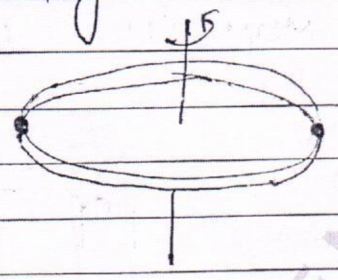
→ $I_1\omega_1 = I_2\omega_2$ ⟵ $\tau_{ext.} = 0$, $L_i = L_f$

$$\frac{2}{5}MR^2 \times \frac{2\pi}{24} = \frac{2}{5}M\left(\frac{3R}{4}\right)^2 \times \frac{2\pi}{T_2}$$

$$\frac{1}{24} = \frac{9}{16} \times \frac{1}{T_2}$$

$$T_2 = 9 \times \frac{24}{16} = \frac{54}{4} = 13.5 \text{ hrs.}$$

Q. A ring of mass 'M' and radius 'R' is rotating in Hz plane about geometrical axis with angular velocity ω, Now 2 mass particle each of small m' are gently placed over it at diametrically opp. position. Find angular velocity of ring.

→ 

$\tau_{ext.} = 0$

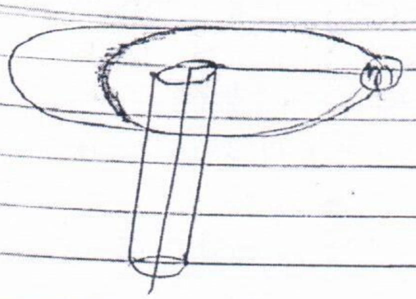
$L_i = L_f$

$I_1\omega_1 = I_2\omega_2$

$MR^2\omega = [MR^2 + 2mR^2]\omega_2$

$$\omega_2 = \frac{M\omega}{(M+2m)}$$

Q. A stone of mass 1 kg is tied with 1 end of string and revolved in hz. plane with speed 4 m/s in a circle of radius 2 mt. The string is passing through vt. pipe a/c to fig. Now other end of string is gradually pulled until radius of the circle becomes 1 mt. Find new ~~angular~~ speed, angular speed, ratio of KE. in 1st & 2nd case, & Tension.



$\tau_{ext.} = 0.$

$m_1 v_1 r_1 = m_2 v_2 r_2.$

$v_2 = \frac{v_1 r_1}{r_2}$

$= \frac{4 \times 2}{1} = 8 \ m/s.$

$\omega_2 = \frac{v_2}{r_2} = \frac{8}{1} = 8 \ rad/s.$

$K = \frac{1}{2} m v^2.$

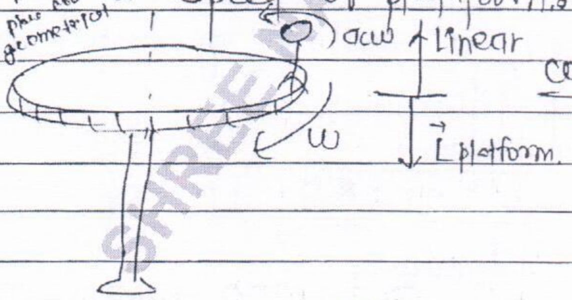
$\frac{K_1}{K_2} = \left(\frac{v_1}{v_2}\right)^2 = \left[\frac{4}{8}\right]^2 = \frac{1}{4}$

$T = \frac{m v_2^2}{r_2} = \frac{1 \times (8)^2}{1} = 64 \ N.$

Circular Platform concept →

<u>case I</u>: platform is @ rest. And person is standing on edge. NOW, the person starts walking with speed 'v' rel. to ground, Find angular speed of platform.

A circular disc of rad. R & mass m. is mounted over a vert. axel & free to rotate in the plane about geometrical axis.



COAM → $\tau_{ext.} = 0$

$\vec{L_i} = \vec{L_f}$

$0 = mvR - I\omega, \Rightarrow I\omega = mvR$

$$\boxed{\omega = \frac{mvR}{I}}$$

$\vec{v}_{mp} = \vec{v}_m - \vec{v}_p$

$\vec{v}_m = \vec{v}_{mp} + \vec{v}_p$

$v = v_{mp} - R\omega.$

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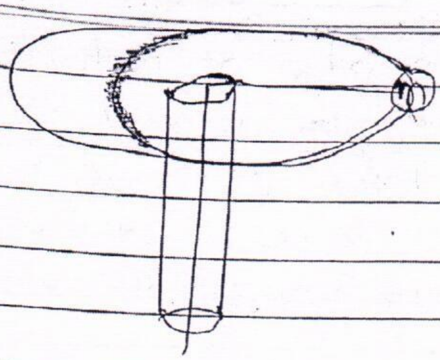

$$\tau_{ext.} = 0.$$

$$m_1 v_1 r_1 = m_2 v_2 r_2.$$

$$v_2 = \frac{v_1 r_1}{r_2}$$

$$= \frac{4 \times 2}{1} = 8 \text{ m/s.}$$

$$\omega_2 = \frac{v_2}{r_2} = \frac{8}{1} = 8 \text{ rad/s.}$$

$$K = \frac{1}{2} m v^2.$$

$$\frac{K_1}{K_2} = \left(\frac{v_1}{v_2}\right)^2 = \left[\frac{4}{8}\right]^2 = \frac{1}{4}$$

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Circular Platform concept →

case I: platform is @ rest. And person is standing on edge. Now, the person starts walking with speed 'v' rel. to ground, find angular speed of platform.

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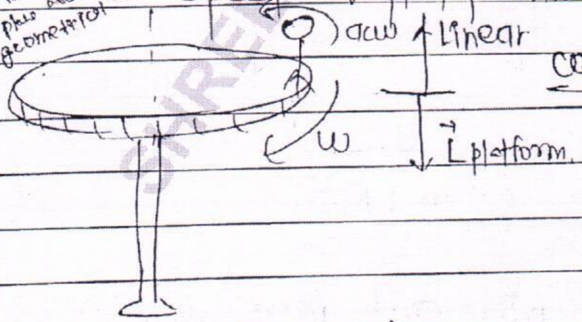

$\tau_{ext.} = 0$

COAM $\vec{L_i} = \vec{L_f}$

$0 = mvR - I\omega$, $\Rightarrow I\omega = mvR$

$$\boxed{\omega = \frac{mvR}{I}}$$

$$\vec{V}_{mp} = \vec{V}_m - \vec{V}_p$$

$$\vec{V}_m = \vec{V}_{mp} + \vec{V}_p$$

$$v = V_{mp} - R\omega.$$

VAIBHAV KR.

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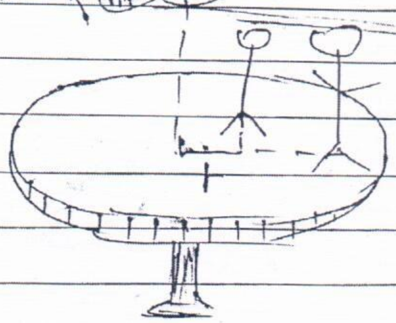
Case II : Person is standing on the edge of platform which is rotating with angular velocity 'ω' which is geometrical axis. Now person starts walking towards axis of rotatⁿ & stops @ distance 'r' from axis. then new angular velocity of platform.

opp case - Agar person pehle AOR pe ho fir edge pe jae to ($mR^2=0$)



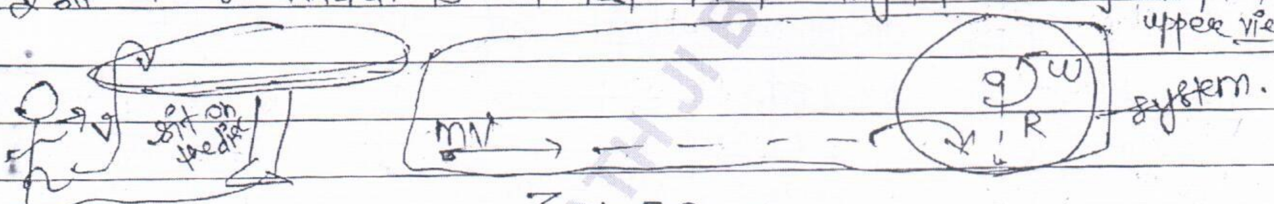
$\tau_{ext} = 0$.

$\vec{L_i} = \vec{L_f}$

$\left[\frac{MR^2}{2} + mR^2\right]\omega = \left[\frac{MR^2}{2} + mr^2\right]\omega_2$.

$$\boxed{\omega_2 = \frac{R^2[M+2m]\,\omega}{[MR^2 + 2mr^2]}}$$

Case III : Person is running on ground and jumps over a platform & sits on its edge which is at rest. Find final angular velocity of platform.



COAM, $\tau_{ext} = 0$

$\vec{L_i} = \vec{L_f}$ $I\omega$

$mvR + 0 = \left[\frac{MR^2}{2} + mR^2\right]\omega$

$$\boxed{\omega = \frac{2mv\cancel{R}}{(M+2m)R^{\cancel{2}}}}$$

Q. A disc of radius 2 mts and M.O.I. 200 kg m² is mounted over a frictionless Vt. axel. A person of mass 50 kg is standing on its edge. now the person starts walking with speed 1 m/s rel. to ground find angular speed of platform & time taken by man to complete 1 revolution.

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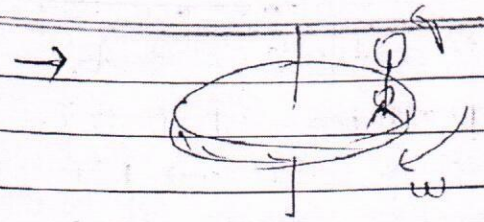


$$0 = mvR - I\omega$$

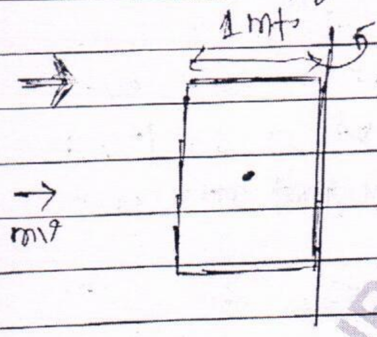
$$\omega = \frac{mvR}{I} = \frac{5\cancel{0} \times 1 \times \cancel{2}}{\frac{\cancel{2}\cancel{0}\cancel{0}}{400}}$$

$$= \frac{1}{2} \text{ rad/s.}$$

$$\omega_{man} = \frac{v}{R} = \frac{1}{2} \text{ rad/s.}$$

$$T = \frac{2\pi}{\omega_{rel.}} = \frac{2\pi}{\omega_p + \omega_m} = \frac{2\pi}{\frac{1}{2}+\frac{1}{2}} = 2\pi s.$$

Q. A bullet of mass 50 gm is fired with muzzle velocity 500 m/s towards a door which is 1 mt wide & mass is 12 kg & it is hinge by one side. V to Bullets hit @ the centre & get embedded. Find angular velocity of door just after embedding the bullet.



$$L_i = L_f$$

$$mvr = \left[\frac{Ml^2}{3} + mr^2\right]\omega.$$

$$\cancel{50} \times 10^{-\cancel{3}} \times 0.5 \times 5\cancel{00} = \left[\frac{\overset{4}{\cancel{12}} \times 1}{\cancel{3}} + 50 \times 10^{-3} \times 0.25\right]\omega$$

$$12.5 = [4 + 0.0125]\,\omega$$

$$\omega = \frac{12.5}{4.0125} \text{ rad/s.}$$

* angular spr = COAM
* ~~angule~~ vel. of com = COLM
collision

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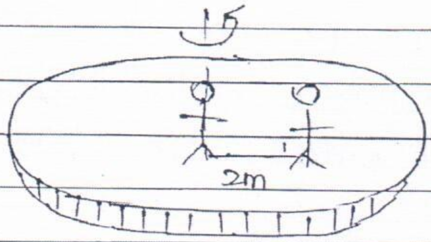
Q. A person of mass 100 kg is standing at the centre of a rotating circular platform of m.o.I. 1200 kgm^2. Platform is rotating at the rate of 1 revolutn in 10 sec. Now person starts walking towards the edge & stops at a distance 2 mt. from axis. Find new angular speed.

$10s \to 1$ rev
$1s \to \frac{1}{10}$ rev.

→ 

$L_i = L_f$

$I_1\omega_1 = I_2\omega_2$

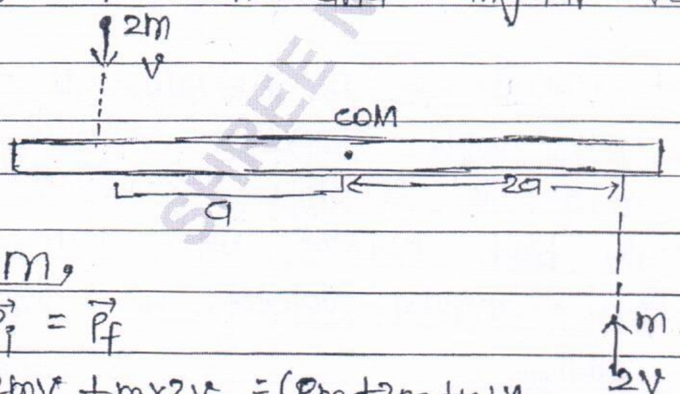
$1200 \times \frac{2\pi}{10} = [1200 + 100 \times (2)^2]\,\omega_2$

$\omega_2 = \frac{240\pi}{1600} = \frac{3\pi}{20} = \frac{3 \times 3.14}{20}$

$= \frac{9.42}{20} = 0.471$ rad/s

JEE M

Q. A uniform rod of mass 8m and length '6A' is placed on frictionless floor. Two particles of mass '2m' and 'm' moving with speed 'V' & 2V a/c to fig. colloide with the rod and sticks to it, find vel. of com and angular velocity just after collision.



particle st-line
pe to
angular mom
= mvr

COLM,

→ $\vec{P_i} = \vec{P_f}$

$-2mv + m \times 2v = (8m + 2m + m)V_{com}$

$V_{com} = 0.$

COAM,

$\vec{L_i} = \vec{L_f} \Rightarrow 2mva + m(2v)(2a) =$

$\left[\frac{8m(6a)^2}{12} + 2m(a)^2 + m(2a)^2\right]\omega_2$

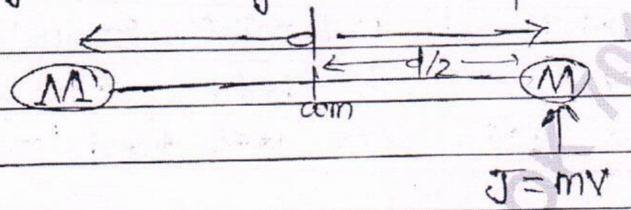
$$6mva = \left[\frac{8m \times \cancel{36}^{\,3} a^2}{\cancel{12}} + 6ma^2\right]\omega_2$$

$$6mva = 30^{5}\, ma^2 \omega_2$$

$$\boxed{\omega_2 = \frac{v}{5a}}$$

JEE Mains

Q. Two pt. mass, each of mass 'm' are connected with a thin rod of length 'd'. An impulse 'J = mv' is given to one particle a/c to the figure. Find vel. of com and angular velocity after impulse.



→ COLM,

$P_i = P_f$

$mv = 2m V_{com}$

$V_{com} = \frac{v}{2}$

COAM

$L_i = L_f$

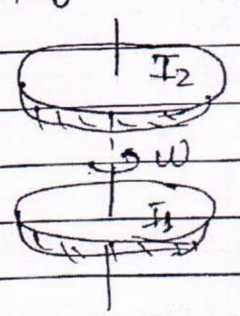
$mv\frac{d}{2} = 2m\left(\frac{d}{2}\right)^2 \omega_{com}$

$\omega_{com} = \frac{v}{d}$

Q. A uniform disk of M.O.I. 'I_1' is rotating in H.Z plane about geometrical axis with angular velocity 'ω'. Now another disk of M.O.I 'I_2' is dropped over I^{st} gently co-axially eventually both rotates with same angular velocity. Find final angular velocity & loss in K.E. during relative sliding.

work done against friction

→



$\tau_{ext} = 0$

COAM,

$\vec{L_i} = \vec{L_f}$

$I_1 \omega_1 = (I_1 + I_2)\omega_2$

$$\omega_2 = \frac{I_1 \omega_1}{I_1 + I_2}$$

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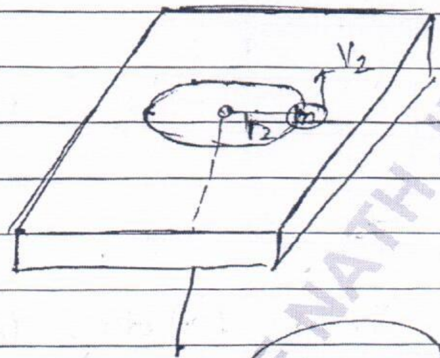
$$\boxed{\Delta K = \frac{1}{2}\frac{I_1 I_2}{I_1+I_2}(\vec{\omega_1}-\vec{\omega_2})^2}$$

$$\Delta K = \frac{1}{2}\frac{m_1 m_2}{(m_1+m_2)}(\vec{V_1}-\vec{V_2})^2$$

$$\boxed{\Delta K = \frac{1}{2}\frac{I_1 I_2}{I_1+I_2}\omega^2}$$

Q. A particle of mass 4 kg tied with a string rotated in a circle of radius 1 mt. with speed 4 m/s on a frictionless Hz surface a/c to the fig. string is passing through a hole at the centre and pulled by the other end. Find min^m radius of the circle upto which it can be pulled without breaking if the string can withstand a tension of 600 N.



$$T_{max.} = \frac{mV_2^2}{r_2}$$

$$\overset{150}{\cancel{600}} = \frac{\cancel{4}\times V_2^2}{r_2}$$

$$V_2 = \sqrt{150\, r_2}$$

→ COAM

Eq.4

$$L_i = L_f$$

$$\cancel{m_1}V_1 r_1 = \cancel{m_2}V_2 r_2$$

$$4\times 1 = \sqrt{150 r_2}\times r_2$$

$$r_2^3 = \frac{16}{150}$$

$$r_2 = \left[\frac{8}{75}\right]^{1/3} m.$$

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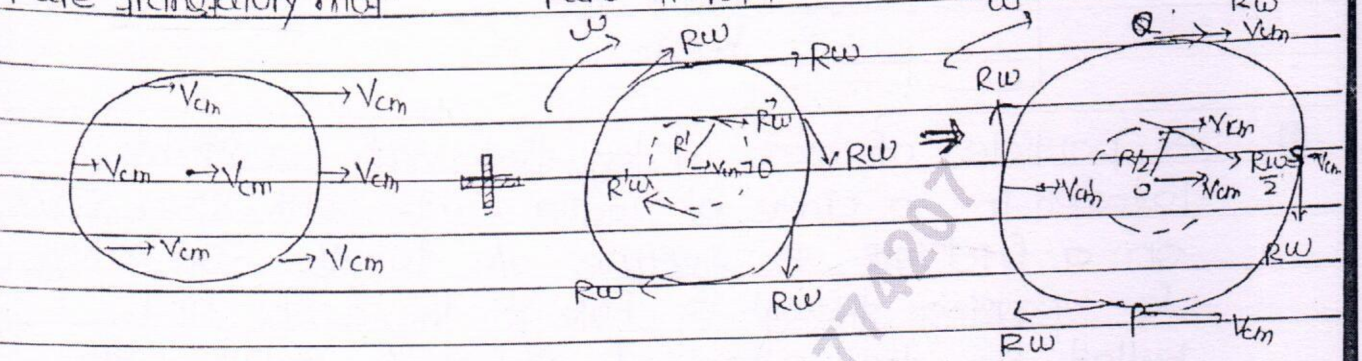
Rolling Motion :-

Pure translatory motn. Pure rotational motn Rolling motn.



$$\vec{V}_{net} = \vec{V}_{trans} + \vec{V}_{rot}$$

$$|\vec{V}_{net}| = \sqrt{V_{tran}^2 + V_{rot}^2 + 2V_{tran}V_{rot}\cos\theta}$$

$$\vec{V}_{net} = \sqrt{V_{cm}^2 + (R\omega)^2 + 2V_{cm}(R\omega)\cos\theta}.$$

$$\boxed{V_P = V_{cm} - R\omega}$$

$$\boxed{V_Q = V_{cm} + R\omega.}$$

$$\boxed{V_S = \sqrt{V_{cm}^2 + (R\omega)^2}}$$

Pure Rolling motn-

Rolling without slipping →

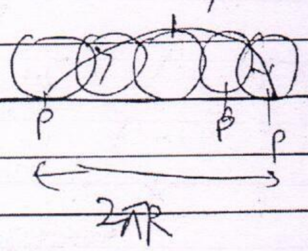
In pure rolling motn, the lowest pt is always to rest relative to ground,

means, $V_P = 0$

$$0 = V_{cm} - R\omega$$

$$\boxed{V_{cm} = R\omega}$$

Cycloid path



$|\vec{V}_{net}| = \sqrt{V_{cm}^2 + V_{cm}^2 + 2V_{cm}^2\cos\theta}$
$= V_{cm}\sqrt{2(1+\cos\theta)}$

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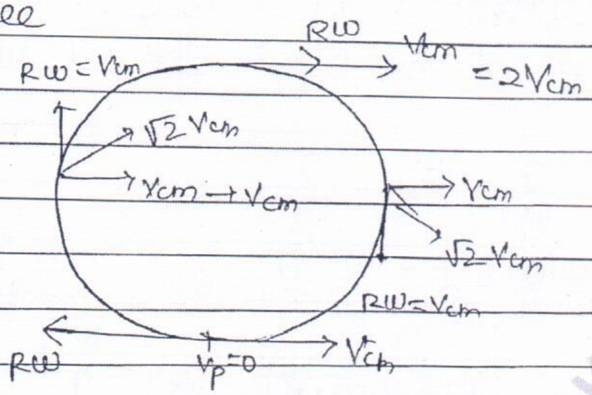
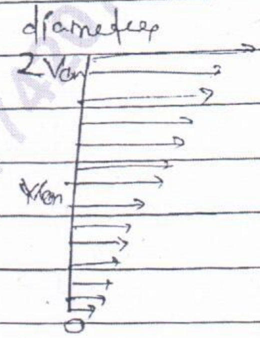
Pure rolling Motn $V_{cm} = R\omega$ distance travelled in one complete revolutn,
$x = 2\pi R$

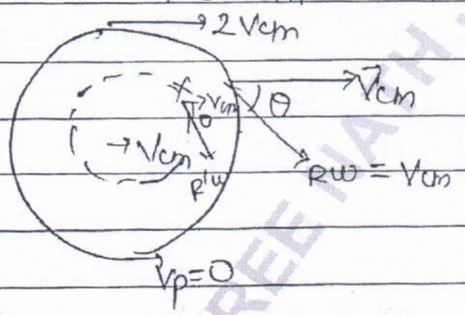
Rolling with slipping → $V_{cm} < R\omega$ · $x < 2\pi R$ wheel in mud.
→ $V_{cm} > R\omega$ $x > 2\pi R$ sudden breaks are applied.

Pure





* on pure rolling motn magnitude of velocity of a pt. on a circumference.

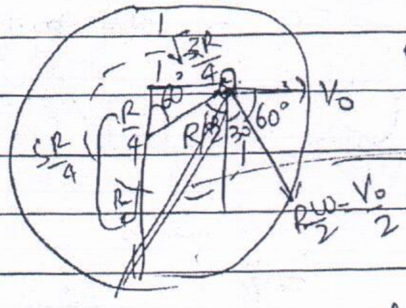


$\boxed{|\vec{V}_{net}| = 2V_{cm}\cos\theta/2}$

$\boxed{V_x = \sqrt{V_{cm}^2 + (R'\omega)^2 + 2V_{cm}(R'\omega)\cos\theta}}$

$R'\omega = V_0$

Q. Find magnitude of velocity of pt. 'Q'. Bodies in pure rolling motn. $V_{com} = V_0$



$V_Q = \sqrt{V_0^2 + (R/2\,\omega)^2 + 2V_0^2\cos 60^\circ}$ $\frac{V_0^2}{4}$

$= \sqrt{}$ → $\sqrt{\frac{25R^2}{16} + \frac{3R^2}{16}}$

$V_0 = R\omega$.

$= \sqrt{\frac{28}{16}R^2} = \frac{\sqrt{7}R\omega}{2}$

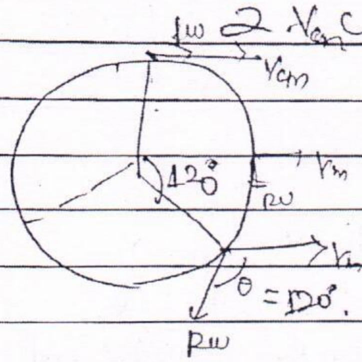
$V_Q = \frac{\sqrt{7}V_0}{2}$

$$V_Q = \sqrt{V_0^2 + \frac{V_0^2}{2} + \frac{2V_0^2}{2}\cos 60^\circ}$$

$$= V_0\sqrt{1+\frac{1}{4}+\frac{1}{2}} = V_0\sqrt{\frac{4+1+2}{4}} = \frac{\sqrt{7}}{2}V_0.$$

Q. Find pt. on circumference of a body rolling purely such that the speed of the pt. = the speed of com.

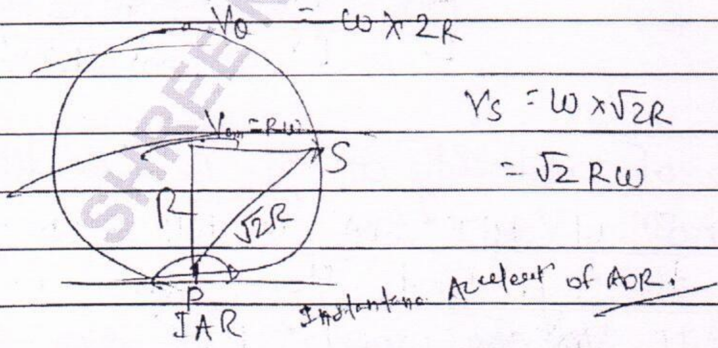
→ $2V_{cm}\cos\frac{\theta}{2} = V_{cm}$



$\cos\frac{\theta}{2} = \frac{1}{2} \Rightarrow \frac{\theta}{2} = 60^\circ \Rightarrow \theta = 120^\circ$

Vt. line से 120° पे.

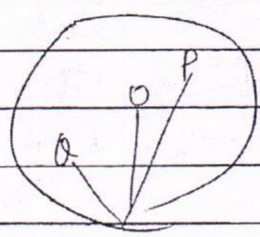
* If contact pt. with the ground is assumed to be axis of rotatn. then, the entire body can be considered in pure rotatn motn about this pt.



$V_S = \omega \times \sqrt{2}R$

$= \sqrt{2}R\omega$

Q.



