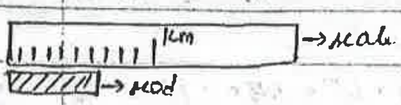
19/09/19

Unit, Dimension, Measurement & Error Analysis

• **Errors.**



∴ Least Count:-

10 division = 1cm

1 division = $\frac{1}{10}$ cm = 1mm.

$\Rightarrow$ L.C. = 1mm.

Error ⇒ $l = 6 \pm 1$ mm

→ Max^m possible error in a measurement will be equal to least count of measuring Instrument.

$$\boxed{\text{Accuracy} \propto \frac{1}{\text{Least Count}}}$$

(1). Systematic errors

→ Source of the systematic errors are known

→ These errors can be rectified completely by proper adjustment in calculation.

(2). Random Errors.

→ Source of the random errors are unknown.

→ These errors can not be rectified completely but we can reduce these errors by repeating the experiment a number of times.

Value ← $l \pm \Delta l$ → absolute error

$l \longrightarrow \Delta l$

$1 \longrightarrow \frac{\Delta l}{l}$ → fractional error OR Relative error

$\frac{\Delta l}{l} \times 100$ = Percentage ~~Relative~~ error

\# l_1, l_2, l_3, l_4 } four measurements of length

$l_m = \frac{l_1 + l_2 + l_3 + l_4}{4}$ (l_m → Avg value or mean value)

∴ $\Delta l_1 = |l_1 - l_m|$

$\Delta l_2 = |l_2 - l_m|$

$\Delta l_3 = |l_3 - l_m|$

$\Delta l_4 = |l_4 - l_m|$

Now, $\Delta l_m = \frac{\Delta l_1 + \Delta l_2 + \Delta l_3 + \Delta l_4}{4}$

Now, $\boxed{l' = l_m \pm \Delta l_m}$

eg:- $\mu_1 = 1.60$ F/o - ① μ_m ② $\Delta\mu_m$ ③ Relative error

$\mu_2 = 1.65$ ④ % error.

$\mu_3 = 1.75$

$\mu_4 = 1.70$

① $\mu_m = \frac{1.60+1.65+1.75+1.70}{4} = \frac{6.50}{4} = 1.625$

$\mu_m = 1.62$

②. $\Delta\mu_1 = |\mu_1 - \mu_m| = |1.60 - 1.62| = 0.02$

$\Delta\mu_2 = |1.65 - 1.62| = 0.03$

$\Delta\mu_3 = |1.75 - 1.62| = 0.07$

$\Delta\mu_4 = |1.70 - 1.62| = 0.08$

$\therefore \Delta\mu_m = \frac{0.02+0.03+0.07+0.08}{4} = \frac{0.2}{4} = 0.05$

③ Relative error $= \frac{0.05}{1.62} = \frac{5}{162}$

④ % error $= \frac{5}{162} \times 100 = \frac{500}{162}$

- Propogation of Errors

Let $x \pm \Delta x$ and ① $A = xy$ Then $\Delta A = ?$ ③ $C = x^p y^q$

$y \pm \Delta y$ ② $B = \frac{x}{y}$ $\Delta B = ?$ ④ $D = \frac{x^p y^q}{z^r}$

$\Rightarrow A = xy$

$\log A = \log x + \log y$

differentiate both side.

$\frac{dA}{A} = \frac{dx}{x} + \frac{dy}{y}$

$100 \times \frac{dA}{A} = \frac{dx}{x} \times 100 + \frac{dy}{y} \times 100$

$\Rightarrow \boxed{\% \Delta A = \% \Delta x + \% \Delta y}$

④ $\% \Delta D = p(\% \Delta x) + q(\% \Delta y) + r(\% \Delta z)$

② $\% \Delta B = \% \Delta x + \% \Delta y$

③ $\% \Delta C = (\% \Delta x)p + q(\% \Delta y)$.

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Ques:- ① $V = 20 \pm 1$ volt F/o- ① $R = ?$

$I = 50 \pm 0.5$ amp. (ii) $\% \Delta R = ?$

$\Delta R = ?$

Soln:- ① $R = \frac{V}{I}$ ~~$\pm \% \Delta R$~~

$R = \frac{20}{5.0} = 4\Omega$.