$V_P > V_O > V_Q.$

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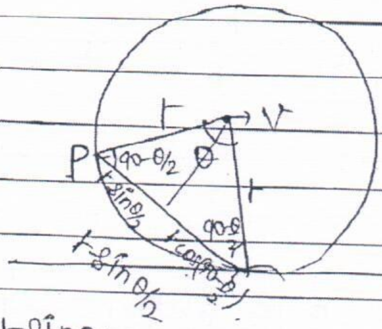
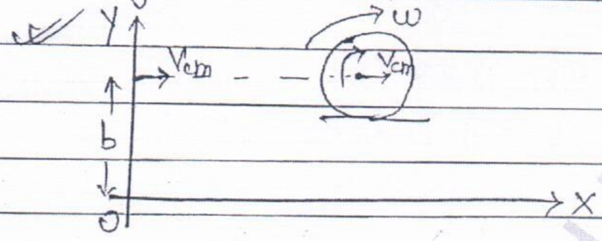
Q. Find magnitude of velocity of pt. 'P'.



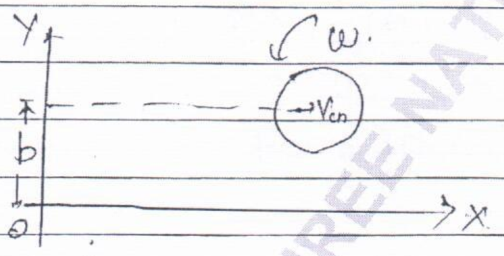
$= 2r\sin\theta/2$

$$V_P = \omega \times 2r\sin\theta/2$$
$$V_P = 2r\omega\sin\theta/2$$
$$V_P = 2v\sin\theta/2.$$

Angular momentum in Rolling Moth w.r.t. 'O'.

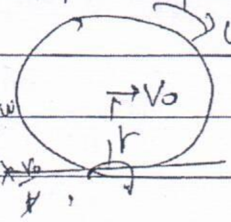


$$\vec{L_O} = \vec{L}_{trans.} + \vec{L}_{rot.}$$
$$\vec{L_O} = mV_{cm}b(-\hat{k}) + I_{cm}\omega(-\hat{k})$$
$$\vec{L_O} = (mV_{cm}b + I_{cm}\omega)(-\hat{k}).$$



$$\vec{L_O} = mV_{cm}b(-\hat{k}) + I_{cm}\omega(+\hat{k})$$
$$L_O = [I_{cm}\omega - mV_{cm}b](+\hat{k})$$

Q.1. A hollow sphere of mass 'm' is rolling on the surface purely a/c to fig. Find its angular momentum w.r.t. a pt. on ground, radius 'r'.



M.M.I.

$L = I_P\omega$

$= \frac{2}{3}mr^2 + mr^2 \omega$

$= \frac{5}{3}mr^{2} \times \frac{V_0}{r}$

$$\vec{L_O} = \vec{L}_{trans.} + \vec{L}_{rot.}$$
$$\vec{L_O} = \cancel{mV_0 r(-\hat{k}) + \frac{2}{3}mr^2(-\hat{k})}$$
$$= \cancel{mr\left(\frac{V_0}{1} + \frac{2r}{3}\right) = \frac{}{3}}$$

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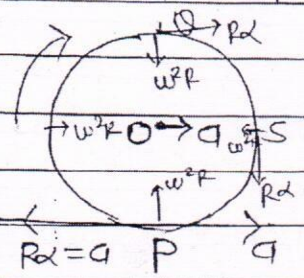
$$L_o = \left[mV_0 r \right] \times \frac{2}{3} m r^2 \times \frac{V_0}{r} \Big] (-\hat{k})$$

$$L_o = \left[\frac{5}{3} m V_0 r \right] (-\hat{k}).$$

$$L_o = \frac{5}{3} m V_0 r.$$

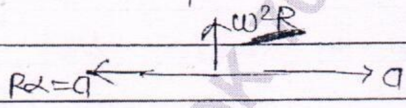
Q. A body of radius 'R' is in pure rolling motⁿ over the surface, accⁿ of center 'a'. Find accⁿ of pt. P, Q, S.



pure rolling motⁿ.

$a = r\alpha$.

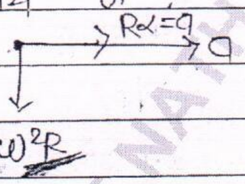
(1) At point 'P'.



$$\vec{a}_P = a\hat{i} - a\hat{i} + \omega^2 R\hat{j}.$$

$$\vec{a}_P = \omega^2 R\hat{j} \qquad |\vec{a}_P| = \omega^2 R.$$

(2) At point 'Q'

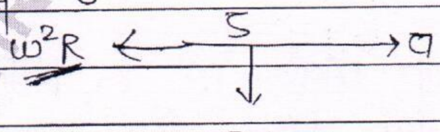


$$\vec{a}_Q = a\hat{i} + a\hat{i} - \omega^2 R\hat{j}$$

$$= 2a\hat{i} - \omega^2 R\hat{j}.$$

$$|\vec{a}_Q| = \sqrt{(2a)^2 + (\omega^2 R)^2}.$$

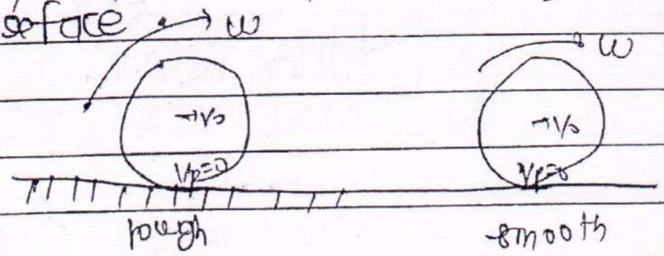
(3) At point 'S'



$$\vec{a}_S = a\hat{i} - \omega^2 R\hat{i} - a\hat{j}.$$

$$= (a - \omega^2 R)\hat{i} - a\hat{j}.$$

* Body may be in pure rolling motⁿ on a smooth the surface



pure rolling.

rough smooth
No फर्क
body roll करेगी.

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K.E. in Rolling motion :-

$$\underset{\frac{1}{2}mv^2}{K.E._{trans.}} + \underset{\frac{1}{2}I\omega^2}{KE_{rot.}}$$

$$\frac{1}{2}\frac{mk^2v^2}{R^2} \Rightarrow \boxed{KE_{roll.} = \frac{1}{2}mv^2\left[1+\frac{k^2}{R^2}\right]}$$

$$\frac{1}{2}mv^2\left(\frac{k^2}{R^2}\right)$$

$$\boxed{\begin{aligned} I &= cmR^2 \\ \cancel{m}k^2 &= c\cancel{m}R^2 \\ \frac{k^2}{R^2} &= c \end{aligned}}$$

	Ring	disc	solid sphere	hollow sphere	s. cylinder	h. cyli.
$\frac{k^2}{R^2}$	1	$1/2$	$2/5$	$2/3$	$\frac{1}{2}$	1.
	1	0.5	0.4	0.66	0.5	1

$$K.E._{trans.} : KE_{rot.} : KE_{roll.}$$
$$1 \quad : \quad k^2/R^2 \quad : \quad 1+\frac{k^2}{R^2}.$$

$$\text{Fract}^n \text{ of translatory} = \frac{1}{1+k^2/R^2}.$$

$$\text{Fract}^n \text{ of rotat}^n = \frac{k^2/R^2}{1+k^2/R^2}.$$

Q. A hollow sphere is rolling purely on H.z surface, find %age of rot. K.E.

$$\text{Fract}^n \text{ of rot}^n = \frac{2/3}{1+2/3}\times 100 = \frac{2}{5}\times 100$$

$$= 40\% \text{ rot.}$$

so 60 % trans.

ASP sem

Vaibhav

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Q. A body of mass 10 kg is rolling without slipping on the surface. velo of com 2 m/s. If its KE is 33.2 Joule identify the body.

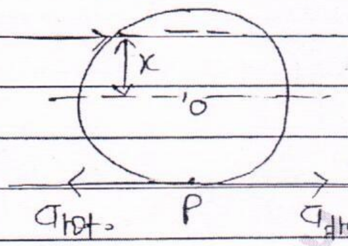
→ $$KE = \frac{1}{2}mv^2\left[1+\frac{k^2}{R^2}\right]$$

$$\cancel{33.2}^{16.6} = \frac{1}{2}\times 10\times 4^2\left[1+\frac{k^2}{R^2}\right]$$

$$1.66 = 1+\frac{k^2}{R^2}$$

$$\frac{k^2}{R^2} = 0.66 = \frac{2}{3}$$ Hollow sphere.

Dirn of frictn :-



$$a_{trans.} = \frac{F}{M}$$

$$a_{rot.} = R\alpha = R\frac{\tau}{I} = \frac{R\times Fx}{Mk^2}$$

$$a_{rot.} = \frac{F}{M}\frac{Rx}{k^2}$$

$$1 - \frac{Rx}{k^2} > 1$$

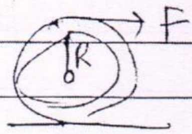
$$1 > \frac{Rx}{k^2}$$

$$|\vec{a}_p| = \frac{F}{m} - \frac{Rx}{k^2}$$

$$= \frac{F}{m}\left[1-\frac{Rx}{k^2}\right]$$

$\boxed{k^2 > Rx}$	frictn opposite to F or backward.
$\boxed{k^2 < Rx}$	frictn along F or forward.
$\boxed{k^2 = Rx}$	No frictn.

$x = 0$, $a_{net} = \frac{F}{m}$ backward



$$\frac{k^2}{R^2} = 1 \qquad \frac{Rx}{R^2}$$

$$k^2 = R^2$$

Agar force com के नीचे then body hamesha पीछे जायेगा . so fricⁿ आगे होगी,

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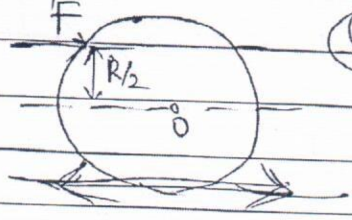
Q. Find the dirn of fricⁿ in the following situatⁿ, if the body is ring, disc, solid sphere and hollow sphere.

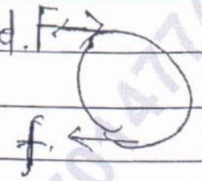


(1) Ring $\quad k^2 \quad Rx$

$\frac{k^2}{R^2} = 1, \quad k^2 = R^2 \quad R \times \frac{R}{2} = \frac{R^2}{2}$

$k^2 > Rx$.

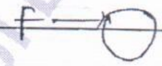
backward.



(2) disc $\quad k^2$

$\frac{k^2}{R^2} = \frac{1}{2} \quad k^2 = \frac{R^2}{2}, \quad k^2 = R \quad Rx \quad \frac{R^2}{2}$

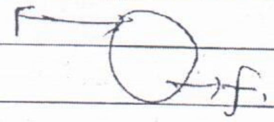
$k^2 = Rx \rightarrow$ No fricⁿ.



(3) solid sphere,

$\frac{k^2}{R^2} = \frac{2}{5} \quad Rx \quad \frac{R^2}{2} \quad k^2 < Rx$ forward.

$k^2 = 0.4R^2. \quad = 0.5R^2$



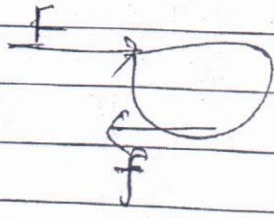
(4) Hollow sphere.

$\frac{k^2}{R^2} = \frac{2}{3} \quad k^2 = \frac{2}{3}R^2$

$= 0.66R^2. \quad 0.5R^2$

$k^2 > Rx$

backward,



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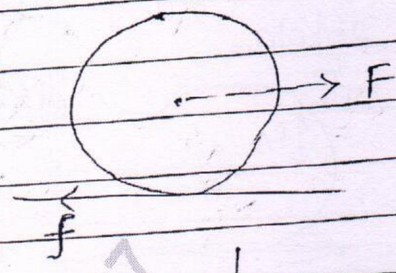
Q. If force 'F' is applied on a solid sphere a/c to fig. Which is rolling purely. find –

(1) dirn of fricth.

(2) value of fricth.

(3) acch of center.



$F - f = ma$ —(1)

$F - \frac{2}{5} ma = Ma$.

$F = \frac{7}{5} ma$.

$$\boxed{a = \frac{5}{7} \frac{\cancel{ma}F}{m}}$$

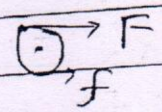
$\tau = I\alpha$

$f\cancel{R} = \frac{2}{5} MR^{\cancel{2}} \frac{a}{\cancel{R}}$

$f = \frac{2}{5} Ma$ —(2)

$f = \frac{2}{5} \cancel{m} \times \frac{5}{7} \frac{F}{\cancel{m}}$

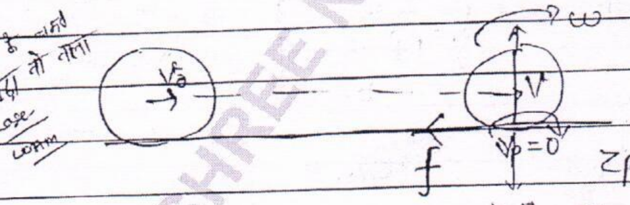
$$\boxed{f = \frac{2}{7} F}$$



$F + f = ma$ —(1)

$FR - fR = I\alpha$.

**Q. A solid sphere is projected on rough surface with speed V_0. after sometime it starts rolling purely. When it starts pure rolling motn. Find speed of center.



$\tau_p = 0$

$\tau_{ext.} = 0$

about P

COAM.

$\vec{L_i} = \vec{L_f}$

$mV_0R = I_p\omega$.

$mV_0R = \left[\frac{2}{5} mR^2 + mR^2\right]\frac{V}{R}$

$\cancel{m}V_0\cancel{R} = \frac{7}{5} \cancel{m}R^{\cancel{2}} \times \frac{V}{\cancel{R}}$

$$\boxed{V = \frac{5V_0}{7}}$$

vaibhav KF

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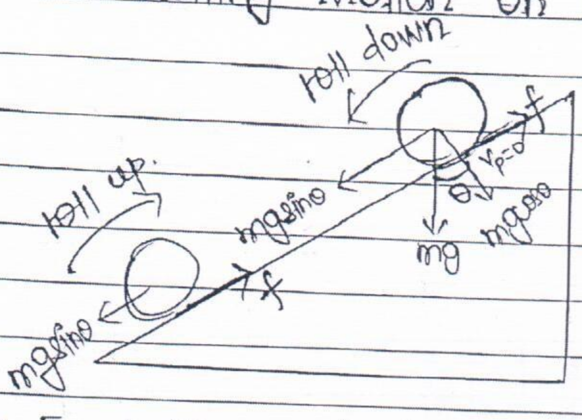
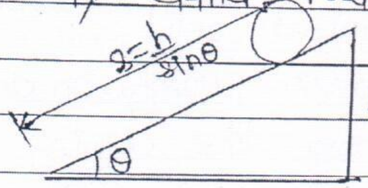
Pure Rolling Motion on inclined plane :-



→ For rolling without slipping on inclined plane, frictⁿ (static) is necessarily reqd.

→ Work done by frictⁿ is zero, because point of actⁿ has not relative displacement.

→ Body either roll up or roll down, frictⁿ always upwards.



(1) speed on reaching bottom-

COME.

Loss in P.E. = Gain in KE_{roll}.

$$mgh = \frac{1}{2}mv^2\left[1+\frac{K^2}{R^2}\right]$$

$$\boxed{V_{roll.} = \sqrt{\frac{2gh}{1+\frac{K^2}{R^2}}}}$$

$\boxed{V_{roll} < V_{slide}}$

$$V_{slide} = \sqrt{2gh}.$$

(2) accⁿ.

$$v^2 = u^2 + 2as.$$

$$\frac{2gh}{1+\frac{K^2}{R^2}} = 0 + 2a \times \frac{h}{\sin\theta}.$$

$\boxed{a_{roll} < a_{slide}}$

$$\boxed{a_{roll.} = \frac{g\sin\theta}{1+K^2/R^2}}$$

$$a_{slide} = g\sin\theta$$

3. time taken to reach bottom -

$$s = ut + \frac{1}{2}at^2.$$

$$\frac{h}{\sin\theta} = 0 + \frac{1}{2}\,\frac{g\sin\theta}{1+\frac{K^2}{R^2}}\,t^2.$$

$$\boxed{t_{roll.} = \frac{1}{\sin\theta}\sqrt{\frac{2h}{g}\left(1+\frac{K^2}{R^2}\right)}}$$

$$t_{sliding} = \frac{1}{\sin\theta}\sqrt{\frac{2h}{g}}.$$

$$\boxed{t_{roll.} > t_{slide.}}$$

Q. A Ring, disc, solid sphere, hollow sphere, solid cylinder, hollow cylinder are left to roll down purely on an inclined plane from same ht. arrange the coming first to last on bottom.

→ $t \propto \sqrt{1+\frac{K^2}{R^2}}$

first

Solid sphere > disc/~~hollow~~ solid cylind. > H. sphere > Ring/hollow cylind >

Ring

last

Q. A hollow sphere rolls down purely on an rough inclined plane and then slides down on an smooth identical inclined plane. Find ratio of accn in both case -

→ $a = \frac{g\sin\theta}{1+2/3}.$

$$\frac{a_{roll} = \frac{3}{5}\cancel{g\sin\theta}}{a_{slide.} = \cancel{g\sin\theta}} = \frac{3}{5}$$

comm roll 9:30 - 11:30 Andy.

OFFENCE IS THE BEST DEFENCE.

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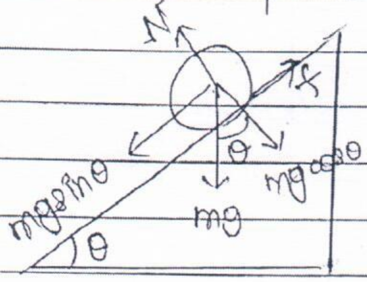
**

* condition for pure rolling motⁿ on inclined plane -



$$mg\sin\theta - f = ma.$$

$$mg\sin\theta - \frac{mg\sin\theta}{1+\frac{K^2}{R^2}} = f.$$

$$f = mg\sin\theta\left[\frac{1+K^2/R^2 - 1}{1+K^2/R^2}\right]$$

$$= mg\sin\theta\left[\frac{1}{\frac{1+K^2/R^2}{K^2/R^2}}\right]$$

$$f = mg\sin\theta\left[\frac{1}{\frac{1}{K^2/R^2}+1}\right]$$

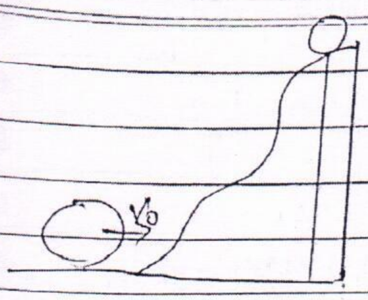
$$f \le f_l$$

$$mg\sin\theta\left[\frac{1}{R^2/K^2+1}\right] \le \mu mg\cos\theta.$$

$$\frac{\tan\theta}{R^2/K^2+1} \le \mu.$$

$$\boxed{\mu \ge \frac{\tan\theta}{1+R^2/K^2}} \qquad \boxed{\mu \ge \frac{\tan\theta}{1+\frac{1}{K^2/R^2}}}$$

Q. A body is rolling purely on Hz. surface, speed V_0, now it rolls up on an irregular inclined plane upto ht. $3V_0^2/4g$. identify the body.



COME,

Loss in $KE_{roll.}$ = Gain in P.E.

$$\frac{1}{2} m V_0^2 \left[1 + \frac{K^2}{R^2}\right] = mg \times \frac{3V_0^2}{4g}.$$

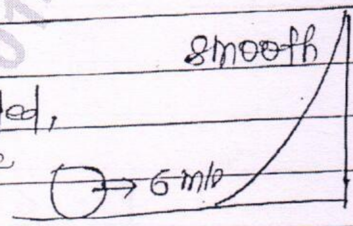
$$1 + \frac{K^2}{R^2} = \frac{3}{2}$$

$$\frac{K^2}{R^2} = \frac{1}{2} \Rightarrow \text{disc/ solid cylinder.}$$

Q. A disc is rolling purely on Hz. surface, speed of center 6 m/s. Find the max^m ht. upto which it move up on smooth inclined plane.

⇓

so, rotational K.E. doesnot affected, only translatory k.E. is lost due to work done by gravity.



COME,

$$mgh = KE_{trans.}$$

$$mgh = \frac{1}{2} m (6)^2$$

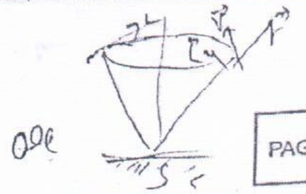
$$h = \frac{1}{2 \times 10} \times 36 = 1.8 \text{ m.}$$

[pt. to ponder 1424 JEE-main-14.]

Q. A simple Pendulum of length 'l'. mass of bob small 'm' is moving as a conical pendulum, a/c to fig. with const. angular velocity 'ω' about vertical. It

Angular momentum related to pt. of suspension continuously changes its orientation in space, so it is variable but its magnitude is const.

EX-II max fTY↓



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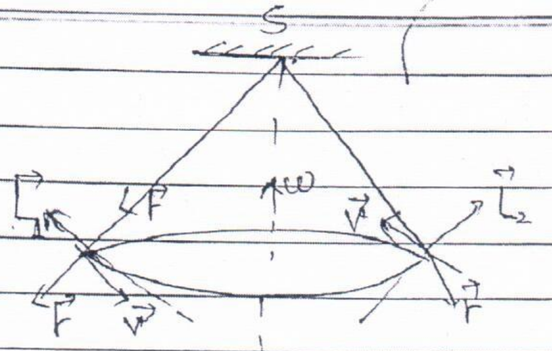
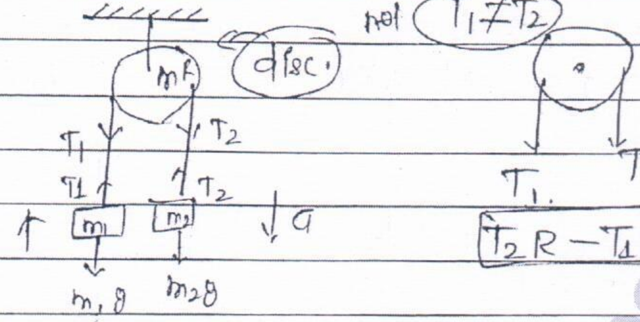


L = variable, due to change in dirⁿ.
but mag. is const.
$\vec{L}_S = mvl\,(\hat{n})$.
$\hat{n}$ is ⊥ʳ to the plane of $\vec{r}$ & $\vec{v}$.

so, $\vec{L}$ & $\vec{\omega}$ may not be in same directⁿ.

#

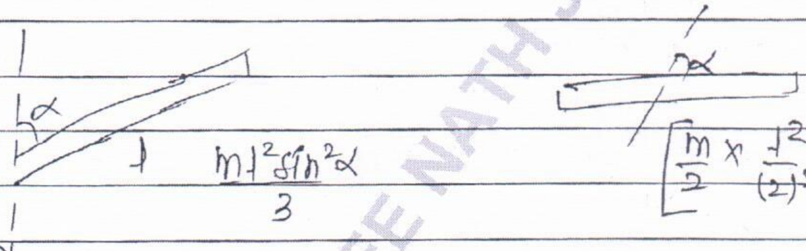


not $T_1 \neq T_2$

$\boxed{T_2 R - T_1 R = I\alpha}$

$T_1 m_1 g = m_1 a$ $\quad \boxed{m_2 g - T_2 = m_2 a}$

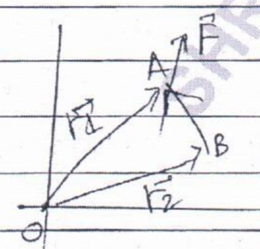
#



$\frac{ml^2 \sin^2\alpha}{3}$

$\left[\frac{m}{2} \times \frac{l^2}{(2)^2} \frac{\sin^2\alpha}{3}\right] \times 2$

EX-I



$\vec{F}, \vec{r}$

$r_2 + \vec{BA} = \vec{r}_1$

$\vec{BA} = \vec{r}_1 - \vec{r}_2$

$\vec{BA} \times \vec{F}$

$\vec{F} = 2\hat{i} - 3\hat{k}$

$\vec{r}_1 = 0.5\hat{j} - 2\hat{k}$

$\vec{r}_2 = 2\hat{i} - 3\hat{k}$

$2\hat{i} + 0.5\hat{j} + \hat{k}$

$2\hat{i} + 0\hat{j} - 3\hat{k}$

$= \hat{i}(-4.5 - 0) - \hat{j}(6 - 2) + \hat{k}(0 - 1)$.

GRAVITATION

⇒ Natural Phenomenon in which 2 physical bodies attract each other

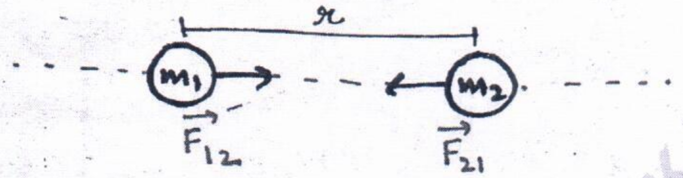
Newton's Law of Gravitation

↳ Any 2 particles [point-sized mass] in the universe exert mutually attractive force on each other it follows Newton's 3rd law

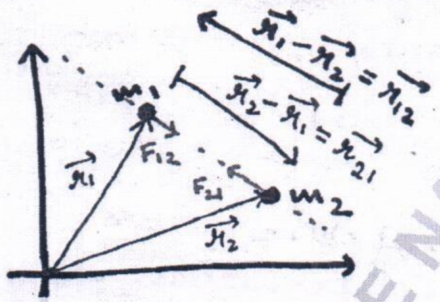
↳ Always attractive.

↳ Magnitude of force $\propto$ product of masses.
$\propto \dfrac{1}{\text{sq. of dist. b/w the masses}}$ #

↳ gravitation force b/w 2 particle always acts along the line joining the 2 particles.



$$|\vec{F}_{12}| = |\vec{F}_{21}| \propto \frac{m_1 m_2}{r^2} = \frac{G m_1 m_2}{r^2}$$

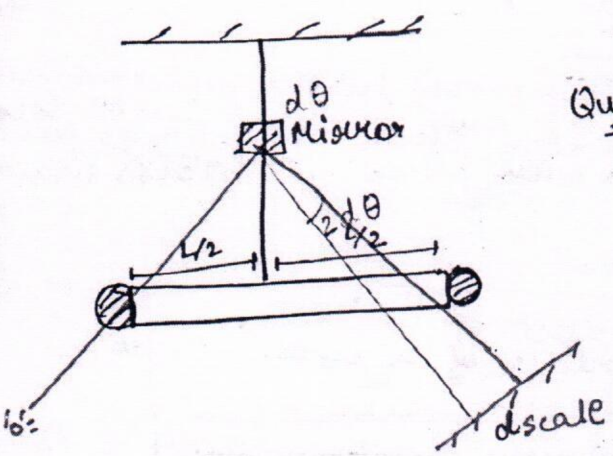


$$F_{12} = \frac{G M_1 M_2}{|\vec{r}_{21}|^2} = F_{21} = \frac{G M_1 M_2}{|\vec{r}_{12}|^2}$$

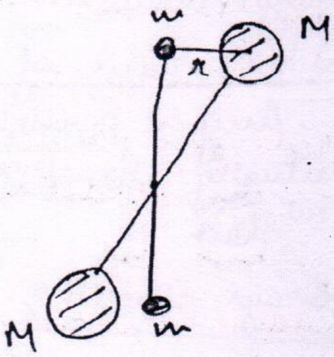
$$\boxed{\vec{F}_{12} = \frac{G M_1 M_2}{|r_{21}|^3} \cdot \vec{r}_{21}} \qquad \boxed{\vec{F}_{21} = \frac{G M_1 M_2}{|r_{12}|^3} \cdot \vec{r}_{12}}$$

$$G = 6.67 \times 10^{-11} \frac{N\text{-}m^2}{Kg^2} = 6.67 \times 10^{-8} \frac{dyne\text{-}cm^2}{gm^2}$$

Cavendish $\Rightarrow$ determined value of G.



Quartz



$$F = -Kx$$
$$\tau = -C\theta$$

$$2\theta x = d$$
$$\theta = \frac{d}{2x}$$

$$2 \times \frac{FL}{2} = -C\frac{d}{2x}$$

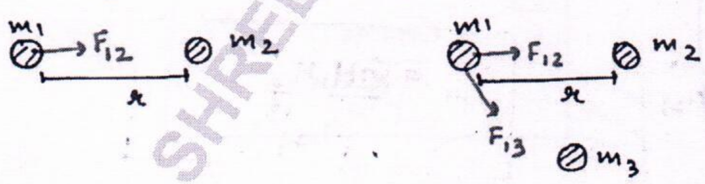
$$\frac{Gm_1m_2}{r^2} \cdot L = \frac{-C \times d}{2x}$$

$$\boxed{\therefore G = \frac{-Cdr^2}{2xm_1m_2}}$$

Properties of G. Force

↳ doesn't depend on medium

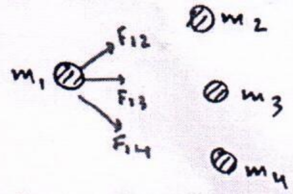
↳ gravitation force is not affected by (+) of other masses.



$\therefore$ Net force on particle changes.

But F_{12} remains same.

#

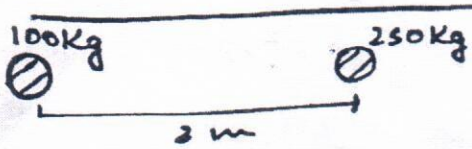


$\Rightarrow$ G. Force follows principle of superposition.

↳ It is a conservative force.

↳ It is a weak force
↳ $\bar{c}$ long range order
range = ∞

Q Find acc. of both particles.

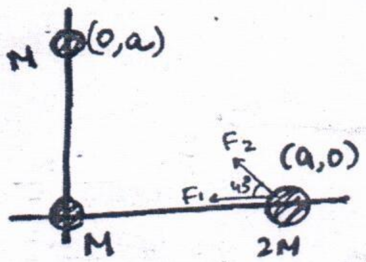


$$F = \frac{6.67\times10^{-11}\times100\times250}{4} = 6.67\times6.25\times10^{-8} =$$

$$a_1 = \frac{F}{100} = 6.67\times6.25\times10^{-6}$$

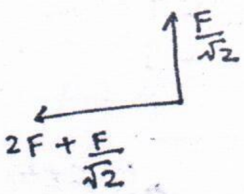
$$a_2 = \frac{F}{250} = 6.67\times0.25\times10^{-7}$$

Q Find force on 2m.



$$F_1 = \frac{G\,2m\cdot m}{a^2} = \frac{G\,2m^2}{a^2} = 2F$$

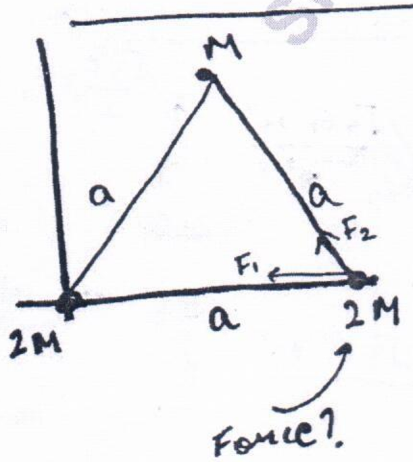
$$F_2 = \frac{G\,2m\cdot m}{2a^2} = \frac{Gm^2}{a^2} = F$$



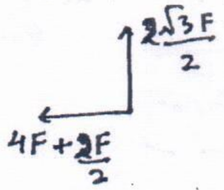
$$\vec{F} = -\left(2F+\frac{F}{\sqrt{2}}\right)\hat{i} + \frac{F}{\sqrt{2}}\hat{j}$$

$$= -\left(\frac{2\sqrt{2}\,GM^2}{\sqrt{2}a^2} + \frac{Gm^2}{}\right)\hat{i} + \left(\frac{Gm^2}{a^2\sqrt{2}}\right)\hat{j}$$

Q



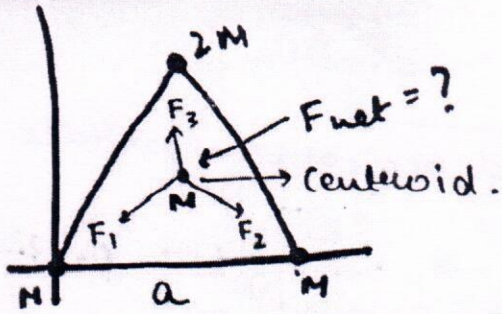
$$F_1 = \frac{G\,4m^2}{a^2} = 4F \quad F_2 = \frac{2GM^2}{a^2} = 2F$$



$$\vec{F} = -\left[4F+1F\right]\hat{i} + \left[\frac{3\sqrt{3}F}{2}\right]\hat{j}$$

$$= -\frac{5GM^2}{a^2}\hat{i} + \frac{3\sqrt{3}GM^2}{a^2}\hat{j}$$

Q



$$F_1 = \frac{GM^2}{a^2} \times 3 \Rightarrow F$$

$$F_2 = \frac{GM^2 3}{a^2} \Rightarrow F$$

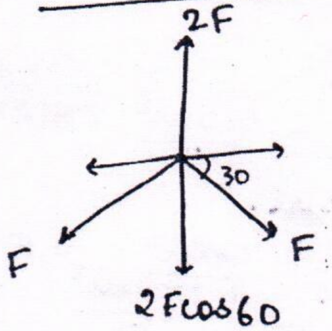
$$F_3 = \frac{G 2M^2}{a^2} \times 3 \Rightarrow 2F.$$

$$\sqrt{a^2 - \frac{a^2}{4}} = \frac{a}{2}\sqrt{3} \times \frac{2}{3}$$

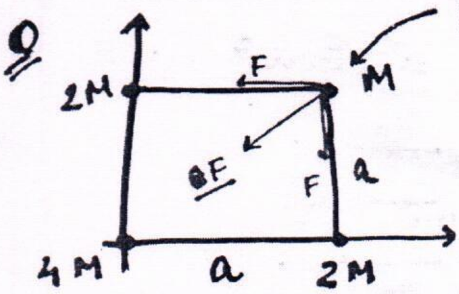
$$= \frac{a}{\sqrt{3}}$$

$$\vec{F}_{net} = 2F - 2F\cos 60$$

$$= F = \left(\frac{3GM^2}{a^2}\right)\hat{j}$$



Q



$$\frac{F+F}{\sqrt{2}} \qquad \frac{F}{\sqrt{2}} + F$$

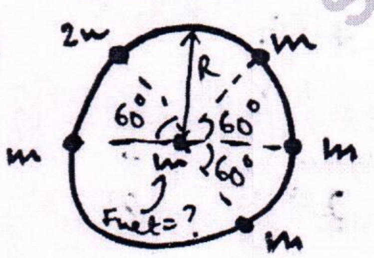
$$F = \frac{G\,2M^2}{a^2}$$

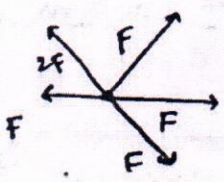
$$\frac{F}{\sqrt{2}} = \frac{GM^2}{\sqrt{2}\,a^2}$$

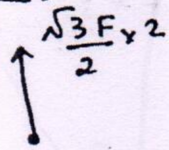
$$\vec{F}_{net} = -$$

$$= \frac{-2GM^2}{a^2}\left[1 + \frac{1}{\sqrt{2}}\right]\left[\hat{i} + \hat{j}\right]$$

Q



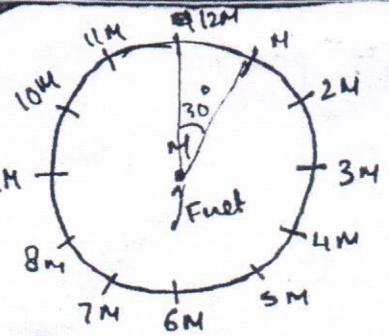


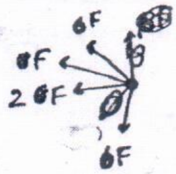


$$F = \frac{Gm^2}{R^2}$$

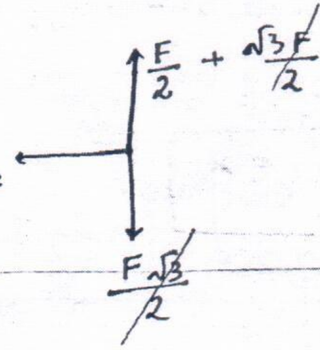
$$\sqrt{3}F\hat{j} = F_{net} = \frac{\sqrt{3}\,G\,m^2}{R^2}\hat{j}$$

Q





$$F = \frac{6M^2G}{a^2}$$

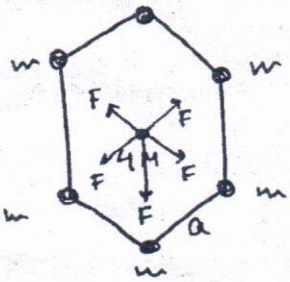


$$\frac{6F+\sqrt{3}F}{2} = \frac{F}{2} + \frac{F}{2} + \frac{\sqrt{3}F}{2} + 2F$$

$$\frac{\overset{18}{\cancel{36}} + \overset{3}{\cancel{6}}\sqrt{3}}{2}$$

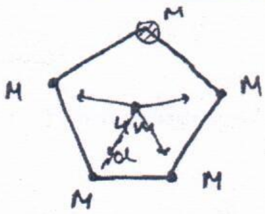
$$\vec{F}_{net} = \left(\frac{3M^2G}{a^2}\right)\hat{j} - \left(\frac{(18+3\sqrt{3})GM^2}{a^2}\right)\hat{i}$$

Q



$$\vec{F}_{net} = \frac{4M^2G}{a^2}\,\hat{j}$$

Q



$$F = \frac{4GM^2}{d^2}$$

If one more mass (+)

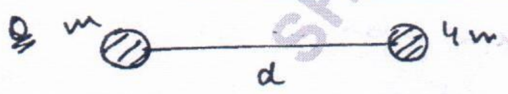
$F_{net} = 0$.

$$\vec{F_1} + \vec{F_2} + \vec{F_3} + \vec{F_4} + \vec{F_5} = 0$$

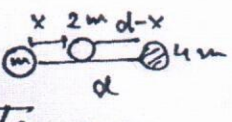
$$\vec{F_1} \ldots \vec{F_4} = -\vec{F_5}$$

$$\vec{F}_{net} = -\frac{4GM^2}{d^2}$$

Q



where should a 3rd mass = 2m be placed so that it stays in eqm?

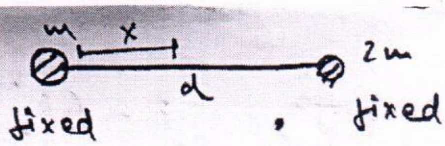


$$\frac{G8m^2}{(d-x)^2} = \frac{G2m^2}{x^2}$$

$$\frac{4}{(d-x)^2} = \frac{1}{x^2}$$

$$2x = d - x$$

$$x = d/3$$

Q 

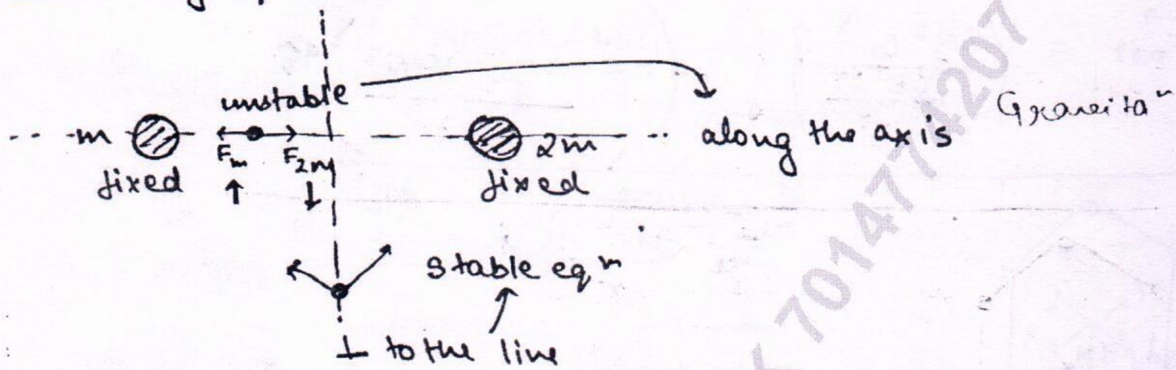
Find eq^m $posi^n$.

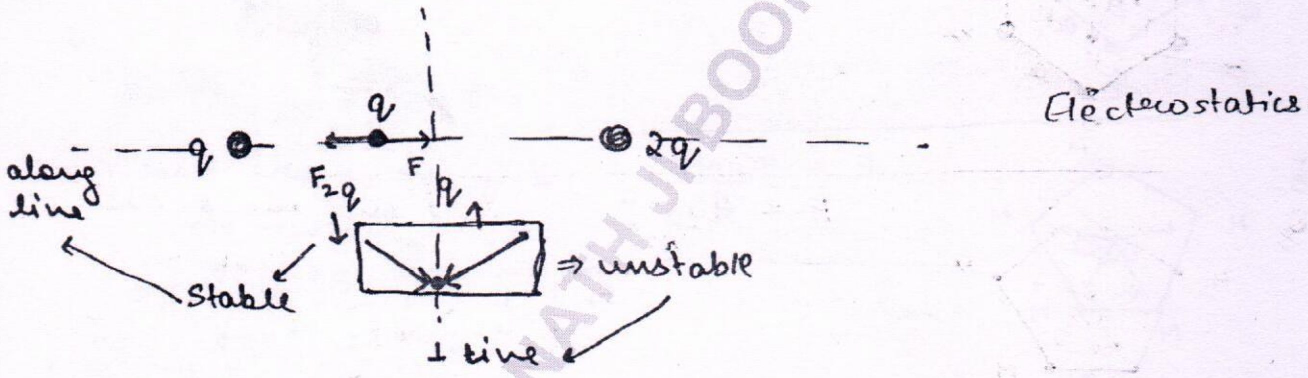
$$\frac{Gm}{x^2} = \frac{G2m}{(d-x)^2}$$

$$d-x = \sqrt{2}\,x$$

$$\boxed{\frac{d}{1+\sqrt{2}} = x}$$

<u>Nature of Eq^m</u>





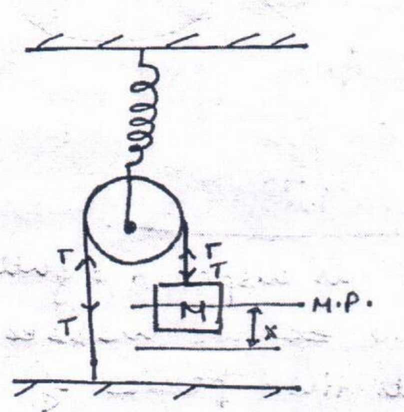
To find charge to put in eq^m b/w 2 charges →

a) Find eq^m $posi^n$.

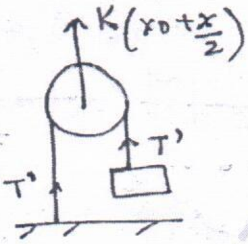
b) Balance forces on either of 2 given charges.

Time Period of oscilla^n.

$$T = 2\pi\sqrt{\frac{M}{K}}$$



$T = Mg$, $2T = Kx_0 \rightarrow mg = \frac{Kx_0}{2}$



$$2T' = K\left[x_0 + \frac{x}{2}\right]$$

1. Mean Posi^n Force rel^n
2. Displace from M.P.
3. Convert Fnet in the form of $[-Kx]$

$\underbrace{-K}$ Const. of SHM = Force const.

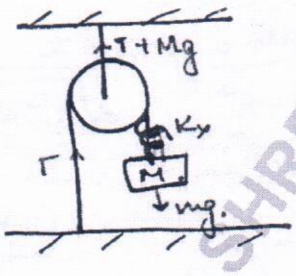
$$F_{net} = Mg - T'$$

$$= \cancel{Mg} - \left[\cancel{\frac{K}{2}x_0} + \frac{Kx}{4}\right]$$

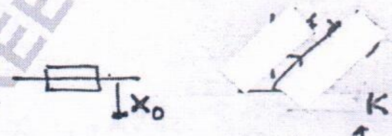
$$F_{net} = \boxed{\frac{-K}{4}}x$$

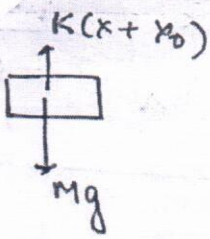
$$\rightarrow T = 2\pi\sqrt{\frac{m}{K/4}}$$

Q Find time period oscilla^n. Spring const. $= K$.



$Mg = Kx$

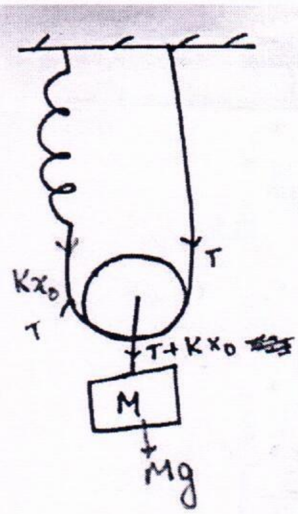




$$F_{net} = Mg - K(x_0 + x)$$

$$F_{net} = -Kx$$

$$\therefore T.P = 2\pi\sqrt{\frac{m}{K}}.$$

Q 

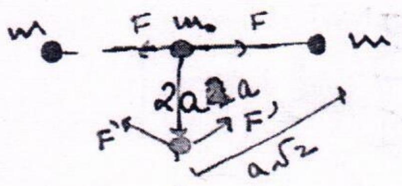
Find time Period.

$T + Kx_0 = Mg = 2T = 2Kx_0$

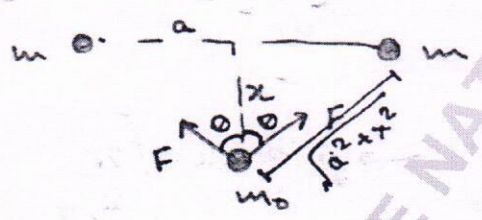
$\Rightarrow 2K(x_0 + 2x) - Mg = F_{net}$

$\Rightarrow \quad 4Kx = F_{net}$

$TP = 2\pi\sqrt{\frac{m}{4K}}$

Q 

If the m_0 is displaced slightly $\perp$ the line ~~the~~ two masses then find its time period of oscillation.



$F_{net} = 2F\cos\theta$

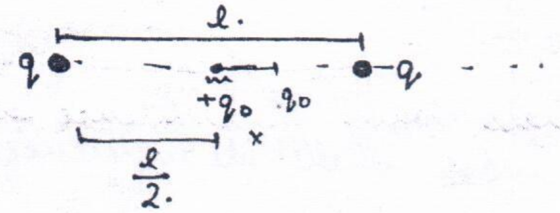
$= 2\,\frac{GMm_0}{a^2+x^2}\cdot\frac{x}{\sqrt{a^2+x^2}}$

$= \frac{2GMm_0\cdot x}{(a^2+\cancel{x^2})^{3/2}} = \frac{2Gmm_0}{a^3}\cdot x$

$\therefore K = \frac{2GMm_0}{a^3}$

$T = 2\pi\sqrt{\frac{a^3}{2GM}}$

Q



$$\frac{Kqq_0}{\left(\frac{l}{2}-x\right)^2} - \frac{Kqq_0}{\left(\frac{l}{2}+x\right)^2} = F_{net}$$

$$Kqq_0\left[\frac{4}{(l-2x)^2} - \frac{4}{(l+2x)^2}\right] = F_{net}$$

$$4Kqq_0\left[\frac{(l+2x)^2-(l-2x)^2}{[(l-2x)(l+2x)]^2}\right] = F_{net}$$

$$4Kqq_0\left[\frac{(l+\cancel{2x}+l-\cancel{2x})(\cancel{l}+2x-\cancel{l}+2x)}{[l^2-4x^2]^2}\right] = F_{net} = -K'x$$

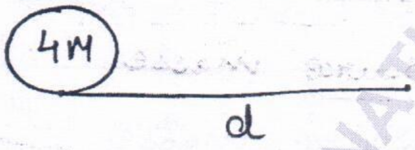
$$4Kqq_0\left[\frac{2l[4x]}{[l^2-\cancel{4x^2}]^2}\right] = -K'x$$

$$\frac{4Kqq_0 2l \times 4}{l^{4}3}[x] = -K'x$$

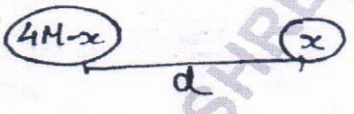
$$K' = -\frac{32Kqq_0}{l^3}$$

$$T = 2\pi\sqrt{\frac{m \cdot l^3}{32Kqq_0}}$$

Q



What amt. should be detached from 4M and placed at a dist. d so that Force b/w 2 fragments = max.



$$\Rightarrow \frac{Gm_1m_2}{r^2} = \frac{G[4M-x][x]}{d^2}$$

$$= \frac{G[-x^2+4Mx]}{d^2}$$

$$\boxed{-x^2+4Mx} \Rightarrow max.$$

$$\frac{dF}{dx} = -2x + 4M = 0$$

$$4M = 2x$$

$$\boxed{2M = x}$$

GRAVITATIONAL FIELD

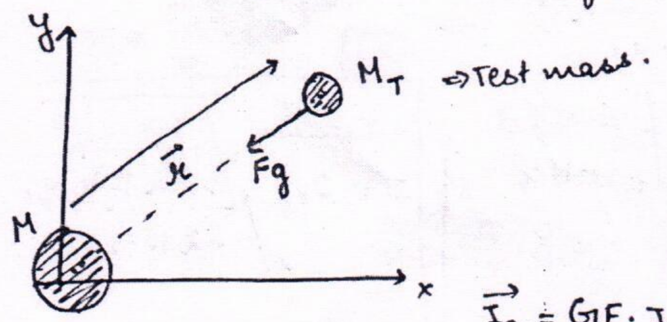
⇒ It is a region around a mass where its influence can be felt.

Gravitaⁿ field intensity / Strength $[I_g] \simeq [g]$

1. ⇒ Graviⁿ F.I = gravitaⁿ force per unit test mass.

   ⇒ Its direcⁿ always towards source mass.

2. ⇒ G.F.I. is numerically = G.Force on a unit mass in the graviⁿ field of source mass for the same distance.



$$F_G = \frac{GMM_T}{r^2} = \frac{GMM_T(-\vec{r})}{r^3}$$

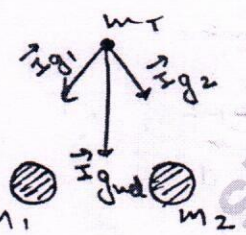
$$\vec{I_g} = GF.I = \frac{\vec{Fg}}{M_T} = \frac{GM}{r^2} = \vec{g}$$

Can be defined for a region [$\bar{c}$ out mass]

Identical but not same

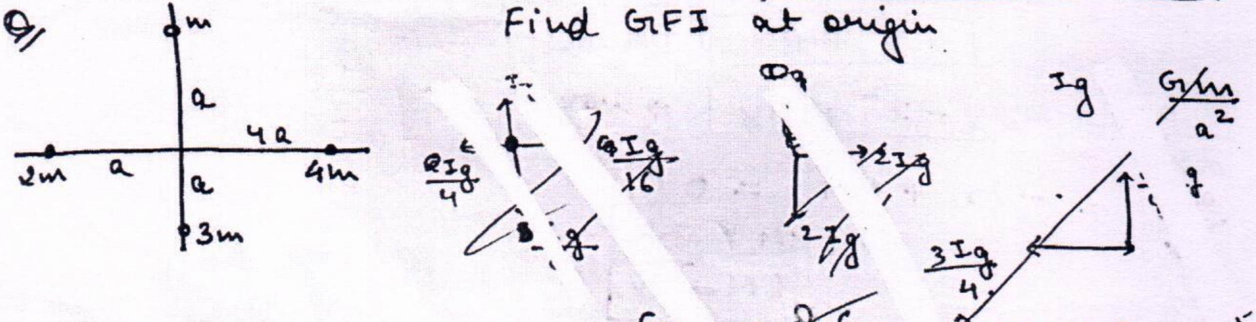
⇒ can be defined only for a mass

$\vec{F}_{net}$ on test mass in a sys. of source masses

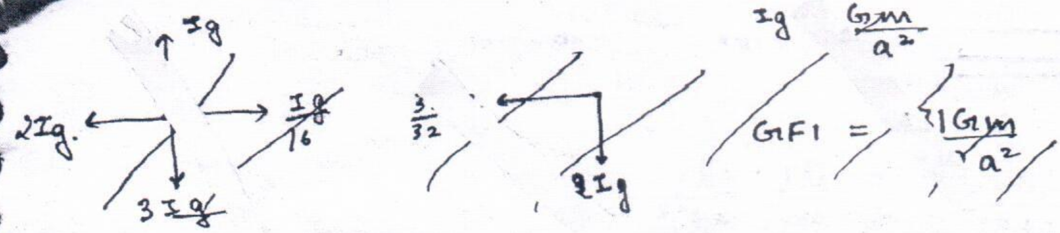


$$\vec{F}_{net} = \vec{I}_{gnet} \times m_T$$

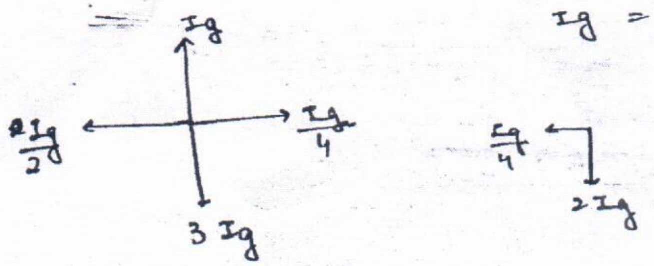
Q/ Find GFI at origin



$G = \frac{2Gm}{} \hat{i} - 2Gm \ldots$

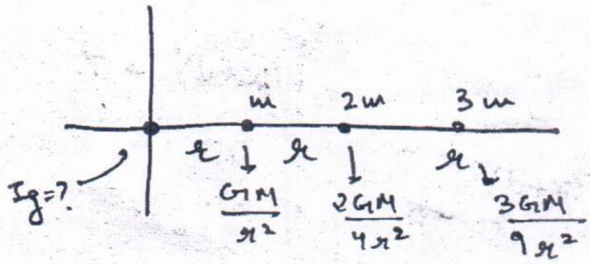


$$I_g = \frac{Gm}{a^2}$$



$$= -\frac{Gm}{4a^2}\hat{i} + \frac{2GM}{a^2}\hat{j}$$

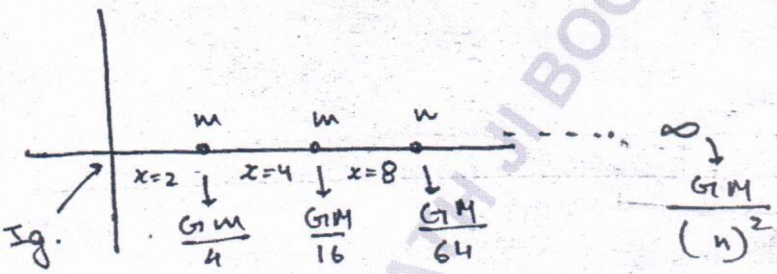
Q/



$$I_{g\,net} = \frac{GM}{r^2}\left[1 + \frac{1}{2} + \frac{1}{3}\right]\hat{i}$$

$$= \frac{11\,GM}{6r^2}\hat{i}$$

Q/

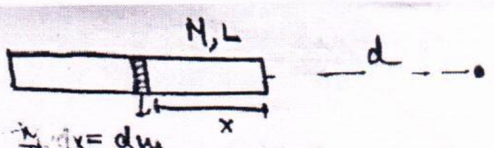


$$I_{g\,net} = \left[\frac{GM}{4} + \frac{GM}{16} - - - \frac{GM}{(n)^2}\right]\hat{i}$$

$$= GM\left[\frac{1}{4} + \frac{1}{16} - - - - \infty\right]\hat{i}$$

$$= GM\left[\frac{1/4}{3/4}\right]\hat{i} = \frac{GM}{3}\hat{i}$$

#



$\frac{M}{L}dx = dm$

$$dI_g = \frac{G\,dm}{(x+d)^2} = \frac{GM}{L}\cdot\frac{dx}{(x+d)^2}$$

$$I_g = \frac{GM}{L}\int_0^L (x+d)^{-2}\cdot dx$$

$$= \frac{GM}{L}\int_0^L \frac{dz}{z^2} = \frac{GM}{L}\int_0^L z^{-2}\cdot dz$$

$(x+d) = z$

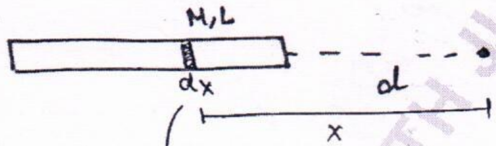
$\frac{dz}{dx} = 1$

$dz = dx$

$$= \frac{G}{L}\cdot\left[\frac{z^{-1}}{-1}\right]_0 = \frac{GM}{L}\left[-(x+d)^{-1}\right]_0^L$$

$$= -\frac{GM}{L(\ \ +d)}$$

OR



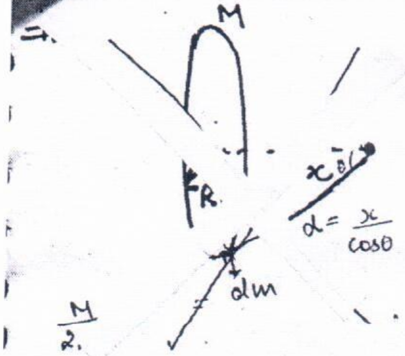
$$dm = \frac{M}{L}\cdot dx$$

$$dI_g = \frac{GM}{x^2}$$

$$I_g = \frac{GM}{L}\int_d^{d+L}\frac{1}{x^2}\cdot dm$$

$$\boxed{I_g = \frac{GM}{L}\left[\frac{1}{d} - \frac{1}{d+L}\right]} = \frac{GM}{L}\left[\frac{\cancel{d}+L-\cancel{d}}{d(d+L)}\right]$$

$$I_g \boxed{= \frac{GM}{d(d+L)}}$$



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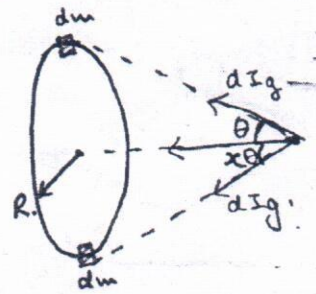


$$\vec{I_g} = \int dI_g \cos\theta$$

$$= \int \frac{G\,dm}{(R^2+x^2)} \cdot \frac{x}{(R^2+x^2)^{1/2}}$$

$$= \frac{GMx}{(R^2+x^2)^{3/2}}$$

Electrostatics

↓

Electric field for a ring $= \dfrac{KQx}{(R^2+x^2)^{3/2}}$

Max field

$$\vec{I_g} = \frac{GMx}{(R^2+x^2)^{3/2}} \qquad \frac{d\vec{I_g}}{} = \frac{\frac{GM}{(R^2+\ \ )^{\ 2}} + \frac{3}{2}\frac{GMx}{(R^2+x^2)^{1/2}}}{(R^2+\ \ )^3}$$

$$0 = \frac{G\ \ \ \frac{3}{2} GMx\ \ \ +x^2)}{(R^2+x^2)}$$

$$GM = 3GM(R^2\ \ x^2)$$

$$= R^2 + x^2$$

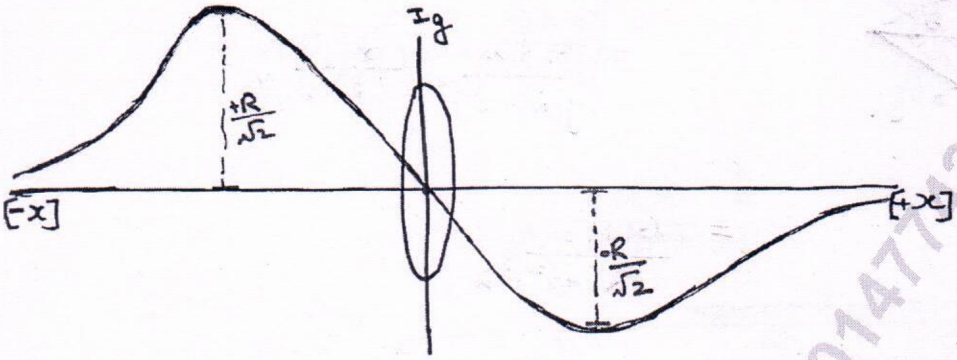
$$d\left(\frac{u}{v}\right) = \frac{v\,du - u\,dv}{v^2}$$

$$0 = \frac{(R^2+x^2)^{3/2} - \frac{3}{2}(R^2+x^2)^{1/2}\cdot 2x\cdot x}{A}$$

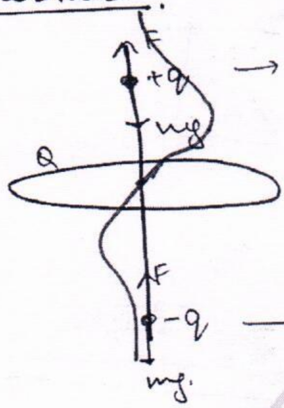
$$R^2 + x^2 = \frac{3}{\cancel{2}}\,\cancel{2}x\cdot x$$

$$R^2 = 2x^2$$

$$R = \pm\sqrt{2}\,x \quad\Rightarrow\quad \boxed{x = \pm\frac{R}{\sqrt{2}}}$$



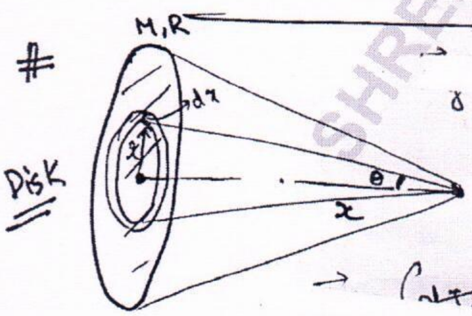
Electrostats



$+q \longrightarrow eq^n = +\frac{R}{\sqrt{2}}$ $\uparrow$ stable

$-q \longrightarrow eq^n = -\frac{R}{\sqrt{2}}$ $\downarrow$ unstable.

Disk



$$\int_0^R \frac{\ldots}{x^2+r^2}$$

$$\int_0^R \frac{\ldots\, x}{(x^2+r^2)\ldots}$$

$$-\int dI_g = \int_0^R \frac{\quad}{(\ldots + x^2)^{3/2}} = \ldots x$$

$$dm = \sigma \cdot 2\pi r \cdot dr.$$

$$dI_g = \frac{G \cdot dm \cdot x}{(R^2 + x^2)^{3/2}}$$

$$I_g = G\sigma\pi \cdot x \int \frac{2r\,dr}{(r^2 + x^2)^{3/2}} \qquad r^2 + x^2 = t, \quad 2r\,dr = dt$$

$$= G\sigma\pi \cdot x \int \frac{dt}{t^{3/2}}$$

$$= -2G\sigma\pi x \left[\frac{1}{\sqrt{r^2+x^2}}\right]_0^R$$

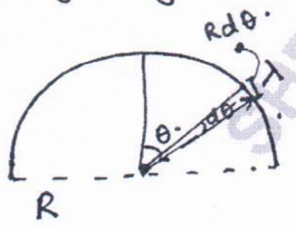
$$I_g = \frac{+2\pi G M x}{\pi R^2} \left[\frac{1}{x} - \frac{1}{\sqrt{R^2+x^2}}\right]$$

$$= \frac{2GM}{R^2}\left[1 - \frac{x}{\sqrt{R^2+x^2}}\right]$$

$\boxed{I_g = \frac{2GM}{R^2}\left[1 - \cos\theta\right]}$.

For infinite disk.
$1 - \cos\theta = 0$
$I_g = 2GM/R^2$
$\Downarrow$
$\boxed{2G\sigma\pi}$

Half Ring.



I_g at center.

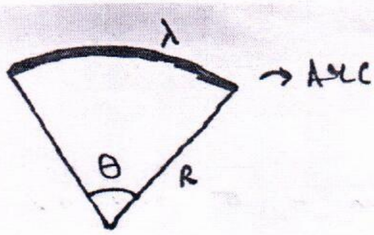
$\lambda \pi R = m.$

$$\ldots = \ldots \frac{\lambda G R}{R^2} \ldots \frac{\lambda G}{R}.$$

$$I_g = \int dI_g \sin\theta$$

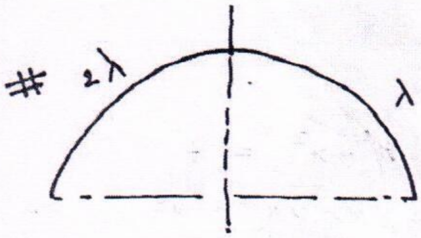
$$= \int \frac{G\lambda R\,d\theta}{R^2} \sin\theta = \frac{G\lambda}{R}\int_0^\pi \sin\theta \cdot d\theta = \frac{2G\lambda}{R}$$

#



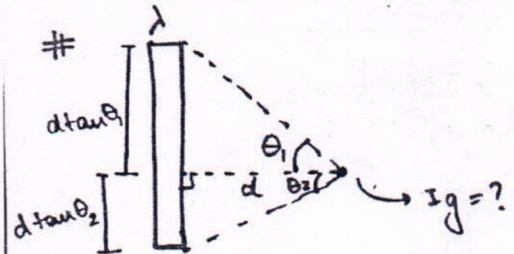
$$\frac{2G\lambda \sin\theta}{R}$$

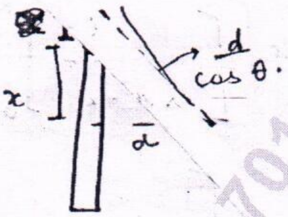
#



Hay sing.

#

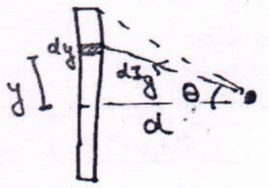


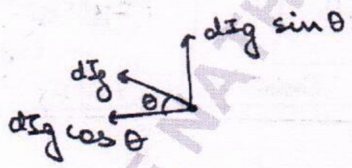


$$\int dI_g = \int \frac{G \cdot dm}{d^2} \cdot \sin^2\theta.$$

$$I_g = \frac{G\, d \tan\theta_1 \cdot \lambda}{d^2} \int_0^{\theta_1} \sin^2\theta$$

$$I_g = -\frac{G \tan\theta_1 \cdot \lambda}{d} \cos^2\theta_1$$





$$\tan\theta = \frac{y}{d}$$

$$dy = d \sec^2\theta \cdot d\theta$$

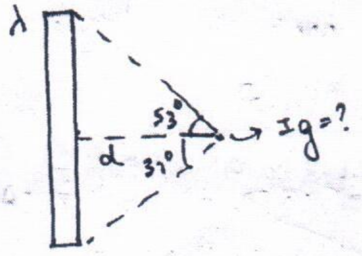
$$dI_g = \frac{G\lambda \cdot dy}{d^2 \sec^2\theta}.$$

$$I_g = \frac{G\lambda}{d} \cdot d\theta$$

$$I_{g\perp} = \int dI_g \cos\theta = \frac{G\lambda}{d} \int_{-\theta_2}^{\theta_1} \cos\theta \cdot d\theta = \boxed{\frac{G\lambda}{d}\left[\sin\theta_1 + \sin\theta_2\right]}$$

#

$$I_{g_{\parallel}} = \int_{-\theta_2}^{\theta_1} dI_g \sin\theta = \boxed{\frac{G\lambda}{d}\left[\cos\theta_2 - \cos\theta_1\right]} \ \#$$

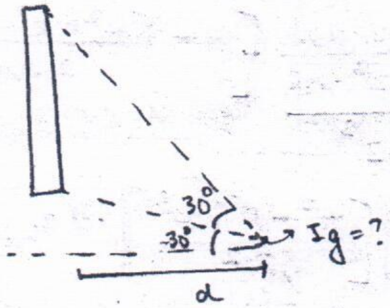
Q 

$$I_{g_\perp} = \frac{G\lambda}{d}\left[\sin 53^\circ + \sin 37^\circ\right].$$

$$= \frac{7G\lambda}{5d}$$

$$I_{g_{\parallel}} = \frac{G\lambda}{d}\left[\cos\theta_2 - \cos\theta_1\right]$$

$$= \frac{G\lambda}{d}\left[\cos 37^\circ - \cos 53^\circ\right] = \frac{G\lambda}{5d}$$

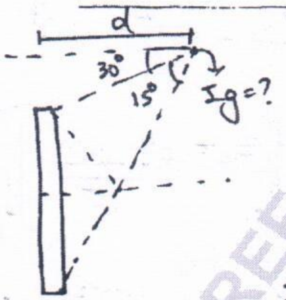
Q 

$$I_{g_\perp} = \frac{G\lambda}{d}\left[-\sin 30 + \sin 60\right]$$

$$= \frac{G\lambda}{2d}(1-\sqrt{3})$$

$$I_{g_{\parallel}} = \frac{G\lambda}{d}\left[+\cos 30 - \cos 60\right]$$

$$= \frac{G\lambda\left[+1-\sqrt{3}\right]}{d\,2}$$

Q 

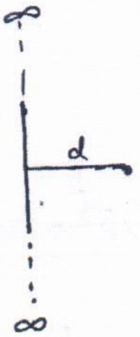
$$I_{g_\perp} = \frac{G\lambda}{d}\left[\sin 45 - \sin 30\right]$$

$$= \frac{G\lambda}{d}\left[\frac{1}{\sqrt{2}} - \frac{1}{2}\right] = \frac{\left[2-\sqrt{2}\right]G\lambda}{2\sqrt{2}\,d}$$

$$I_{g_{\parallel}} = \frac{G\lambda}{d}\left[\cos 30^\circ - \cos 45^\circ\right] = \frac{\left[1-\sqrt{2}\right]G\lambda}{2\ d}$$

Q

infinite wire.

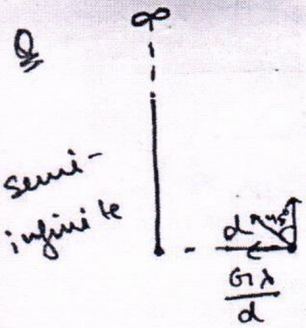


$$\theta_1 = \theta_2 \simeq 90^\circ$$

$$I_{g_{\parallel}} = 0$$

$$I_{g_\perp} = \frac{2G\lambda}{d}$$

Q



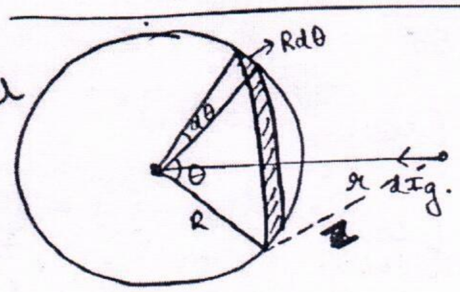
$\theta_1 = 90$

$\theta_2 = 0$

$I_{g\perp} = \frac{G\lambda}{d} = I_{g_{\parallel}}$

$I_g = \frac{\sqrt{2}\, G\lambda}{d}$

Shell



$dm = \sigma\, dA$

$= \frac{M}{4\pi R^2} \cdot 2\pi R \sin\theta \cdot R\, d\theta.$

$= \frac{M}{2} \sin\theta \cdot d\theta$

$\vec{z} = \vec{r} + \vec{R}$

$z = \sqrt{r^2 + R^2 + 2rR\cos(\pi - \theta)}$

$z^2 = r^2 + R^2 - 2rR\cos\theta$

$2z \cdot dz = 2rR \sin\theta \cdot d\theta.$

$\sin\theta \cdot d\theta = \cancel{2\pi R}\, \frac{2z \cdot dz}{2rR}$

$= \frac{z \cdot dz}{rR}$

$dm = \frac{M}{2} \sin\theta \cdot d\theta. = \frac{M}{2} \frac{z \cdot dz}{rR}$

$dI_g = \frac{G\, dm\, [r - R\cos\theta]}{z^3}$

$dI_g = \frac{GM\; z \cdot dz\; [r - R\cos\theta]}{2z^3 \cdot rR}$

$= \frac{\frac{GM}{2} \frac{z \cdot dz}{z^3 \cdot rR}}{z^3} \left[r - \frac{[r^2 + R^2 - z^2]}{2r} \right]$

$= \frac{GM}{4r^2 R} \left[\frac{r^2 - R^2 + z^2}{z^2} \right] \cdot dz$

$= \frac{GM}{4r^2 R} \left[1 + \frac{r^2 - R^2}{z^2} \right] dz$

$I_g = \frac{GM}{4r^2 R} \left[z + \frac{R^2 - r^2}{z} \right]_{r-R}^{r+R}$

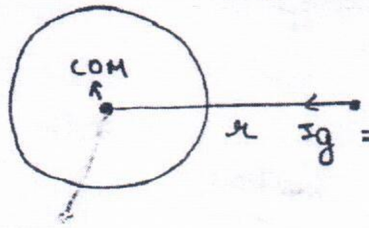
$= \frac{GM}{4r^2 R} \left[\frac{(R+r)^2 + R^2 - r^2}{r+R} - \frac{(R-r)^2 + R^2 - r^2}{r-R} \right]$

$I_g = \frac{GM}{4r^2 R} \left[\frac{2R^2 + 2Rr}{r+R} - \frac{(2R^2 - 2Rr)}{r-R} \right]$

$= \frac{GM}{4r^2 R} \left[\frac{2\cancel{R^2 r} - 2R^3 + 2Rr^2 - \cancel{2R^2 r} - \cancel{2R^2 r} + 2Rr^2 - 2R^3 + \cancel{2R^2 r}}{r^2 - R^2} \right]$

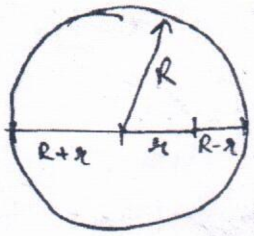
$= \frac{GM}{\cancel{4}r^2 R} \left[\frac{\cancel{4}Rr^2 - \cancel{4}R^3}{r^2 - R^2} \right] = \frac{GM}{r^2 R} \left[\frac{R[r^2 - R^2]}{(r^2 - R^2)} \right] = \boxed{\frac{GM}{r^2}}$

Shell. [from a pt. on outside]



$$I_g = \frac{GM}{r^2}$$

Shell [from a pt. inside]

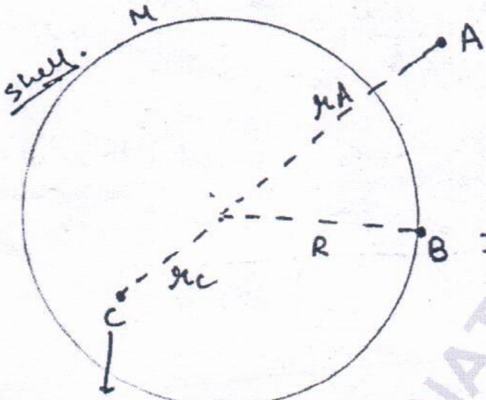


$$I_g = \frac{GM}{4r^2R}\left[z + \frac{R^2-r^2}{z}\right]_{R-r}^{R+r} \Rightarrow 0$$

$$I_g = 0$$

Field intensity inside a shell $\Rightarrow$ zero.

CONCLUSION



$$I_{g_A} = \frac{GM}{r_A^2}$$

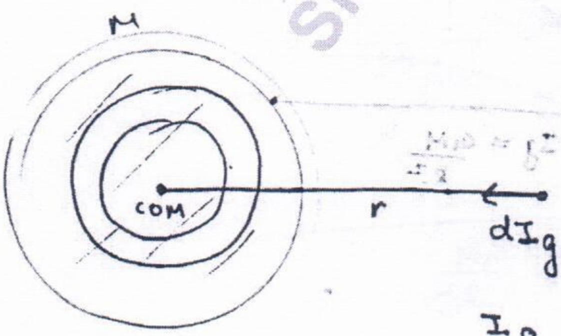
$$I_{g_B} = \frac{GM}{R^2}$$

$I_{g_C} = 0$

For $r \geq R$
consider COM at center

For $r < R$
$I_g = 0$

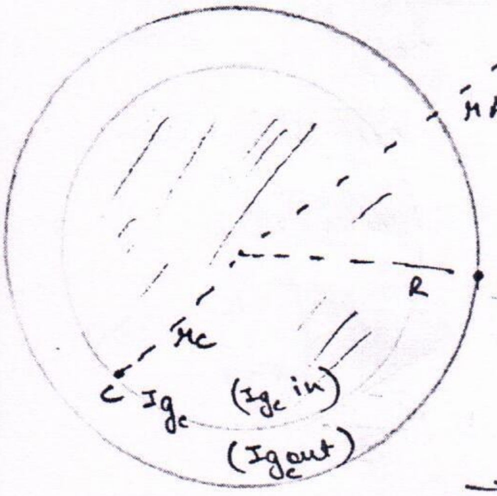
Solid sphere



$$dI_{g_1} + dI_{g_2} - \text{---} \infty$$

$$I_g = \frac{GM}{r^2}$$

CONCLUSION



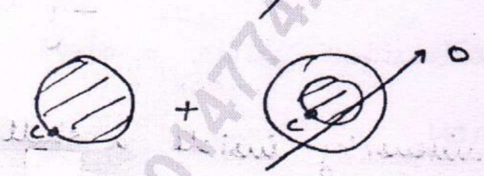
$Ig_A = \frac{GM}{r_A^2}$

$Ig_B = \frac{GM}{r_B^2}$

$\vec{Ig_c} = \vec{Ig}_{c\ in} + \cancelto{0}{\vec{Ig_c}\ out}$..

$M' = \rho\ Vol.$

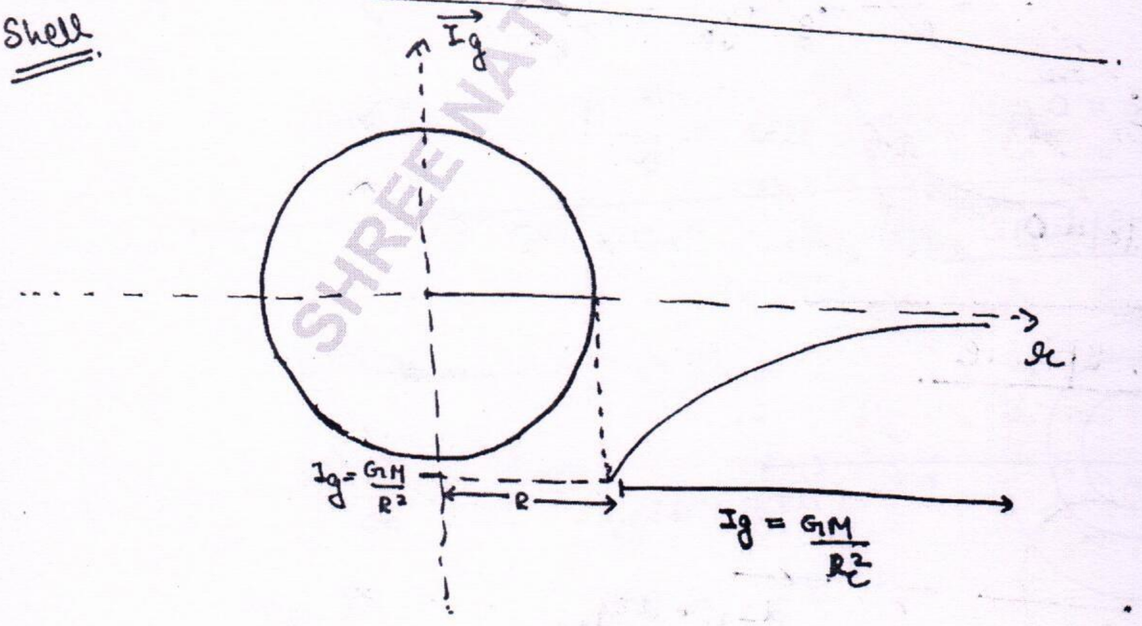
$= \frac{M}{\frac{4}{3}\pi R^3} \cdot \frac{4}{3}\pi r^3 = \frac{Mr^3}{R^3}$

$=$ 

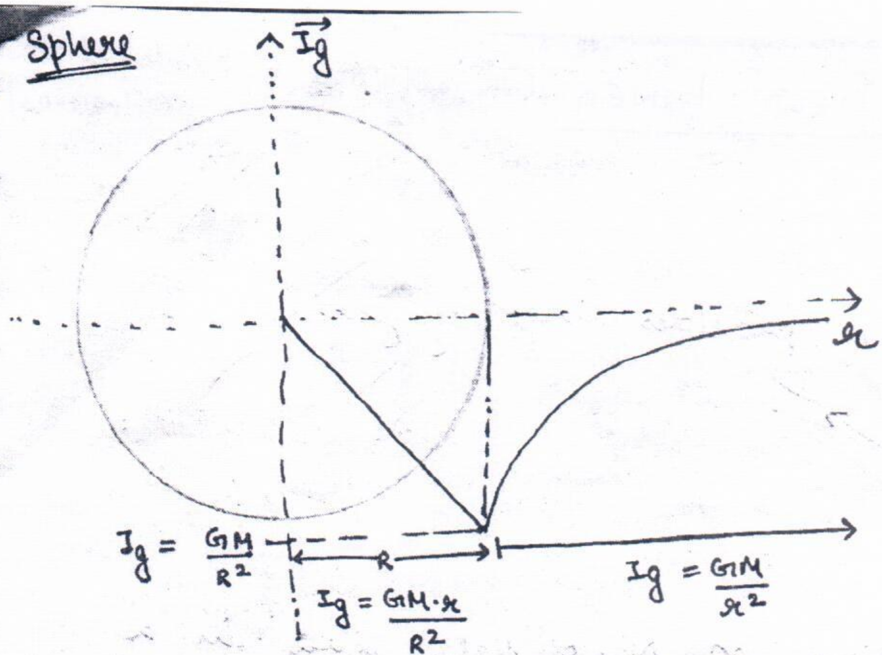
$\vec{Ig_c} = \frac{GM'}{r^2} = \frac{GMr^3}{R^3 r^2}$

\# $\boxed{Ig_c = \frac{GMr}{R^3}}$

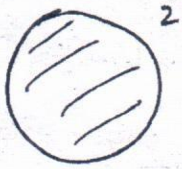
Shell



Sphere



Q



2M, 2R
solid sphere.

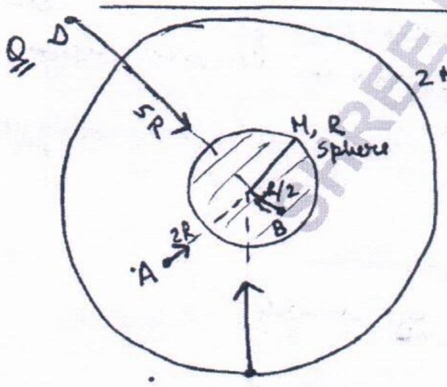
$I_g = ?$ a) $r = R/2$
b) $r = R$
c) $r = 4R$.

a) $I_g = \frac{GMr}{R^3} = \frac{G2MR}{2 \cdot 8R^3} = \frac{GM}{8R^2}$

b) $I_g = \frac{GMr}{R^3} = \frac{2GMR}{8R^3} = \frac{GM}{4R^2}$

c) $I_g = \frac{GM}{r^2} = \frac{2GM}{16R^2} = \frac{GM}{8R^2}$

Q



2M, 4R. shell $I_g = ?$ A, B, C, D.

$A \Rightarrow I_g = \frac{GM}{4R^2} + 0$

$B \Rightarrow I_g = \frac{GMR}{2 \cdot R^3} + 0 = \frac{GM}{2R^2}$

$C \Rightarrow I_g = \frac{GM}{16R^2} + \frac{2GM}{16R^2} = \frac{3GM}{16R^2}$

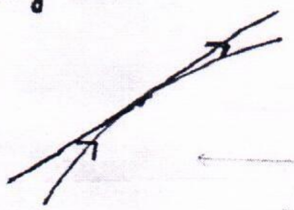
$D \Rightarrow I_g = \frac{GM}{25R^2} + \frac{G \cdot 2M}{25R^2} = \frac{3GM}{25R^2}$

PROPERTIES OF FIELD LINES [in analogy c̄ electrostatics field lines]

1. originates from ∞

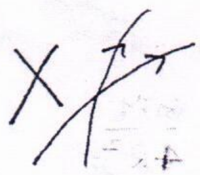
2. terminates at mass

3. tangent at field line provides direc^n of force.



4. Strength of field $\propto$ No. of field lines in a region

5. Field lines do not intersect.



→ at a pt. there can't be 2 direc^n of force.

6.

$dw = F \cdot dx$

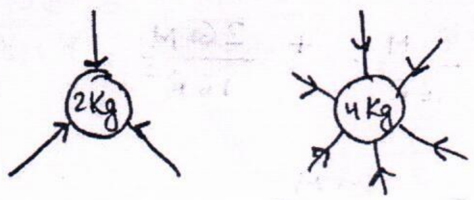
⇒ Electrostatics ✗

⇒ Electric may / may not

Gravita^n Force is a conservative force.

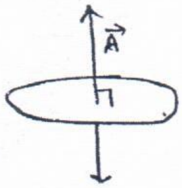
⇒ They don't form a closed loop.

7. No. of field lines $\propto$ magnitude of mass



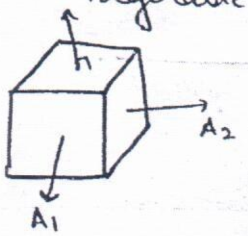
Gauss - Flux theorem for Gravitaⁿ

Area $\rightarrow$ vector qty.

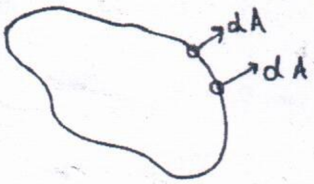


open surface.

area can be considered in any direcⁿ $\perp$ to surf.
Preferable towards observer.



Closed surface



Electric Flux.

$$d\phi_E = \vec{E} \cdot \vec{dA}$$

$$\phi_E = \oint \vec{E} \cdot \vec{dA}$$

Magnetic Flux.

$$d\phi_B = \vec{B} \cdot \vec{dA}$$

$$\phi_B = \oint \vec{B} \cdot \vec{dA}$$

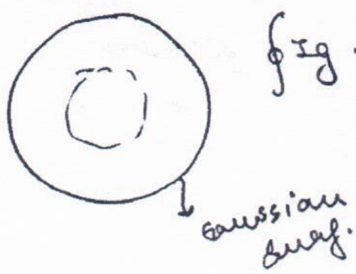
Gravitational Flux

$$d\phi_G = \vec{I_g} \cdot \vec{dA}$$

$$\phi_G = \oint \vec{I_g} \cdot \vec{dA}$$

$$\left.\begin{aligned} \phi_E &= \oint \vec{E} \cdot \vec{dA} = \frac{q_{enclosed}}{\epsilon_0} \\ \phi_G &= \oint \vec{I_g} \cdot \vec{dA} = -4\pi G\, M_{enclosed} \end{aligned}\right\} \text{Gauss Theorem}$$

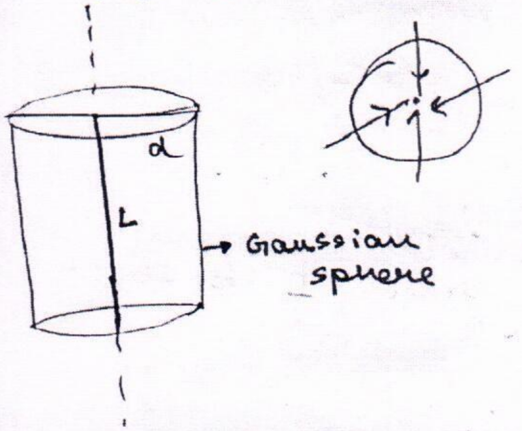
Proof of shell by Gauss Law $\Rightarrow$ USELESS



$$\oint I_g \cdot dA = -4\pi G \cdot \cancel{M_{en}}^{\,0}$$

$$I_g = 0$$

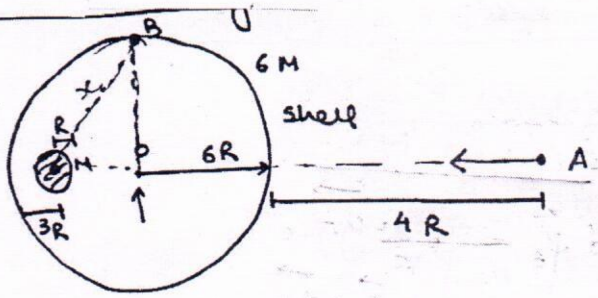
Proof of infinite wire by Gauss law $\Rightarrow$ USELESS



$$-I_g \cdot 2\pi d \cdot L = -4\pi G(\lambda L)$$

$$I_g = \frac{2G\lambda}{d}$$

Q



$I_{g_A} = ?$

$I_{g_O} = ?$

$I_{g_B} = ?$

$$\boxed{I_{g_0} = \frac{GM}{9R^2}}$$

$$I_{g_A} = \frac{G6M}{100R^2} + \frac{GM}{169R^2}$$

$$= \frac{GM}{R^2}\left[\frac{3}{25} + \frac{1}{169}\right] = \frac{[507+25]\,GM}{25\times169\,R^2}$$

$$\boxed{\frac{532\,GM}{R^2\cdot 25\times169}}$$

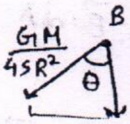
$$\vec{I_{g_B}} = \left(\frac{G6M}{36R^2}\right) + \left(\frac{GM}{x^2}\right)$$

$$\left[x^2 = 36R^2 + 9R^2 = 45R^2\right]$$

$$\cancel{= \frac{6GM}{R^2}\left[\frac{1}{6} + \frac{1}{45}\right]}$$

$$\cancel{= \frac{(15+2)GM}{90R^2}} = \cancel{\boxed{\frac{17GM}{90R^2}}}$$

$I_2 = \frac{GM}{45R^2}$

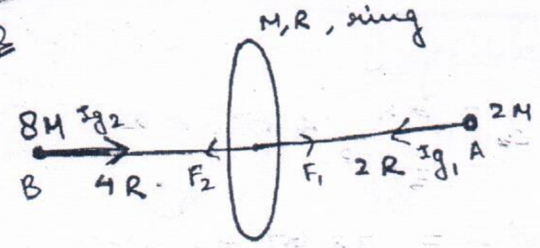


$\frac{GM}{6R^2} = I_1$

$$\cos\theta = \frac{6}{\sqrt{45}}$$

$$I_g = \sqrt{I_1^2 + I_2^2 + 2I_1I_2\cos\theta}$$

Q



Net force on the ring.

$$Ig_A = \frac{GM\,2R}{(R^2+4R^2)^{3/2}} = \frac{2GMR}{(5R^2)^{3/2}}$$

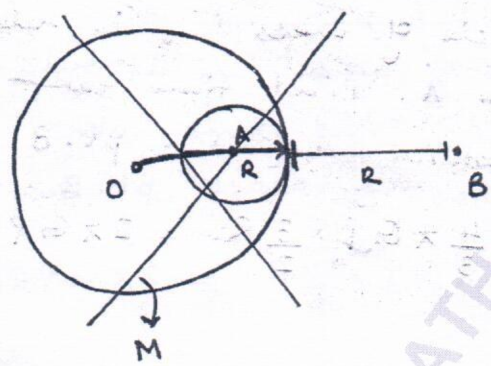
$$= \frac{2GM}{5^{3/2}\cdot R^2}$$

$$Ig_B = \frac{GM\,4R}{(R^2+16R^2)^{3/2}} = \frac{4GMR}{17^{3/2}\cdot R^3} = \frac{4GM}{17^{3/2}\cdot R^2}$$

$$\vec{F}_{net} = Ig_1 \times M_{T_1} + Ig_2 \times M_{T_2} = \left[\frac{2GM\cdot 2M}{5^{3/2}\cdot R^2} - \frac{4GM\cdot 8M}{17^{3/2}\cdot R^2}\right]$$

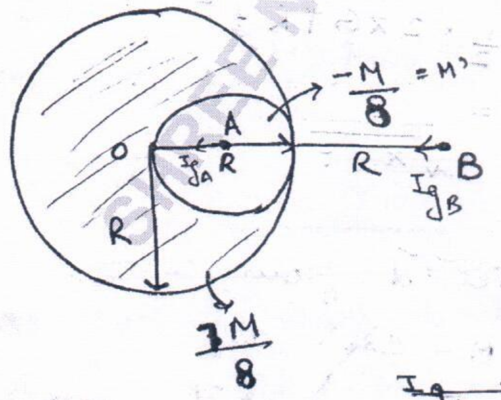
$$= \frac{4GM^2}{5^{3/2}\cdot R^2} - \frac{32GM^2}{17^{3/2}\cdot R^2} = \frac{4GM^2}{R^2}\left[\frac{1}{5^{3/2}} - \frac{8}{17^{3/2}}\right]$$

Q



A cavity is cut from a sphere of Mass M as shown. Find Ig at A, B.

$$\rho = \frac{M}{\frac{4}{3}\pi R^3} \qquad M' = \frac{4}{3}\pi\frac{R^3}{8} \times \frac{M}{\frac{4}{3}\pi R^3} = \frac{M}{8}$$



$$\cancel{Ig_B = \frac{3GM}{4R^2} - \frac{GM}{8\times\left[\frac{3R}{2}\right]^2}}$$

$$\cancel{= \frac{3GM}{4R^2} - \frac{GM}{4\times 9R^2}}$$

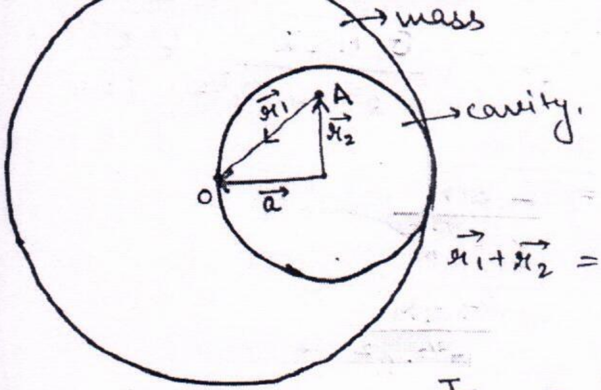
$$\cancel{= \frac{27GM}{36R^2}}$$

$$\cancel{Ig_A = \frac{3GMR}{4\times 2\times R^3} = \frac{3GM}{8R^2}}$$

$$Ig_A = \frac{GM}{R^3}\cdot\frac{R}{2} = \frac{GM}{2R^2}$$

$$Ig_B = \frac{GM}{4R^2} - \frac{GM \times 4}{\cancel{8}_2 \times 9R^2} = \frac{(9-2)\,GM}{36R^2} = \frac{7GM}{36R^2}$$

Field Strength at any pt. inside cavity.



$$I_{g_{a}} = \frac{GM}{\frac{4}{3}\pi R^3} \cdot r \quad \times \frac{4}{3}\pi$$

$$= \frac{4\pi}{3} G\rho \cdot r .$$

$\vec{r_1} + \vec{r_2} = \vec{a}$ → mag. = length of line joining centres of cavity & sphere
→ direcⁿ ⇒ towards center of sphere.

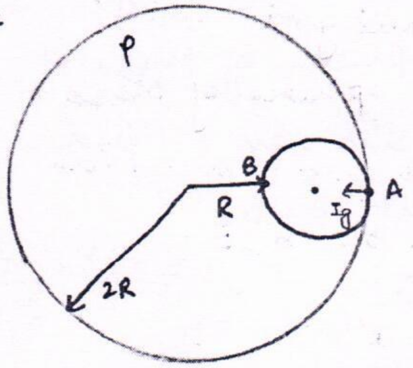
$$I_{g_A} = \vec{I_g}_{sphere} + \vec{I_g}_{cavity = (-ve\ mass)}$$

$$I_{g_A} = \frac{4}{3}\pi G\rho \vec{r_1} + \frac{4}{3}\pi G\rho \vec{r_2}$$

$$= \frac{4}{3}\pi G\rho [\vec{r_1} + \vec{r_2}]$$

$$\boxed{I_{g_A} = \frac{4\pi G\rho}{3} \vec{a}} \ \#$$

Q. A particle of mass m_0 is released from A. Find time taken by the particle to reach pt. B.



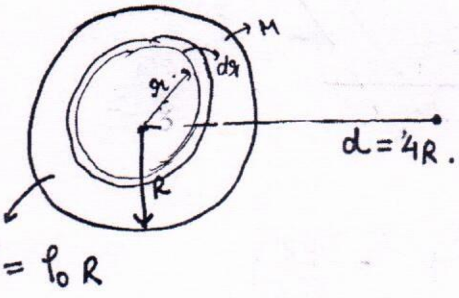
$$I_g = \frac{4}{3}\pi G\rho \times \frac{3}{2} R = 2\pi G\rho R . = acc$$

$$S = \frac{1}{2} a t^2$$

$$\cancel{R} = \frac{1}{\cancel{2}} \times \cancel{2}\pi G\rho \cancel{R}\, t^2$$

$$t = \frac{1}{\sqrt{\pi G\rho}}$$

Q Find field strength at a dist. = d from center.



$\rho = \rho_0 R$

$$dM = \frac{4}{3}\pi R^3 \cdot \rho_0 dR .$$

$$M = \frac{4}{3}\pi \rho_0 \left[\frac{R^4}{4}\right]_0^R = \frac{\pi \rho_0 R^4}{3}$$

$$I_g = \frac{GM}{R^2} = \frac{G\pi \rho_0 R^4}{3R^2 \times 16} = \frac{G\pi \rho_0 R^2}{48}$$

$\cancel{dM = \frac{4\pi}{3}}$. $dM = \rho_0 r \cdot 4\pi r^2 \cdot dr$

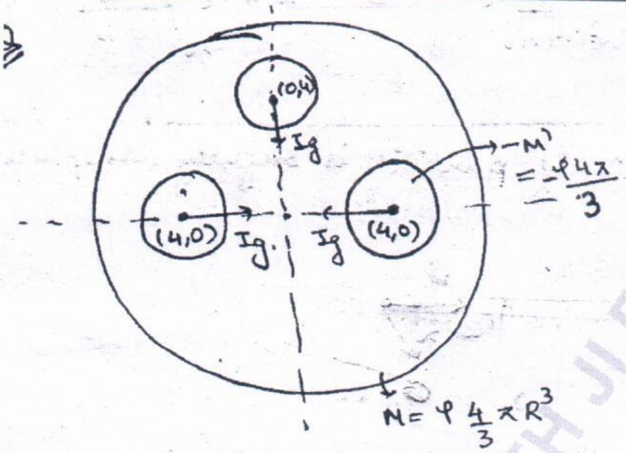
$$M = 4\pi \rho_0 \int_0^R r^3\, dr = \pi \rho_0 R^4$$

$$I_g = \frac{GM}{d^2} = \frac{G\pi\rho_0 R^4}{16 R^2} = \frac{G\pi\rho_0 R^2}{16}$$

I_g at $R/2$ $= ?$

$$M = 4\pi\rho_0 \int_0^{R/2} r^3 \cdot dr = 4\pi\rho_0 \cdot \left[\frac{r^4}{4}\right]_0^{R/2} = \frac{\pi \rho_0 R^4}{16}$$

$$I_g = \frac{GM}{r^2} = \frac{G\pi\rho_0 R^4}{16 \cdot \frac{R^2}{4}} = \frac{G\pi\rho_0 R^2}{4}$$



If 3 small spheres are removed from a large sphere of mass M, radius = 8m. Find I_g at center of large sphere if radius of small spheres = 1m.

$$M = \rho \frac{4}{3}\pi R^3 = 512 \frac{\rho 4\pi}{3}$$

$I_{g\,\text{small spheres}}$

$$= \frac{GM}{d^2} = -G\rho\frac{4\pi}{3} \times \frac{1}{16} = \frac{-\rho G\pi}{4\times 3} = \frac{-\rho G\pi}{12}$$

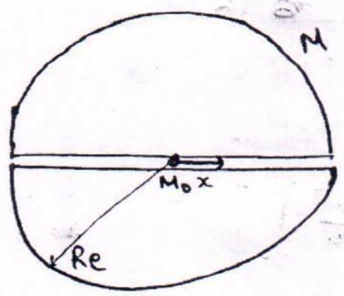
$I_{g\,\text{sphere}} = 0$

$$\boxed{\rho = \frac{M}{\frac{4}{3}\pi 512}}$$

$$I_{g\,\text{sys.}} = \frac{-\rho G\pi}{12} = \frac{3M}{4\pi 512} \cdot \frac{G\pi}{12}$$

$$= \frac{-GM}{8192}$$

If a small particle of mass = m_o is at the center of the earth as shown. If it is displaced slightly then find time period of oscillaⁿ of particle



$\cancel{F = \frac{GMM_o}{x^2}}$ $\vec{F} = m \cdot \vec{I_g}$

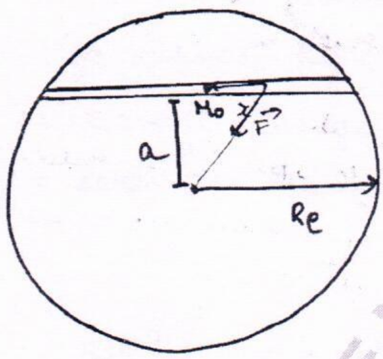
$\vec{F} = m_o \cdot \frac{GMx}{R_e^3} = \frac{GMM_o \cdot x}{R_e^2}$

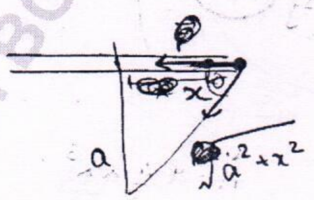
$\vec{F} = kx$

$\therefore k = \frac{GMM_o}{R_e^3}$

$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{m_o \cdot R_e^3}{GMM_o}} = 2\pi R_e\sqrt{\frac{R_e}{GM}}$

Q. Find time period of oscillaⁿ if particle is slightly displaced.





$\vec{F}\cos\theta = m \cdot \vec{I_g}$

$= M_o \cdot \frac{GNM_o}{(a^2+x^2)^{3/2}} \cdot x\cos\theta$

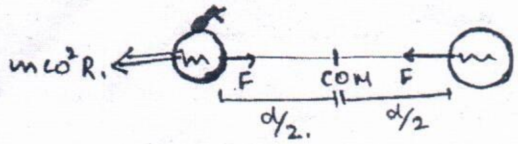
$= M_o GMM_o \cdot x$

$F\cos\theta = \frac{GMM_o}{R_e^3} \cdot \cancel{\sqrt{x^2+a^2}} \cdot \frac{x}{\cancel{\sqrt{x^2+a^2}}}$

$= \frac{GMM_o}{R_e^3} \cdot x$ $T = 2\pi R_e\sqrt{\frac{R_e}{GM}}$

DOUBLE STAR SYS.

Q 2 masses [equal], seperated by dist. 'd' starts moving in circular path due to mutual gravitaⁿ force. Find speed of each mass.

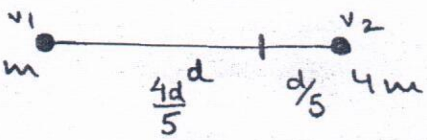


$$\vec{F} = \frac{Gm^{\cancel{2}}}{d^2} = \cancel{m}\omega^2 \frac{d}{2}$$

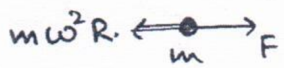
$$\omega = \sqrt{\frac{2GM}{d^3}}$$

$$v = \sqrt{\frac{2GM}{d^3} \cdot \frac{d^2}{4}} = \sqrt{\frac{GM}{2d}}$$

Q Find speed of each particle if they move due to mutual gravitaⁿ force.



$$x_{cm} = \frac{4md}{5m} = \frac{4d}{5}$$



$$\vec{F} = \frac{G\,4m^{\cancel{2}}}{d^2} = \cancel{m}\omega^2 \frac{4d}{5}$$

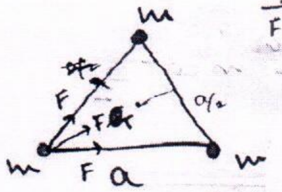
$$\omega = \sqrt{\frac{20GM}{4d^3}} = \sqrt{\frac{5GM}{d^3}}$$

$$v_1 = \sqrt{\frac{5GM \cdot d^2 \cdot 16}{d^3 \cdot 25}} = 4\sqrt{\frac{GM}{25d}}$$

$$v_2 = \sqrt{\frac{5GM \cdot d^2}{d^3 \cdot 25}} = \sqrt{\frac{GM}{25d}}$$

TRIPLE STAR SYS.

Q 3 Particles of equal mass starts moving in a circular due to mutual gravitaⁿ force. Find speed of each particle.



$\vec{F_T} = \vec{F} + \vec{F}$

$\sqrt{\frac{3a^2}{4} \times \frac{4}{9} 3}$ $\frac{a}{\sqrt{3}}$

$$\vec{F} = \frac{Gm^2}{a^2}$$

$$\vec{F_T} = \sqrt{F^2 + F^2 + 2F_1F_2\cos 60}$$

$$= F\sqrt{3}.$$

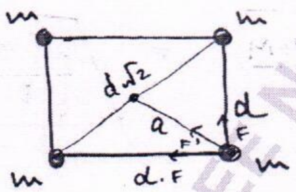
$$\frac{Gm^2\sqrt{3}}{a^2} = \frac{mv^2 \cdot \sqrt{3}}{a}$$

$$v = \sqrt{\frac{Gm}{a}}$$

---

4 STAR SYS.

Q Find speed of each particle



$a = \frac{d}{\sqrt{2}}.$

$\vec{F_T} \longleftarrow m \Longrightarrow \frac{mv^2}{a}$

$$\cancel{\vec{F'} = \frac{Gm^2}{2d^2}}$$

$$F\sqrt{2} = \frac{Gm^2}{d^2}\sqrt{2}$$

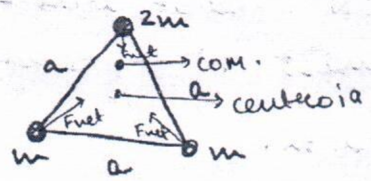
$$\frac{mv^2 2}{d^2} = \frac{Gm^2}{2d^2} + \frac{Gm^2\sqrt{2}}{d^2}$$

$$2v^2 = \frac{Gm + \sqrt{2}\,Gm \cdot 2}{2}$$

$$4v^2 = GM(1 + 2\sqrt{2})$$

$$v = \frac{\sqrt{GM(1+2\sqrt{2})}}{2}$$

#



For a sys. to rotate in a given arrangement. $\vec{F}_{net}$ due to mutual gravita^n force "on each particle" should be towards COM.

ACCELERA^N DUE TO GRAVITY [g]

g →
- at surface
- $h \le 5\%$ of R_e
- $h \geq 5\%$ of R_e
- depth d below surf.

Q If weight of body on earth surface = 700N. What will be the body at surf. of a planet whose mass = $\frac{1}{7}$th of earth's mass. and radius is $\frac{1}{2}$ of radius of earth.

$$g' = G\,\cancel{700}^{10}\,\frac{M}{7} \times \frac{4}{R^2}$$

$$g' = \frac{40GM}{R^2} \qquad \frac{GM70}{R^2} = g.$$

$$\frac{g'}{g} = \frac{4\cancel{0}}{7\cancel{0}}.$$

$$g' = \cancel{\tfrac{40}{70}}\ \frac{4}{7} \times 9.8 = 5.6.$$

$$mg' = 5.6 \times 70 = 392$$

$$g = \frac{GMe}{Re^2} = \frac{GMe \cdot \frac{4}{3}\pi Re}{\frac{4}{3}\pi Re^2 \cdot Re} = \frac{4}{3}\pi Re\rho G$$

Q If range of a projectile on earth's surf = 20m

Find the ~~range~~ ~~time to~~ range ~~of~~ on a planet whose mass = 10 times Me. & radius = 2Re.

$$g' = \frac{G\,10Me}{4Re^2} = 2.5\,g = 25.$$

$$R' = \frac{u^2 \sin 2\theta}{2.5g} = \frac{20}{2.5} = 8\text{ m}.$$

Q Find Ig at surf. of a planet whose density is 9 g/cm³ and radius = 14000 Km.

~~Re = 6400~~

$$g = \frac{4}{3} \times \frac{22}{7} \times \cancel{14000}^{2000} \times \frac{\cancel{9000}^{3000}}{\cancel{3}} \times 6.67 \times 10^{-11} \times 10^3$$

$$= 88 \times 2 \times 6.67 \times 3 \times 10^{-5}$$

$$= 3168 \times 10^{-5} = 3.1 \times 10^{-2} \times 10^3 = 31$$

% change in g [< 5%]

$$g = \frac{G M_e}{R_e^2}$$

$$\frac{\Delta g}{g} \times 100 = \left[\frac{\Delta M_e}{M_e} \pm 2\frac{\Delta R_e}{R_e}\right] \times 100$$

% change in g = % change in M_e ± 2(% change in R_e)

Q. If mass of a planet = const.
Radius changed by 3%. Find %age in g.

$$\% \Delta g = \pm 3\%$$

Q. If mass ⇒ increased by 2%
radius ⇒ decreased ~~[illegible]~~ 1%
Find %Δg.

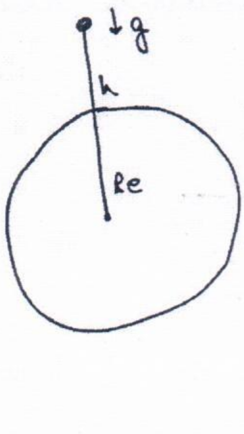
$$\% \Delta g = 2 - 2 = 0.$$

Q. If density of a planet ⇒ Increased by 2%.
Radius ⇒ decreased by 4%.
Find % Δg. $= \left[\frac{\Delta \rho}{\rho} + \frac{\Delta R}{R}\right] \times 100$

$$\% \Delta g = \cancel{[illegible]} \quad 2 - 4 = -2\%$$

ACCELERA^N DUE TO GRAVITY ABOVE SOME HEIGHT 'H'

H < 5% of Re. = 320 Km.



$$\vec{I}_g = g_h = \frac{G M_e}{(R_e + h)^2} = \frac{G M_e}{R_e^2\left(1 + \frac{h}{R_e}\right)^2}$$

$$g_h = g\left[1 - \frac{2h}{R_e}\right]$$

% age change in g with 'h'

$$\Delta g = g - g_h$$

$$\Delta g = g \times \frac{2h}{R_e}$$

$$\frac{\Delta g}{g} = \frac{2h}{R_e}$$

$$\%\text{ change in } g = \frac{2h}{R_e} \times 100$$

[Valid upto → h = 320 Km
→ % Δg = 10%]

Q Find value of 'g' at a height = 256 Km above earth surf.

$$g_h = g\left[1 - \frac{2 \times \cancel{256}^{4}}{\cancel{6400}_{100\,25}}\right] = \cancel{10}^{2}\left(\frac{\cancel{23}}{\cancel{25}}\right) = 0.92g.$$

$$\cancel{= \frac{46}{5} = 9.2\,m/s^2} \quad \boxed{g}$$

Q Find a height above earth surf. where 'g' ⇒ decreases by 8%.

$$8 = \frac{\cancel{2}h}{\cancel{6400}_{32}} \times \cancel{100}$$

$$\boxed{h = 256\,Km}$$

Q Find % age change in g at a height = 128 Km above earth surf.

$$\%\,\Delta g = \frac{2 \times \cancel{128}^{2}}{\cancel{6400}} \times \cancel{100} = 4\%.$$

'g' AT SOME HEIGHT 'H'. [H > 5% of Re]

H > 320 Km.

$$g_h = \frac{G M_e}{(R_e + h)^2}$$

$$g_h = \frac{G M_e}{R_e \left[1 + \frac{h}{R_e}\right]^2} = \frac{g}{\left[1 + \frac{h}{R_e}\right]^2}$$

Q Find value of 'g' at a height $= \frac{1}{2}$ of Re. above earth surf.

$$g_h = \frac{g}{\left[1 + \frac{R_e}{2 \cdot R_e}\right]^2} = \frac{4g}{9} \simeq 4.4.$$

Q Find height above earth surf. where 'g' will be reduced by 75% of its value at earth's surf.

$$\frac{g}{4} = \frac{g}{\left[1 + \frac{h}{R_e}\right]^2} \qquad 2 = 1 + \frac{h}{R_e}$$

$$\boxed{h = R_e}$$

Q Find height where 'g' is reduced to 64% of its value, at $\frac{1}{2}$ of the Re. ~~at a~~

$$g = \frac{G M r}{R^3} = \frac{G M R_e}{2 R_e^3} = \frac{G M}{2 R_e^2}$$

$$\frac{\cancel{64}^{32}}{100} \frac{GM}{\cancel{2} R_e^2} = \frac{g}{\left(1 + \frac{h}{R_e}\right)^2}$$

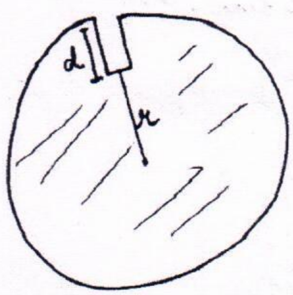
$$\frac{100}{32} = \left(1 + \frac{h}{R_e}\right)^2 \Rightarrow \frac{10 - 4\sqrt{2}}{4\sqrt{2}} = \frac{h}{\cancel{6400}}$$

1600

$800\sqrt{2}$

$$8000\sqrt{2} - 6400 = h = 800\sqrt{2}\,(10 - 4\sqrt{2})$$

'g' AT A DEPTH 'd' from EARTH'S SURF.



$r = R_e - d$

$$g_d = \frac{GM \cdot r}{R_e^3} = \frac{GM}{R_e^2}\left[\frac{R_e - d}{R_e}\right]$$

$$\boxed{g_d = g\left[1 - \frac{d}{R_e}\right]}$$

% Δge change in g $\bar{c}$ depth

$$\frac{\Delta g}{g} = \frac{d}{R_e}$$

$$\%\ \text{change in } g = \frac{d}{R_e} \times 100$$

Q. Find 'g' at a depth of 32 Km below earth surf.

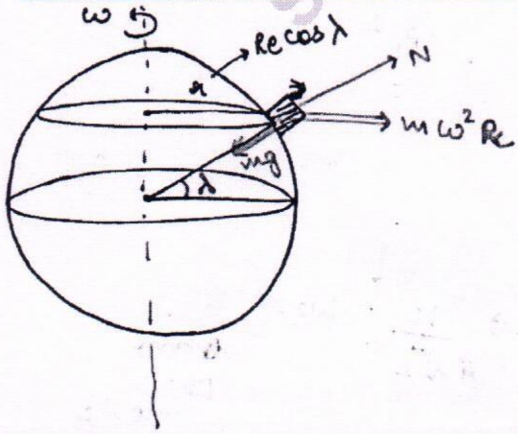
$$g_d = g\left[\frac{\cancel{200} - \cancel{32}}{\cancel{6400}\ 200}\right] \times 100 = \cancel{\frac{168}{2} g = 74g} \cdot \frac{199}{2} g$$

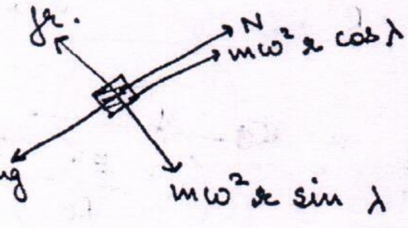
Q. Find 'd' at which g will be reduced by 3%.

$$3 = \frac{d}{6400} \times 100$$

$$\boxed{d = 192\ \text{Km}}$$

EFFECT ON ROTA^N ON 'g'-





$$N = mg - m\omega^2 R_e \cos^2\lambda$$

$$m\,g_{effective} = m\left[g - \omega^2 R_e \cos^2\lambda\right]$$

$$\boxed{g_{eff} = g - \omega^2 R_e \cos^2\lambda}$$

$$g_{equator} = g - \omega^2 R_e \quad [\cos\lambda = 1]$$

$$g_{poles} = g \quad [\cos\lambda = 0]$$

CONDITION OF WEIGHTLESSNESS

$$\omega\uparrow \Rightarrow g_{eff}\downarrow \Rightarrow N\downarrow$$

when $g_{eff} = 0$, $N = 0$ [condiⁿ of wt. lessness]

Q What should be the ω of earth so that body at 60° latitude feels weightless?

$g_{eff} = 0$ $\quad g = \omega^2 R_e \cos^2 60$

$$\sqrt{\frac{4g}{R_e}} = \omega \quad \cancel{= \frac{2}{80}\sqrt{g} = \frac{\sqrt{g}}{40}}$$

$$\cancel{= \sqrt{\frac{10}{1600}} = \frac{1}{\sqrt{160}}} = \sqrt{\frac{4\times10}{64\times10^{5}}} = \frac{1}{800}$$

Q What ~~will~~ should be the angular vel. of earth so that body ~~at equator~~ will feel 64% reducⁿ in its apparent weight. and its wt. at equator

$$\cancel{\frac{36\times N}{100} = m\left[g - \omega^2\, 64\times10^5\right]}$$

$$\cancel{\frac{36g}{100} = g - \omega^2\, 64\times10^5}$$

$$\cancel{\omega^2\, 64\times10^5 = \frac{16g}{25}}$$

$$\cancel{\omega^2 = \frac{g}{10^6}}$$

$$\cancel{\omega = \frac{\sqrt{g}}{10^3}}$$

$$\frac{36}{100}\left[mg - m\omega_0^2 R\right] = mg - m\omega^2 R$$

$$\frac{9}{25}g - \frac{9}{25}\omega_0^2 R = g - \omega^2 R$$

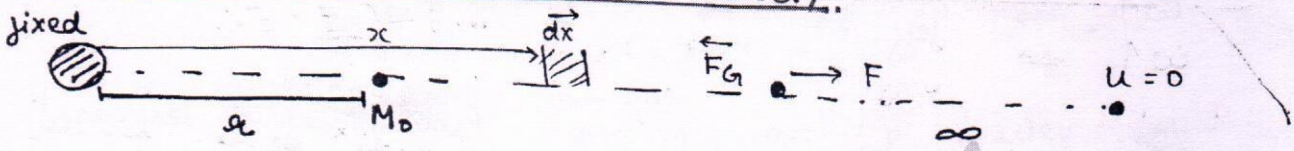
$$\omega^2 R = \frac{16g}{25} + \frac{9}{25}\omega_0^2 R$$

$$\omega^2 = \frac{\left(\frac{16g}{25} + \frac{9}{25}\omega_0^2\, 64\times10^5\right)}{64}$$

a) $\frac{36g}{100} = g - \omega^2 R_e$

b) $\frac{36(g - \omega_0^2 R_e)}{100} = g - \omega^2 R_e$.

GRAVITATIONAL POTENTIAL ENERGY.



$|\vec{F}| = |\vec{F_G}|$
$\vec{F}_{net} = 0$
$a = 0$
$v = const.$
$K_E = const.$

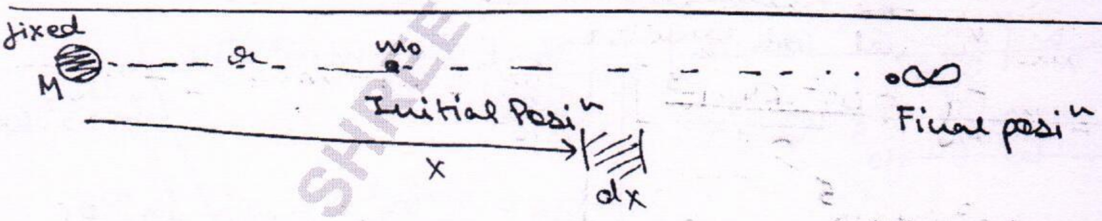
$$dW_F = \int_{\infty}^{R} \frac{GM \cdot m_0}{x^2} \cdot dx \times 1$$

$$\Rightarrow \frac{-GMm_0}{} \left[\frac{1}{x}\right]_{\infty}^{R}$$

$$= -GMm_0 \left[\frac{1}{r} - \cancel{\frac{1}{\infty}}^{\,0}\right] = \boxed{\frac{-GMm_0}{r}}$$

$W D_{all} = \Delta K$
$W \cdot D_{ext} = \cancel{\Delta K}^{\,0} + \Delta u$
$= u_f - \cancel{u_i}^{\,0}$
$W \cdot D \cdot f = u_f$

$\Rightarrow$ Gravitaⁿ potential energy at a pt. in field of a mass [M] is defined as the amt. of work done by a force in bringing an another mass [m_0] from ∞ to that pt, $\bar{c}$ out changing the KE of the mass.

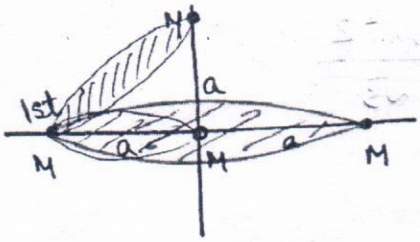


$W \cdot D_{CF} = -\Delta u$

$WD_{GF} = -\Delta u = u_i - \cancel{u_f}^{\,0}$

$$dW_{GF} = \int_{r}^{\infty} \frac{GMm_0}{x^2} \cdot dx \cdot \cos 180 = GMm_0 \left[\frac{1}{r}\right]_{r}^{\infty}$$

$$\boxed{u_i = \frac{-GMm_0}{r}}$$

Q 

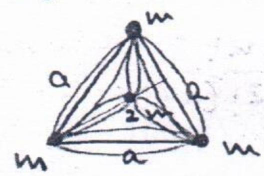
Find a) energy associated $\bar{c}$ 1st mass

b) ——— " ——— sys.

a) $\frac{-GM^2}{\sqrt{2}a} + \left(-\frac{GM^2}{a}\right) + \left(\frac{-GM^2}{2a}\right)$

b) $\left(\frac{-GM^2}{\sqrt{2}a}\right)^2 + \left(\frac{-GM^2}{a}\right)^2 + \left(\frac{-GM^2}{2a}\right)$

Q Energy associated $\bar{c}$ sys.=?

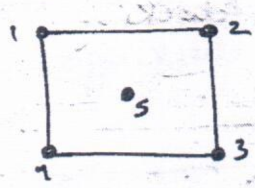


$$U = \left(-\frac{Gm^2}{a}\right)3 + 3\left(-\frac{G2m^2\sqrt{3}}{a}\right)$$

$$= -\frac{Gm^2}{a^2}(3+6\sqrt{3})$$

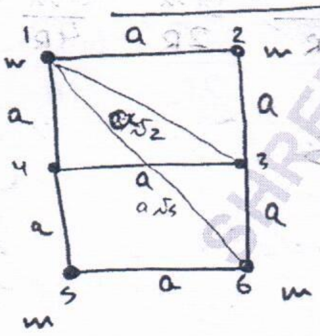
$\frac{\sqrt{3}a}{2} \times \frac{2}{3}$

Q $U_{sys} = ?$



$$U_{sys} = U_{12} + U_{13} + U_{14} + U_{15} + U_{23} + U_{24} + U_{25} + U_{34} + U_{35} + U_{45}$$

$=$

Q 

$U_{sys} = ?$ $= \cancel{\left(\frac{-GM^2}{a}\right) \times} + \cancel{\left(\frac{-GM^2}{a\sqrt{2}}\right)} \times (3+1)$

~~1 + 1 + 3 + 1 + 1 + 1 + 1~~ ×

$\left(\frac{-Gm^2}{a\sqrt{5}}\right) \times 2 + \cancel{\left(\frac{-Gm^2}{2a}\right)} \times 2 +$

$$U_{12} + U_{13} + U_{14} + U_{15} + U_{16}$$
$$U_{23} + U_{24} + U_{25} + U_{26} + U_{34}$$
$$U_{35} + U_{36} + U_{45} + U_{46} + U_{56}$$

$\sqrt{4a^2 + a^2}$

$$= \frac{-Gm^2}{a} + \frac{-Gm^2}{a\sqrt{2}} \cancel{+\frac{-Gm^2}{2a}} - \frac{Gm^2}{2a} - \frac{Gm^2}{a\sqrt{5}}$$

$\downarrow$ 1 + 1 + 2 + 1 + 1 + 1 $\quad$ $\downarrow$ 2 + 1 + 1 – 1 $\quad$ $\downarrow$ $\quad$ $\downarrow$

$$= -\frac{Gm^2 7}{a} - \frac{Gm^2 4}{a\sqrt{2}} - \frac{Gm^2}{a} - \frac{Gm^2 2}{a\sqrt{3}}$$

$$= \frac{-Gm^2}{a}\left[7 + 2\sqrt{2} + 1 + \frac{2}{\sqrt{3}}\right)$$

$$= \frac{-Gm^2}{a}\left[\left(8+2\sqrt{2}\right) + \frac{2}{\sqrt{3}}\right] \cancel{= \frac{-Gm^2}{a}\left[2\left(4+\sqrt{2}\right)+\frac{1}{\sqrt{3}}\right]}$$

$$= \left(8+2\sqrt{2}+\frac{2}{\sqrt{3}}\right)\frac{-Gm^2}{a}$$

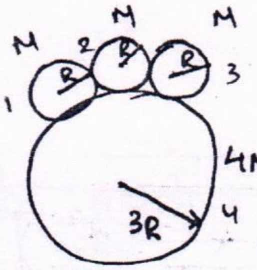
For 'n' particles.

$${}^nC_r \Rightarrow {}^nC_2 = \frac{n!}{r!\,(n-r)!} = \frac{n!}{2!\,(n-2)!}$$

$$= \frac{n!}{2\,(n-2)!} = \frac{n(n-1)\cancel{(n-2)!}}{2\,\cancel{(n-2)!}}$$

No. of sys. $\Rightarrow = \frac{n(n-1)}{2}$ $\Rightarrow$ For cross-checking.

Q



All spheres.

$\cancel{U_{sys.}} = ?$ $U_{4m} = ?$

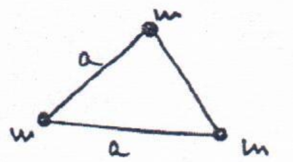
$$\cancel{= U_{12} + U_{13} + U_{14} + U_{23} + U_{24} + U_{34}}$$

$$\cancel{= \frac{-Gm^2}{2R} - \frac{Gm^2}{4R} - \frac{G4m^2}{4R} - \frac{Gm^2}{2R} - \frac{G4m^2}{4R} - \frac{G4m^2}{4R}}$$

$$\cancel{= \frac{-Gm^2}{2R}\times 2 - \frac{Gm^2}{R}\times 3}$$

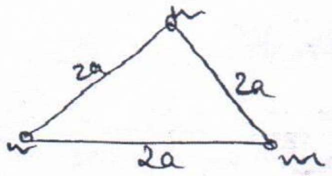
$$\cancel{= \boxed{\frac{-4Gm^2}{R}}}$$

$$= -\frac{Gm^2}{4R}\times 3\times 4 = -\frac{3Gm^2}{R}$$

Q 

→ 3 masses (+) on vertices of equilateral triangle. Find W.D. in extending the side upto $2a$.

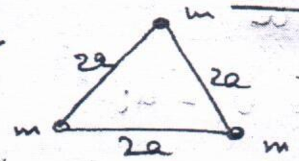
$$U_i = \frac{-Gm^2}{a} \times 3$$



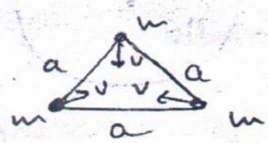
$$\hookrightarrow U_f = -\frac{Gm^2}{2a} \times 3$$

$$W.D._{ext} = \cancel{\Delta K}^{0} + \Delta U$$

$$= -\frac{3Gm^2}{2a} + \frac{Gm^2 3}{a} = \frac{3Gm^2}{2a}.$$

Q 

If the particles are released from the situa^n shown. Find speed of each ~~of~~ particle when the side will be reduced to a.

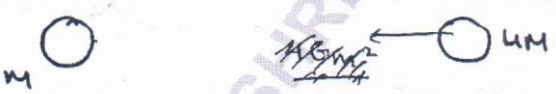


$$\cancel{W.D_{ext}}^{0} = \Delta K + \Delta U$$

$$-0 = \frac{1}{2} m v^2 \times 3 + \left(-\frac{3Gm^2}{2a}\right)$$

$$\frac{3 \cancel{m} v^2}{\cancel{2}} = \frac{\cancel{3} G m^{\cancel{2}}}{\cancel{2} a}$$

$$v = \sqrt{\frac{Gm}{a}}$$

Q 

If 2 masses are released from rest from a large dist. Find there $\vec{v}_{rel}$ of approach when separa^n b/w is R.

$$\cancel{W.D_{ext}}^{0} = \Delta K + \Delta U$$

$$\cancel{\frac{1}{2}\frac{4m^2}{5m}\vec{v}_{rel}^{\,2} + \left(\frac{-G 4m^2}{R}\right) = 0}$$

$$\cancel{\frac{4m}{2}\vec{v}_{rel}^{\,2} = \frac{G 4 m^2}{R}}$$

$$\cancel{\vec{v}_{rel} = \sqrt{\frac{2Gm}{R}}}$$

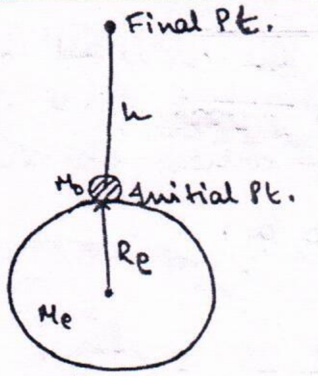
$\Delta K + \Delta U = 0$

$$\frac{1}{2}\mu\left(v_{rel_f}^2 - \cancel{v_{rel_i}^2}^{\,0}\right) + \Delta U = 0$$

$$\frac{1}{2}\frac{4m^2}{5m}\left[\vec{v}_{rel}^{\,2}\right] = \frac{4Gm^2}{R}$$

$$\vec{v}_{rel} = \sqrt{\frac{10GM}{R}}$$

WORK DONE IN RAISING A MASS UPTO HEIGHT 'H' ABOVE EARTH'S SURF.



$$WD_{ext} = \cancel{\Delta K}^{\,0} + \Delta U$$

$$WD_{ext} = \Delta U = U_f - U_i$$

$$= -\frac{GM_eM_0}{R_e+h} + \frac{GM_eM_0}{R_e}$$

$$= -GM_eM_0\left[\frac{R_e - R_e - h}{(R_e+h)R_e}\right]$$

$$= \frac{GM_eM_0h}{R_e(R_e+h)}$$

Q Find W.D. to raise a mass m_0 from surf. to a height 5 times R_e.

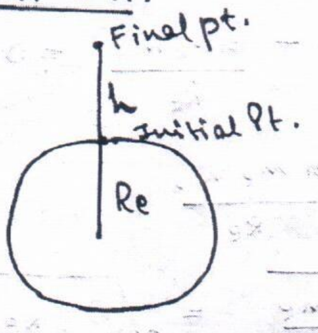
$$W.D. = \frac{GM_eM_0\,5\cancel{R_e}}{R_e \times 6\cancel{R_e}} = \frac{5GM_eM_0}{6R_e} = \frac{5}{6}M_0R_eg$$

Q Find WD in raising a mass upto a height $= 10R_e$ from surface if mass already (+) at a height $= 3R_e$ from earth's center.

$$W.D. = -\frac{GM_eM_0}{11R_e} + \frac{GM_eM_0}{3R_e} = \frac{(11-3)}{33}\frac{GM_eM_0}{R_e}$$

$$= \frac{8}{}\,GM_eM_0$$

VELOCITY REQ. TO THROW A MASS = M_0 FROM SURFACE TO HEIGHT = H.



$$W.D_{ext} = \Delta K + \Delta U$$

$$0 = -\frac{1}{2} m_0 v^2 + \left[-\frac{G M_e M_0}{R_e + h} - \frac{G M_e M_0}{R_e}\right]$$

$$0 = -\frac{1}{2} m_0 v^2 - \frac{G M_e M_0}{R_e + h} + \frac{G M_e M_0}{R_e}$$

$$\frac{1}{2}\cancel{m_0} v^2 = \frac{G M_e \cancel{M_0} h}{R_e (R_e + h)}$$

$$v = \sqrt{\frac{2 G m_e h}{R_e (R_e + h)}}$$

Q Find $\vec{v}$ req. to throw a particle from a height = Re above the earth's surf to a height = 7Re. above earth's surf.

$$W.D = 0 = \Delta k + \Delta u$$

$$0 = -\frac{1}{2} m v^2 + \left[-\frac{G M_e M}{8R_e} + \frac{G M_e M}{2R_e}\right]$$

$$\frac{1}{\cancel{2}}\cancel{m} v^2 = \frac{(8-2)}{16\,8} \frac{G M_e \cancel{M}}{R_e}$$

$$v = \sqrt{\frac{3 G M_e}{4 R_e}}$$

Q Find speed of a particle at earth's surf if it is dropped from a height = 3 Re above earth's surf.

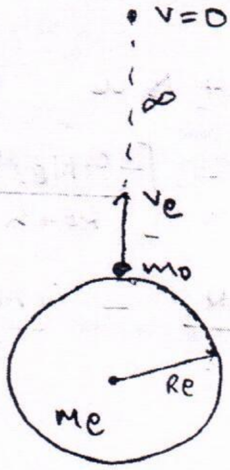
$$\Delta K + \Delta U = 0$$

$$\frac{1}{2} m v^2 + \left[\frac{-G M_e m}{4R_e} + \frac{G M_e m}{R_e}\right] = 0$$

$$\frac{\cancel{m} v^2}{2} = \frac{G m_e \cancel{m}}{R_e} - \frac{G \cancel{m} m_e}{4R_e}$$

$$\frac{v^2}{\cancel{2}} = \frac{3 G m_e}{{}_2\cancel{4}R_e} \Rightarrow v = \sqrt{\frac{3 G m_e}{2 R_e}}$$

ESCAPE VELOCITY.



$$\Delta K + \Delta U = 0.$$

$$-\frac{1}{2} m_0 v_e^2 + \left[0 + \frac{G M_e m_0}{R_e} \right] = 0$$

$$\frac{1}{2} \cancel{m_0} v_e^2 = \frac{G m_e \cancel{m_0}}{R_e}.$$

Escape velocity. ↓ from surf. $= \boxed{v_e} = \sqrt{\frac{2 G m_e}{R_e}} = \sqrt{2 g R_e}$

$= 11.2 \text{ km/s}$

'Ve' from some height $= h$

$$v_e = \sqrt{\frac{2 G M_e}{R_e + h}}$$

Q Find escape vel. from surf. of a planet whose mass $= 4 M_e$. and radius $= 2 R_e$.

$$v_e = \sqrt{\frac{2 G\, 4 M_e}{2 R_e}} = 2 \sqrt{\frac{G M_e}{R_e}} = \frac{11.2}{\sqrt{2}} \times 2 = 11.2 \times 1.4$$

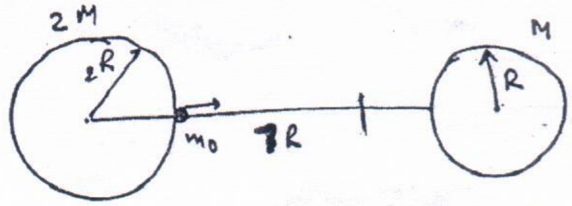
$= 15.68$

Escape vel. in terms of density.

$$v_e = \sqrt{\frac{2 G m_e \times \frac{4}{3} \pi R_e^2}{R_e \times \frac{4}{3} \pi R_e^2}}$$

$$= \sqrt{2 G \rho \frac{4}{3} \pi R_e^2} = \sqrt{\frac{8}{3} \pi G R_e^2 \rho}$$

Q



with what min speed a mass m_0 must be projected from the surf. of larger planet so that it reaches surf. of smaller planet.

$$\Delta K + \Delta U = F_{ext} \cdot = 0$$

$$-\frac{1}{2} m_0 v^2 + \left[-\frac{G m_0 2M}{9R} + \frac{G m_0 2M}{2R} - \frac{G m_0 M}{8R} + \frac{G m_0 M}{R} \right] = 0$$

$$\frac{G m_0 M}{R}\left[\frac{1}{2} - \frac{1}{9} + 1 - \frac{1}{8}\right] = \frac{1}{2} m_0 v^2$$

To get to neutral pt.

$$\frac{G m_0 4M}{(2R+x)^2} = \frac{G m_0 M}{(R+7R-x)^2}$$

$$\frac{2}{2R+x} = \frac{1}{8R-x}$$

$$16R - 2x = 2R + x$$

$$14R = 3x$$

$$x = \frac{3R}{14}$$

To reach neutral pt

$$-\frac{G m_0 4M}{2R+x} = -\frac{G m_0 M}{8R-x}$$

$$32R - 4x = 2R + x$$

$$30R = 5x$$

$$x = 6R.$$

$$\frac{G 4M}{x^2} =$$

$$\Delta K + \Delta U = 0$$

$$-\frac{1}{2} m_0 v^2 + \left[-\frac{G m_0 4M}{2.8R} + \frac{G m_0 4M}{2R} \right] = 0.$$

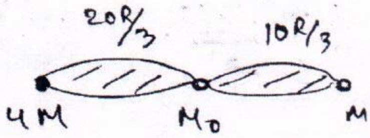
$$\frac{G m_0 M}{2R}\left[4 - 1\right] = \frac{1}{2} m_0 v^2$$

$$v = \sqrt{\frac{3GM}{R}}$$

→ Find neutral pt.
→ Work, energy theorem, $\vec{F}$ on neutral pt.

$$\frac{G4Mm_0}{x^2} = \frac{GM_0M}{(10R-x)^2}$$

$$x = 20R/3.$$



$\Rightarrow \boxed{U_{m_0}}$

$$\Delta K + \Delta U = 0.$$

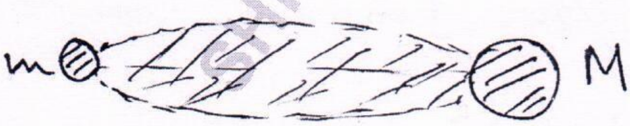
$$-\frac{1}{2}m_0v^2 + \left[\frac{-4Gm_0M\times 3}{20} - \frac{Gmm_0 3}{10R}\right] - \left[\frac{-17GMm_0}{8R}\right] = 0$$

$$\cancel{\frac{18Gm_0m}{20R}}$$

$$\cancel{\frac{1}{2}m_0v^2} = \frac{49}{\cancel{20}} \qquad V = \sqrt{\frac{49}{20}\frac{GM}{R}}$$

GRAVITATIONAL POTENTIAL [V] → scalar q/ty.

⇒ G.P. of a mass is the capacity of that mass to store the gravitaⁿ potential energy. It is defined as gravitaⁿ potential energy stored per unit mass.



$$U_r = -\frac{GMm}{r}.$$

$$V_r = \frac{U_r}{m}$$

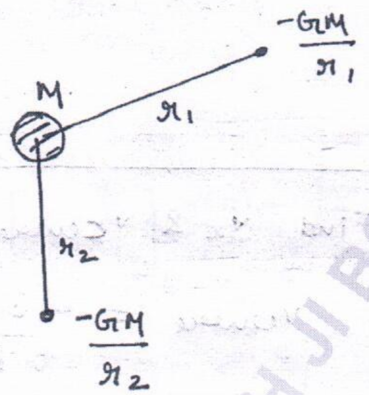
$$V_{r_m} = \frac{U_r}{M} = \frac{-GMm/r}{M} = -\frac{Gm}{r}$$

$$V_{r_M} = \frac{U_r}{m} = \frac{-GMm/r}{m} = -\frac{GM}{r}.$$

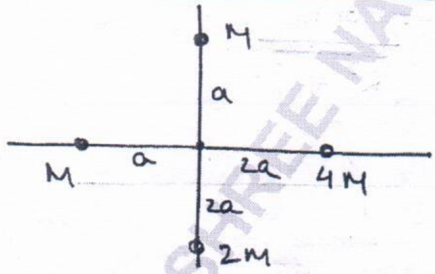
$\Rightarrow$ Gravita^n Potential of a mass at a dist. r is numerically equal to work done in bringing a unit mass from ∞ to dist. r from the source mass

$$W.D. = U_r = -\frac{GM_1M_2}{r} = -\frac{GM \times 1}{r}$$

eg.



Q



Find V_{net} at origin due to sys.

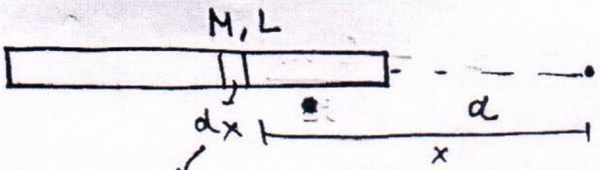
$$V_{net} = -\frac{GM}{a} \times 2 - \frac{GM}{2a} \cdot 4 - \frac{GM}{2a} \cdot 2$$

$$= -\frac{GM}{a}[2+2+1] = -\frac{5Gm}{a}$$

$$\frac{-5GMM_T}{a} = U_{sys}$$

$M_T \to$ Test mass

#



$$dm = \lambda \cdot dx = \frac{M}{L} \cdot dx$$

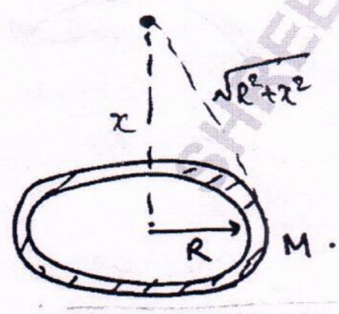
$$\int dV_{dx} = \int \frac{-G\,dm}{(\ \ x)}$$

$$V = \int_{d}^{d+L} \frac{-GM \cdot dx}{L(\ \ x)}$$

$$= \frac{-GM}{L} \int_{d}^{L+d} \frac{1}{x} \cdot dx = \frac{-GM}{L} \Big[\ln x \Big]_{d}^{L+d}$$

$$= \frac{-GM}{L} \left[\ln \frac{(L+d)}{d} \right]$$

Q

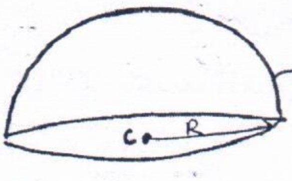


Find V_x & V_{centre}.

$$V_{centre} = -\frac{GM}{R}$$

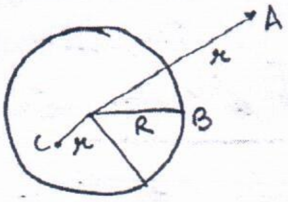
$$V_x = \frac{-GM}{\sqrt{R^2+x^2}}$$

$GP_c = V_c = \dfrac{-GM}{R}$.



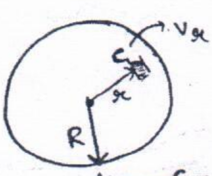
Hollow Hemisphere of mass = m

GP due to shell



$r > R \Rightarrow -\dfrac{GM}{R}$

$r = R \Rightarrow \dfrac{-GM}{r}$



$V_R = \dfrac{-GM}{R}$

$\Rightarrow \Delta V = -\int_r^R \vec{I_g}\cdot\vec{dr}$

$V_R - V_r = 0$

$\boxed{V_R = V_r}$

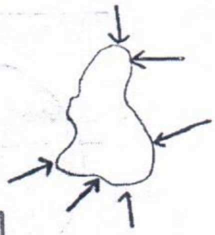
when $r < R$

Relaⁿ b/w G.P. and G.P.E

$dW_{C.F.} = -du = dW_{G.F.}$

$dv = \dfrac{+du}{m} = \dfrac{-dW_{G.F.}}{M}$

$dv = \dfrac{-\vec{F_G}\cdot\vec{dr}}{M}$



$dv = -\vec{I_g}\cdot\vec{dr}$

$\boxed{\Delta V = -\int \vec{I_g}\cdot\vec{dr}}$

$\vec{I_g} = -\dfrac{\partial v}{\partial x}\hat{i} - \dfrac{\partial v}{\partial y}\hat{j} - \dfrac{\partial v}{\partial z}\hat{k}$

Q If G.P. due to a mass distribuⁿ is given as :- $V = x^2 + 2xy$. Find G.Force on a mass of 2Kg. at a pt. 1,2,3.

$\vec{I_g} = -2x\hat{i} - 2y\hat{i} - 2x\hat{j}$

$= (-2x - 2y)\hat{i} - 2x\hat{j}$

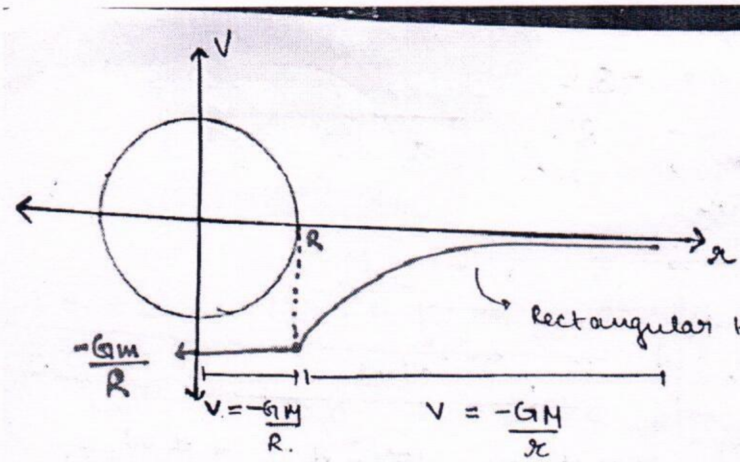
$= (-2-4)\hat{i} - 2\hat{j} = -6\hat{i} - 2\hat{j} \Rightarrow \vec{F} = \vec{I_g}\cdot m$

$= -12\hat{i} - 4\hat{j}$

$= \sqrt{180}$ N

$\Rightarrow$ Gravitaⁿ field lines are always $\perp$ to a equipotential surf..

$\Delta V = 0$



$\Rightarrow$ continuous graph

$\Rightarrow$ non-derivative func.

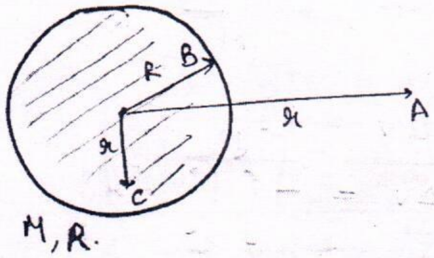
$\Downarrow$

slope not defined

$\downarrow$

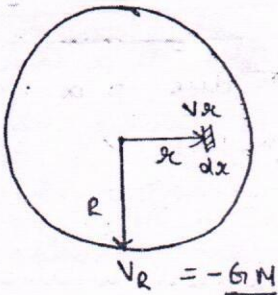
bcz at peak we cant draw tangent.

G.P. of Solid sphere



$$V_A = -\frac{GM}{r}$$

$$V_B = -\frac{GM}{R}$$



$$V_R = -\frac{GM}{R}$$

$$\Delta V = -\int \vec{I_g}\cdot\vec{dr}$$

$$= -\int I_g \cdot dr \cos 180$$

$$= +\int_r^R \frac{GMr}{R^3}\cdot dr$$

$$\Delta V = \frac{GM}{R^3}\left[\frac{r^2}{2}\right]_r^R = \frac{GM[R^2-r^2]}{2R^3}$$

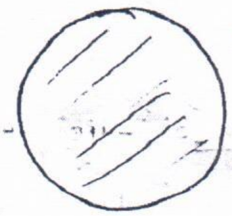
$$\boxed{\therefore V_{centre} = -\frac{3GM}{2R}}$$

$$-\frac{GM}{R} - V_r = \frac{GM\,[R^2-r^2]}{2R^3}$$

$$-\frac{GM}{R} - \frac{GM[R^2-r^2]}{2R^3} = V_r = \frac{GM}{R}\left[\frac{[r^2-R^2]}{2R^2} - 1\right]$$

$$= \frac{GM}{R}\left[\frac{r^2-3R^2}{2R^2}\right] = \frac{GM[r^2-3R^2]}{2R^3} = -\frac{GM}{2R^3}[3R^2-r^2]$$

Q. Find V at a) R/2 b) 2R c) 5R d) 8R.



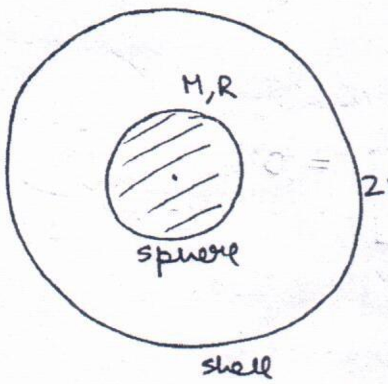
a) $V_{R/2} = \dfrac{-G2M\left[3\times 25R^2 - \frac{R^2}{4}\right]}{2R^3\times 125}$

$= \dfrac{-GM}{R}\left[\dfrac{299}{500}\right]$

b) $V_{2R} = \dfrac{-G2M\left[3\times 25R^2 - 4R^2\right]}{2R^3\times 125} = -\dfrac{GM}{R}\left[\dfrac{71}{125}\right]$

c) $V_{5R} = \dfrac{-G2M}{5R}$ d) $V_{8R} = \dfrac{-2GM}{8R} = -\dfrac{GM}{4R}$

Q.



a) $V_{R/2}$ b) V_{3R} c) V_{4R} d) V_{5R}

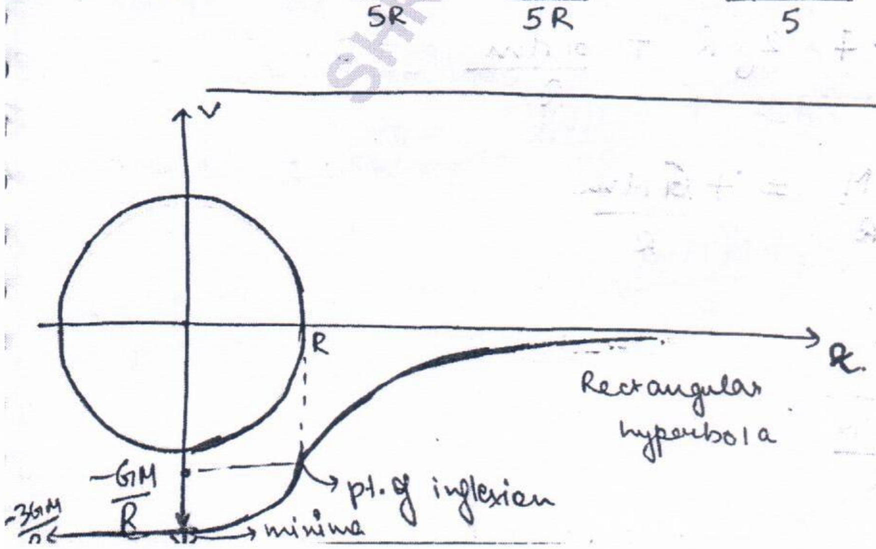
a) $V_{R/2} = \dfrac{-GM}{2R^3}\left[3R^2 - \dfrac{R^2}{4}\right] + \left[\dfrac{-2GM}{4R}\right]$

$= \dfrac{-11GM}{8R} - \dfrac{GM\times 4}{2R\times 4} = \dfrac{-15GM}{8R}$

b) $V_{3R} = \dfrac{-GM}{3R} \dfrac{-2GM}{4R} = \dfrac{-5GM}{6R} = \dfrac{-5GM}{6R}$

c) $V_{4R} = \dfrac{-GM}{4R} \dfrac{-2GM}{4R} = \dfrac{-3GM}{4R}$

d) $V_{5R} = \dfrac{-GM}{5R} \dfrac{-2GM}{5R} = \dfrac{-3GM}{5}$

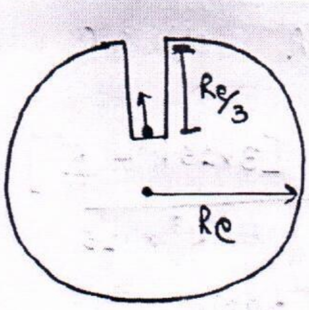


$y = -K\left[a - x^2\right]$

$y = -Ka + x^2K$

$y = x^2 - Ka$

Q. Find escape vel. from a depth $= \frac{R_e}{3}$.



$$\Delta K + \Delta u = 0$$

$$0 - \frac{1}{2}mv^2 + 0\left[\frac{+GMm\left[3R_e^2 - \frac{4R_e^2}{9}\right]}{2R_e^3}\right] = 0$$

$$\frac{1}{2}\cancel{m}v^2 = +\frac{GM_e\cancel{m}}{2R_e^3}\left[\frac{23R_e^2}{9}\right]$$

Use Potential to find ve at a depth.

$$v = +\sqrt{\frac{GM_e}{R_e}\left[\frac{23}{9}\right]}$$

Q. If a body is thrown $\bar{c}$ half of the escape velocity from earth's surf. Find Hmax reached by body, above surf.

$$0 - \frac{1}{2}mv^2 + \left[\frac{-GMm}{R_e+h} + \frac{GMm}{R_e}\right] = 0.$$

$$\frac{1}{2}\cancel{m}v^2 = \frac{GM\cancel{m}h}{R_e+h}$$

$$\frac{\cancel{G}\cancel{M}}{2R^{\cancel{2}}\times 4}\cdot\cancel{R} = \frac{\cancel{G}\cancel{M}h}{R_e+h} \Rightarrow R+h = 8Rh$$

$$R = (8R-1)h$$

$$h = R/2R-1$$

Q. If the body is thrown $\bar{c}$ 2Ve. Find its speed at ∞.

$$\frac{1}{2}mv^2 - \frac{1}{2}\times m\times 4\times 2gR + \frac{GMm}{R} = 0.$$

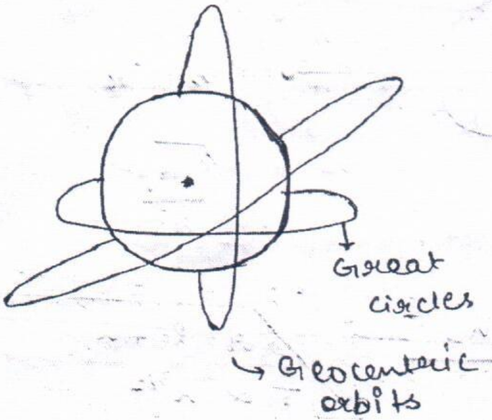
$$-\frac{\cancel{m}v^2}{2} + 4\cancel{m}\frac{GM}{R} = +\frac{GM\cancel{m}}{R}.$$

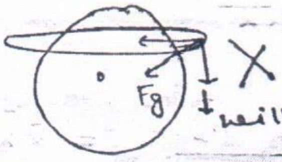
$$\frac{v^2}{2} = \frac{3GM}{R}$$

$$V = \sqrt{\frac{G6M}{R}}$$

SATELLITE

⇒ Centre of orbit should be coincidental $\bar{c}$ the centre of earth for a stable height.





will make satellite unstable

Types of orbit

a) Low orbit (180Km − 2000Km) → for GPS

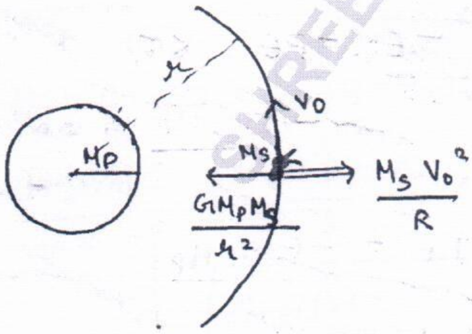
b) Medium orbit (2000 Km − 35786 Km) ≈ 36000 Km.

- → sun-synchronous → once a day → in $\boxed{\text{Polar orbit}}$ → for weather forecast
- → Molenya → elliptical orbit
- → semi-synchronous → crosses same place twice

c) High orbit

- ↳ (≥ 35786 Km)
- ↳ for communicaⁿ, mobile signals

ORBITAL VELOCITY [v_0]



$$\frac{GM_pM_s}{r^2} = \frac{M_s v_0^2}{r}$$

$$v_0 = \sqrt{\frac{GM_p}{r}}$$

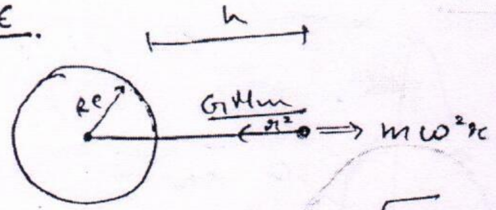
r from the center of earth

$\boxed{V_e = \sqrt{2}\, v_0}$

$$V_e = \sqrt{\frac{2GM_p}{r}}$$

↑ escape velocity from same pt.

TIME PERIOD FOR SATELLITE



$$T = \frac{2\pi}{\omega}$$

$$\omega = \sqrt{\frac{GM_p}{r^3}}$$

$$= \frac{2\pi\sqrt{r^3}}{\sqrt{GM_p}}$$

$$= \frac{2\pi\,[R_p+h]^{3/2}}{\sqrt{GM_p}} = \frac{2\pi\,(R_p+h)^{3/2}}{\sqrt{g R_p^2}}$$

$\Rightarrow r = R_e + h$

$\Rightarrow T \propto r^{3/2}$

$\Rightarrow T^2 \propto r^3$

$\Rightarrow \frac{T_1^2}{T_2^2} = \frac{r_1^3}{r_2^3}$

Time period of near earth satellite

$h = 0.$

$$T = \frac{2\pi (R_p)^{3/2}}{\sqrt{g}\cdot R_p} = 2\pi\sqrt{\frac{R_p}{g}} = 2\pi\sqrt{\frac{640\cancel{0}\times 10^3}{1\cancel{0}}} \quad \cancel{= \frac{2\times 22\times 10^2}{7}}$$

$$= 10^2 \times 8 \times 2 \times \frac{22}{7} = \boxed{84.6\ \text{min}}\ \#$$

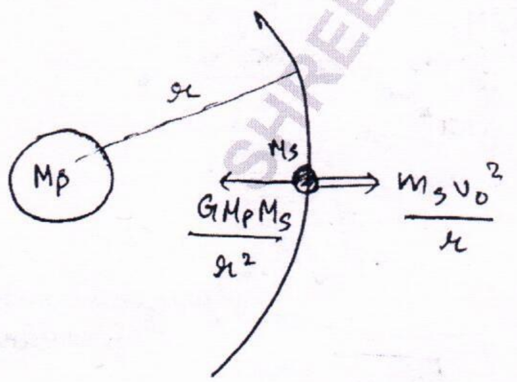
~~$v_e = \sqrt{2}v_0$~~

v_0 for near earth orbit.

$$v_e = \sqrt{2}\, v_0$$

$$\frac{v_e}{\sqrt{2}} = v_0 = \frac{\cancel{11.2}^{\,56}}{\cancel{1.4}_{\,7}} = \boxed{8\ \text{km/s}}\ \#$$

ENERGY IN PLANET - SATELLITE SYS.



$$TE = PE + KE$$

$$= -\frac{GM_sM_p}{r} + \frac{GM_sM_p}{2r}$$

$$\boxed{TE = -\frac{GM_sM_p}{2r}}$$

of planet - satellite sys.

$$\frac{r}{2}\times\frac{GM_pM_s}{r^2} = \frac{m_s v_0^2}{r}\times\frac{r}{2}$$

$$KE = \frac{GM_pM_s}{2r}$$

$$|TE| = \frac{|PE|}{2} = KE$$

BINDING ENERGY [BE]

→ min. energy given to a bound [planet - satellite] sys. to unbind the sys.

$$TE + BE = 0$$

$$\boxed{BE = -TE}$$

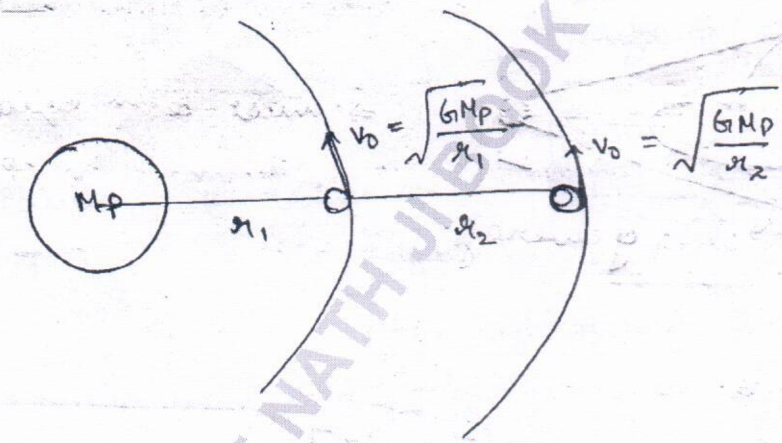
WORK - DONE IN CHANGING THE ORBIT OF SATELLITE

$$WD_{ext} = \Delta K + \Delta u$$

$$= (K_f + u_f) - (K_i + U_i)$$

$$\boxed{W.D_{ext} = T_f - T_i}$$

$$= -\frac{G M_P M_S}{2 r_2} + \frac{G M_P M_S}{2 r_1}$$



Q. Find W.Dext in changing the orbit of a satellite of mass $= \frac{1}{16}^{th}$ Me, if it was initially in a orbit at a height = Re, above the earth's surf. and finally it is at a height = 4Re above the earth's surf.

$$W.D_{ext} = T_f - T_i = -\frac{G M_e \times \frac{1}{16} M_e}{2 R_e \times 5} + \frac{G M_e \frac{1}{16} M_e}{2 R_e \times 2}$$

$$= \frac{G M_e^2}{16}\left[\frac{-1}{10 R_e} + \frac{1}{4 R_e}\right] = \frac{G M_e^2}{16}\left[\frac{10-4}{40 R_e}\right] = \frac{G M_e^2 \times 3 \times R_e}{16 \times 20 \times R_e^2}$$

$$= \frac{g \times M_e \times 3 \times R_e}{16 \times 20} = \frac{10 \times 6 \times 10^{24} \times 3 \times 6400}{16 \times 20} = 36 \times 10^{26}$$

Q A satellite of mass = 1000 Kg is orbiting earth at a h = 2000 Km. Find a) speed b) KE c) PE of Planet - Sat. sys. $= -4.76 \times 10^{10}$

d) Time period $= 7.2365$ s $= 2.01$ hr.

a) $v_0 = \sqrt{\frac{GM_p}{R}} = \sqrt{gR} = \sqrt{9.8 \times 8400 \times 10^3} = \sqrt{98 \times 840 \times 10^3}$

$= 7 \times \sqrt{2 \times 10 \times 4 \times 21} = 7 \times 2 \times 2 \sqrt{210} = 98\sqrt{210 \times 10^3}$

$= 98 \times 10^2 \sqrt{21}$

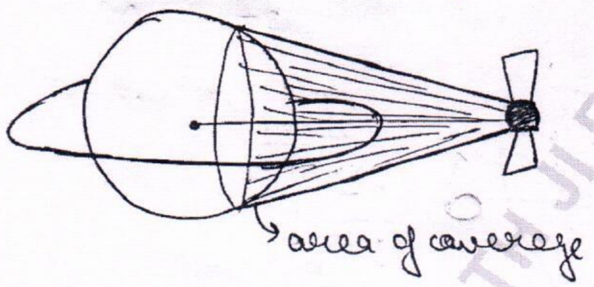
b) $KE = \frac{GM_p M_s \cdot R}{2R \cdot R} = \frac{g \times M_s \times R}{2} = \frac{4.9 \times 1000 \times \cancel{6400}^{3200} \times 10^3}{\cancel{2}}$

$= 4.9 \times 32 \times 10^2 \times 10^6$

$= 156.8 \times 10^8$

GEOSTATIONARY SATELLITE

$\Rightarrow \omega \rightarrow$ same as ω of earth.



$\Rightarrow$ must be in equatorial plane.

$\Rightarrow T = 24$ hrs.

$\omega = \frac{2\pi}{T} \longrightarrow \left[T = \frac{2\pi}{\omega} \right]$

$T = \frac{2\pi [R_e + h]^{3/2}}{\sqrt{g} \cdot R_e}$

$\boxed{h = 35786 \text{ Km} \simeq 36000 \text{ Km}}$

Kepler's Laws

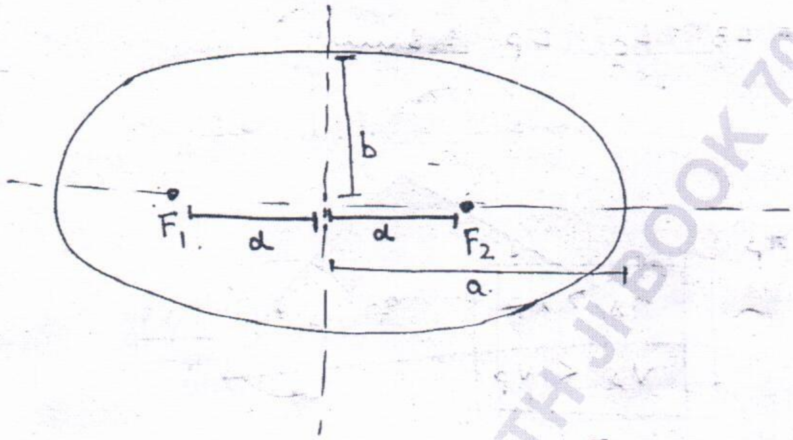
law

1. ⇒ Planets revolve around Sun in elliptical orbits.

⇒ Speed of the planets is max. when it is nearest to the Sun and min. when it is farthest from the Sun.

⇒ Sun is at one of the focus of elliptical orbits.

Ellipse का ज्ञान



represents flatness of ellipse = eccentricity = $\boxed{e} = \dfrac{\text{dist. b/w foci}}{\text{length of major axis}}$

$$= \frac{2d}{2a}$$

$$\boxed{d = ae}$$

a = semi-major axis

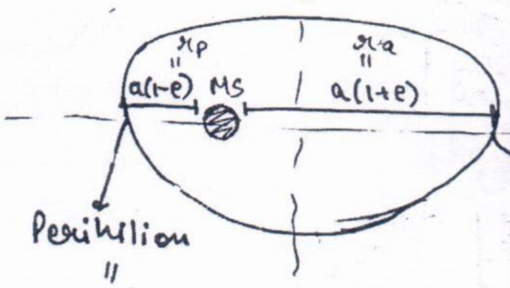
b = semi-minor axis

For ellipse ⇒ $1 > e > 0$

$$b^2 = a^2(1-e^2)$$

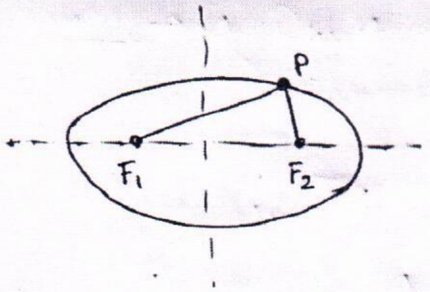
$e = 0 \rightarrow$ circle path

$e = 1 \rightarrow$ straight-line path



→ aphelion = apogee

Perihelion = Perigee



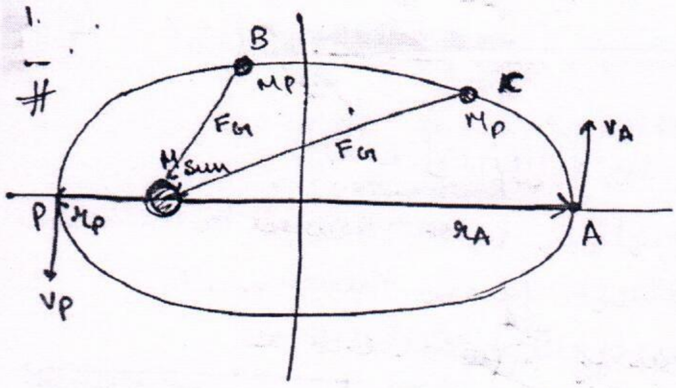
$$PF_1 + PF_2 = 2a$$

$$\text{Mean radius} = \frac{r_p + r_a}{2} = a$$

semi-major axis

1.

#



τ of $\vec{F_G}$ on planet due to Sun $= 0$

↓

'L' of planet about Sun $=$ const.

$$L_A = L_B = L_C = L_P = \text{same}.$$

$$L_A = L_P$$

$$\cancel{m_p} v_A r_A = \cancel{m_p} v_p r_p$$

$$\boxed{\frac{v_A}{v_p} = \frac{1-e}{1+e}} \qquad \boxed{\begin{matrix} r_A > r_P \\ v_A < v_p \end{matrix}} \qquad \Rightarrow v_A = \frac{(1-e)\,v_p}{1+e}$$

$$\Delta K + \Delta u = 0.$$

~~$m_p v_A r_A$~~

$$\frac{1}{2} M_p v_p^2 - \frac{1}{2} M_p v_A^2 + \left[-\frac{G M_p M_s}{a(1-e)} + \frac{G M_p M_s}{a(1+e)}\right] = 0.$$

$$\frac{\cancel{M_p}}{2}\left[v_p^2 - v_A^2\right] = \frac{G \cancel{M_p} M_s}{a}\left[\frac{1}{1-e} - \frac{1}{1+e}\right]$$

$$\frac{(v_p - v_A)(v_p + v_A)}{2} = \frac{G M_s}{a}\left[\frac{1+e-1+e}{1-e^2}\right]$$

$$\left(v_p - \frac{(1-e)v_p}{1+e}\right)\left(v_p + \frac{(1-e)v_p}{1+e}\right) = \frac{2 G M_s}{a} \cdot \frac{2e}{(1-e^2)}$$

$$\frac{\left(v_P\left[(1+e)+(1+e)\right]\right)\left(v_P\left[(1+e)+(1+e)\right]\right)}{(1+e)^3} = \frac{4\,G M_S \cdot e}{a(1-e^2)}$$

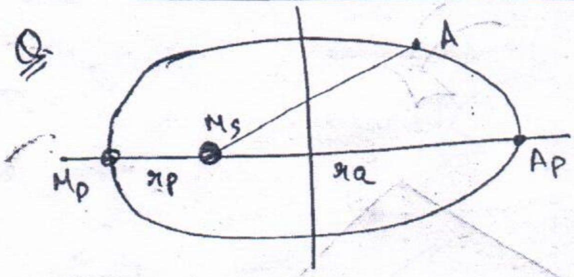
$$\frac{2 v_P e \cdot 2 v_P \cancel{(1+e)}}{(1+e)^{\cancel{3}\,2}} = \frac{4 G M_S \cdot e}{a(1-e^2)}$$

$$4 v_P^2 \cdot \cancel{e}\,(1+e)^2 = \frac{4 G M_S \cdot \cancel{e}}{a(1-e^2)}$$

$$v_P^2 = \frac{G M_S\,(1+e)^2}{a(1-e)^2\,\cancel{(1+e)}}$$

$$\left\{ v_P = \sqrt{\frac{G M_S (1+e)}{a\,(1-e)}} \qquad v_A = \sqrt{\frac{G M_S}{a}\,\frac{(1-e)}{1+e}} \right\}$$

Q.



Find $\vec{L}$ at A.

$$\vec{L}_A = \vec{L}_{AP}.$$

$$= m_P\, v_A\, r_A.$$

$$= m_P \sqrt{\frac{G M_S}{a}\,\frac{(1-e)}{(1+e)}} \cdot r_A.$$

$$= m_P \frac{\sqrt{G M_S (1-e)}}{\sqrt{r_A}} \cdot r_A$$

$$= m_P \cdot \sqrt{r_A} \cdot \frac{\sqrt{G M_S\, a(1-e)}}{\sqrt{a}}.$$

$$= m_P \sqrt{r_A} \cdot \sqrt{r_P} \cdot \sqrt{\frac{G M_S}{a}}$$

$$= m_P \sqrt{r_A \cdot r_P} \cdot \sqrt{\frac{G M_S \cdot 2}{(r_a + r_p)}}$$

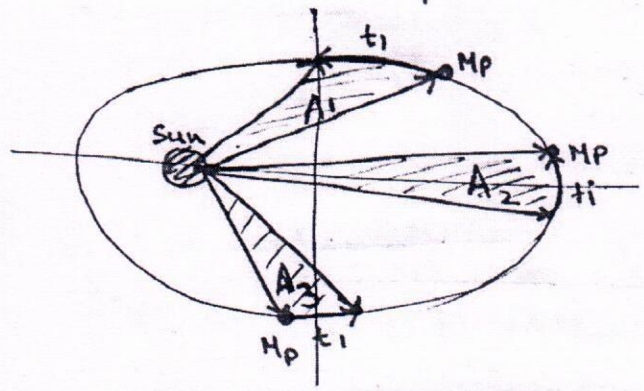
$a(1+e) = r_A.$
$a(1-e) = r_P$
$2a = r_a + r_p.$

$$a = \frac{r_a + r_p}{2}.$$

$$\cancel{= m_P \cdot \sqrt{2 G M_S}}$$

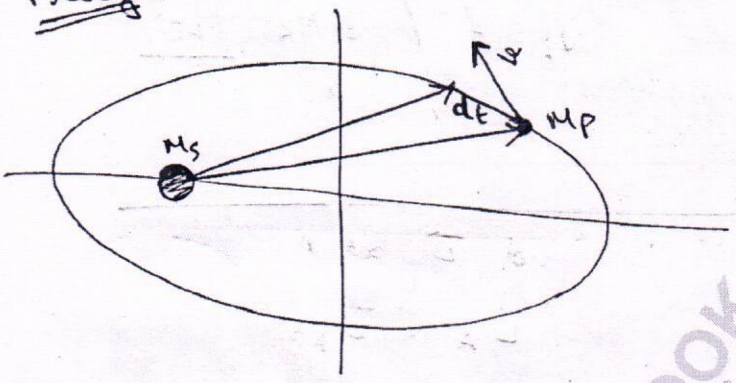
$$\boxed{= m_P \sqrt{\frac{2 G M_S\, r_A \cdot r_P}{r_A + r_P}}}$$

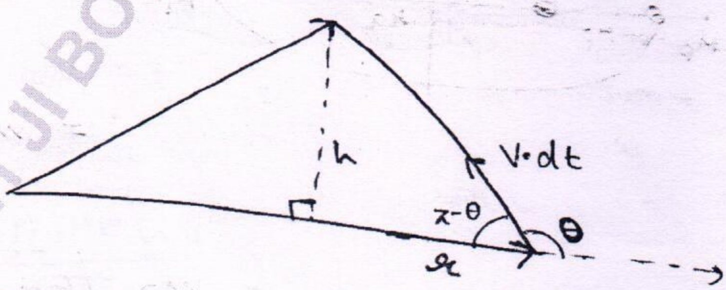
2^n Law $\rightarrow$ A Planet sweeps equal areas in equal time intervals $\bar{c}$ respect to sun.



$A_1 = A_2 = A_3$.

Proof.





$h = v \cdot dt \sin(\pi - \theta)$

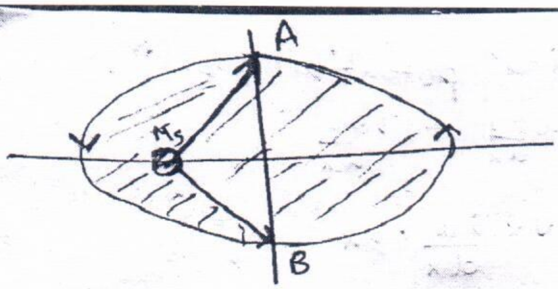
$= v\,dt \sin\theta.$

$dA = \frac{1}{2} r \cdot v\,dt \sin\theta$

$\frac{dA}{dt} = \frac{1}{2} v r \sin\theta \times \frac{M_p}{M_p}$

$\boxed{\frac{dA}{dt}} = \frac{\vec{L}}{2M_p} = \text{const.}$

areal speed (m^2/s)

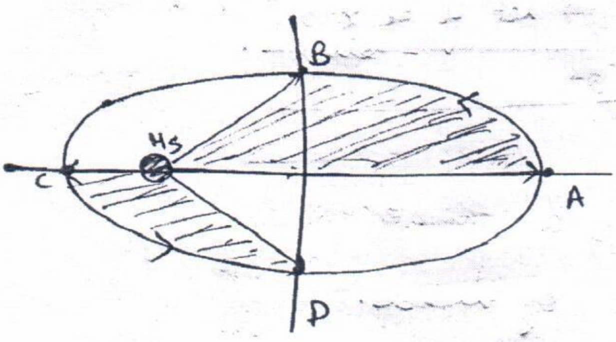


$t_{B\to A} > t_{A\to B}$

↓

bcz

$Area_{B\to A} > Area_{A\to B}$



$t_{A\to B} > t_{C\to D}$

↓

bcz

$\cancel{area}_{A\to B} > \cancel{area}_{C\to D}$

3rd Law → (Time period)² of a planet around sun is proportional to cube of semi-major axis.

$$T = \frac{\text{Total area}}{\text{areal speed}} = \frac{\pi a b \times 2m_p}{L}$$

$$= \frac{\pi a \times a\sqrt{1-e^2} \times 2M_p}{M_p \times a(1+e) \times \sqrt{\frac{GM_s}{a}\frac{(1-e)}{(1+e)}}}$$

$T = a^{3/2}$ [const.] $\cancel{a^{3/2}}$ $\frac{T^2}{a^3} = \frac{4\pi^2}{GM}$

$T \propto a^{3/2} \longrightarrow \boxed{T^2 \propto a^3}$

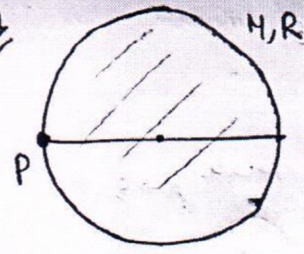
Q. Find time period of a planet whose mean radius is 4 times the mean radius of earth about sun.

$$\frac{T_1^2}{T_2^2} = \frac{a_1^3}{a_2^3} \Rightarrow \frac{365 \times 365}{T_2^2} = \frac{\cancel{a_1^3}}{64\cancel{a_1^3}}$$

$$T_2 = \underset{\text{days}}{8 \times 365} = 8 \text{ yrs}$$

Scanned with CamScanner

Q. Find gravita^n potential at P.



$$** \; \frac{d(uv)}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$

$$uv = \int u \times \frac{dv}{dx}\cdot dx + \int v \times \frac{du}{dx}\cdot dx$$

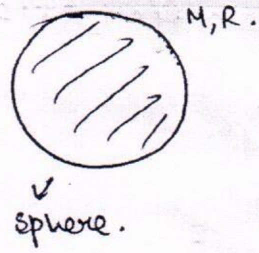
$$\int u\frac{dv}{dx}\cdot dx = uv - \int v\frac{du}{dx}\cdot dx$$ $\Rightarrow$ Let u be x, $v = \sin x$

SELF - ENERGY

→ W.D. by the ext. agent to assemble the config. $\bar{c}$ out changing the KE of mass.

||

Self-energy of the config.



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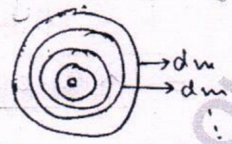
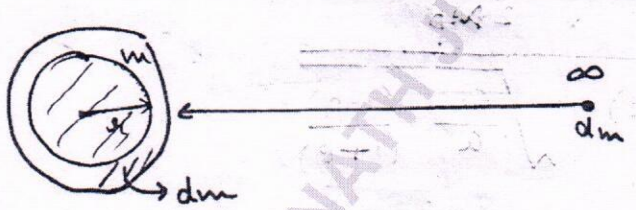




$$W.D = -\frac{Gm\cdot dm}{r} = du.$$

$$= \frac{-G\rho\frac{4}{3}\pi r^{3}\cdot \rho 4\pi r^2 dr}{r} = du$$

$$\cancel{= \frac{-16G\rho^2\pi^2 r^2\cdot dr}{-9} = du}$$

$$\cancel{\frac{-16G\rho^2\pi^2}{9}\int r^2\cdot dr = \int_0^{u} du}$$

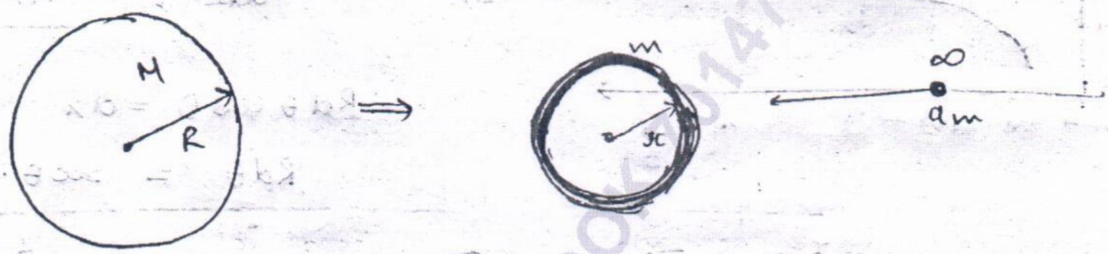
$$\cancel{u = \frac{-16 G \rho^2 \pi^2}{9} \cdot \frac{r^3}{3}}$$

$$dU = \frac{-16}{3} G \rho^2 \pi^2 r^4 \cdot dr \Rightarrow \int dU = \frac{-16}{3} G \rho^2 \pi^2 \left[\frac{r^5}{5}\right]_0^R$$

$$U = \frac{-G \times 16}{3} \times \frac{M^2}{\frac{16\pi^2}{9} R^6} \times \frac{R^5}{5} \times \pi^2$$

$$\boxed{U = -\frac{3}{5} \frac{G M^2}{R}}$$

Self-energy of shell

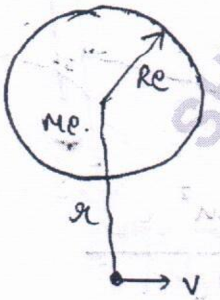


$$\int_0^M \frac{-Gm \cdot dm}{r} = \int dU$$

$$U = -\frac{GM^2}{2R}$$

$$\cancel{-G\, 4\pi r^2 \rho}$$

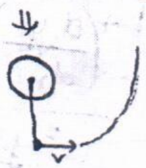
TRAJECTORY



- If $v = 0 \longrightarrow$ straight line.
- $v < v_0 \longrightarrow$ arc. of ellipse ⇒

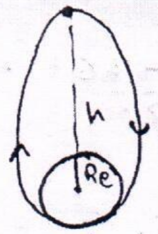
- $\boxed{\#\ v < v_0 \rightarrow \text{far focus}}$ ellipse
- $v = v_0 \longrightarrow$ circle
- $v_e > v > v_0 \longrightarrow$ ellipse (near focus)

- $v = v_e \rightarrow$ Parabolic escape

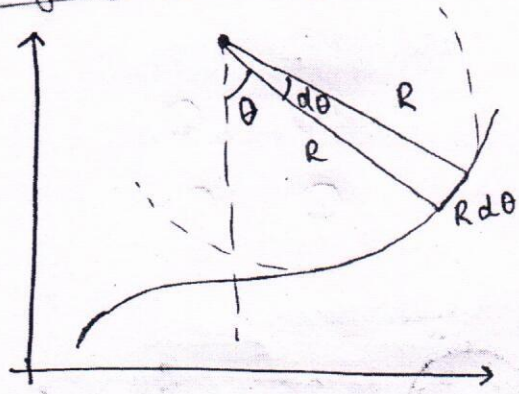


- $v > v_e \rightarrow$ hyperbolic escape

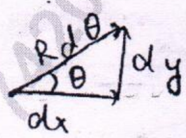
Q Find v_{min} for this trajectory.



Proof of Radius of Curvature.



$$\tan\theta = \frac{dy}{dx}$$



$$R\,d\theta\cos\theta = dx$$

$$R\,d\theta = \sec\theta \cdot dx$$

$$R = \sec\theta \frac{dx}{d\theta} = \frac{\sec^3\theta}{\frac{d^2y}{dx^2}}$$

Differentiating

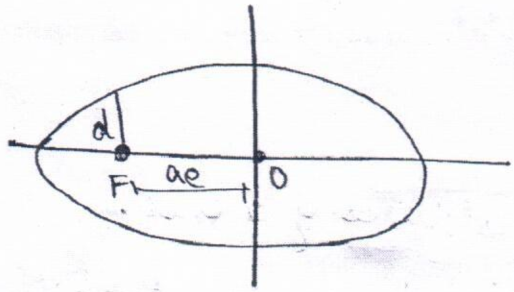
$$\tan\theta = \frac{dy}{dx} \longrightarrow \sec^2\theta \times \frac{d\theta}{dx} = \frac{d^2y}{dx^2}$$

$$\frac{dx}{d\theta} = \frac{\sec^2\theta}{\frac{d^2y}{dx^2}}$$

$$R = \frac{\left[1+\tan^2\theta\right]^{3/2}}{\frac{d^2y}{dx^2}}$$

$$= \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}}$$

#



$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

$$\frac{a^2e^2}{a^2}+\frac{d^2}{b^2}=1$$

$b^2=a^2[1-e^2]$

$$d^2=b^2(1-e^2)$$

$$d^2=a^2(1-e^2)^2$$

$$\boxed{d=a(1-e^2)}$$

$$d=\underbrace{a(1-e)}_{r_1}\overbrace{(1+e)a}^{r_2}\frac{}{\underset{\to \frac{r_1+r_2}{2}}{a}}$$

$$=\frac{2r_1r_2}{r_1+r_2}$$

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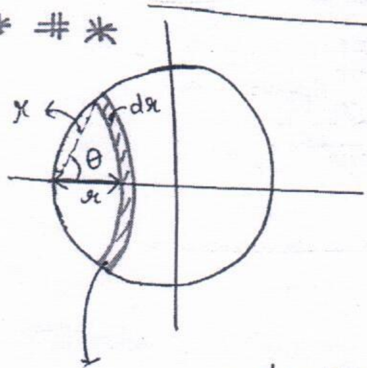
* # *

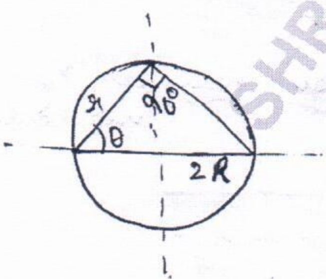


$$dV=-G\frac{[2\sigma\theta r\cdot dr]}{\cancel{r}}$$

$$V=-G\sigma 2\int\theta\cdot dr.$$

$$=+G\sigma 2\int\theta\cdot 2R\sin\theta\cdot d\theta$$

$dm=\sigma\,dr\,2\theta r.$



Let θ=x

$$=4\sigma GR\int_{\pi/2}^{0}\theta\sin\theta\cdot d\theta$$

$$=4\sigma GR\left[-x\cos x-\int -\cos x\cdot 1\cdot dx\right]_{\pi/2}^{0}$$

$$=4\sigma GR\left[-x\cos x+\sin x\right]_{\pi/2}^{0}$$

$$=4\sigma GR[-0+0-0-1]=-4\sigma GR$$

$$=\cancel{4\sigma GR[1-x]}=\cancel{\frac{4M}{\pi R^2}GR[1-\theta]}=\cancel{\frac{4MGR}{\pi R}}$$

$$=\cancel{(4GM/\pi R)\times(1-\theta)}$$

$2R\cos\theta=r$

$-2R\sin\theta\cdot d\theta=dr$

$\sigma=\frac{M}{\pi R^2}$