(ii) $\% \Delta R = \% (\Delta V) + \% (\Delta I)$

$= \left(\frac{1}{20} \times 100\right) + \left(\frac{0.5}{50} \times 100\right)$

$= 5 + 10 = 15\%$

(iii) $\% \Delta R = \frac{\Delta R}{R} \times 100$

$15 = \frac{\Delta R}{4} \times 100$

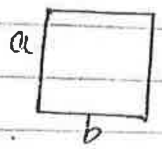
$\Delta R = \frac{15 \times 4}{100} = 0.6\,\Omega$ (Ans)!.

② $A = 200\,cm^2 \pm \Delta A$

$a = 10 \pm 1$ cm F/o - ① $A = ab = ?$

$b = 20 \pm 1$ cm. ② $\% \Delta A = ?$

③ $\Delta A = ?$



Soln:- $\% \Delta A =$

③ $P = 4\sqrt{\frac{x}{y}}$ $x = 2 \pm 0.2$ units $\% \Delta y = ?$

$P = 4 \pm 0.2$ units

Soln:- $\therefore$ ~~$\% (\Delta P) = \frac{1}{2}(\% \Delta x) + \frac{1}{2}(\% \Delta y)$~~

~~$\frac{0.2}{4} \times 100 = \frac{1}{2} \times \frac{0.2}{2} \times 100 + \frac{1}{2}(\% \Delta y)$~~

$5 =$

If changes are very small ($< 8\%$)

$$p^2 = \frac{16x}{y}$$

$$y = \frac{16x}{p^2} \Rightarrow \% \Delta y = \% \Delta x + 2\% \Delta p$$

$$= 10 + 2(5) = 20\% \quad \text{(Ans)}$$

Ques:- (sphere) Radius $= 10 \pm 1\,cm$ ① $\% \Delta A = ?$ ② $\% \Delta V = ?$

Solⁿ:- $\because A = 4\pi R^2$

① $\% \Delta A = 2(\% \Delta R)$

$= 2\left(\frac{1}{10} \times 100\right) = 20\%$

$\because V = \frac{4}{3}\pi R^3$

② $\% \Delta V = 3(\% \Delta R)$

$= 3 \times 10 = 30\%$

Ques:- $\cancel{(PV)^{5/3} = const}$. Volume is increased by 6%. Then f/o % change in Pressure.

$PV^{5/3} = const$

Solⁿ:- $\log P + \frac{5}{3}\log V = \log C$.

$$\frac{dP}{P} + \frac{5}{3}\frac{dV}{V} = 0$$

$$\frac{dP}{P} \times 100 + \frac{5}{3}\frac{\Delta V}{V} \times 100 = 0$$

$$\% \Delta P = -\frac{5}{3}\% \Delta V$$

$$\% \Delta P = -\frac{5}{3}(6\%) = -10\% \quad \text{(Ans)}$$

\# $\begin{bmatrix} x \pm \Delta x \\ y \pm \Delta y \end{bmatrix}$ $A = x + y$, $B = x - y$

$\therefore A = x + y$

$dA = dx + dy$

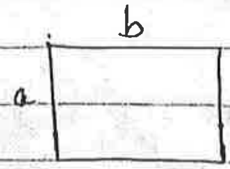
$\boxed{\Delta A = \Delta x + \Delta y}$

$\therefore B = x - y$

$|dB| = |dx| + |dy|$

$\boxed{\Delta B = \Delta x + \Delta y}$

Ques:-



$a = 10 \pm 1\,cm$. F/o - ① $P = 2(a+b) = ?$

$b = 20 \pm 1\,cm$. ② $\% \Delta P = ?$

③ $\Delta P = ?$

Solⁿ:- 5 ④ ∴ $P = 2(10+30) = 60\,cm.$

② $\Delta P = 2(\Delta a + \Delta b) = 2(1+1) = 4\,cm.$

③ $\% \Delta P = \frac{4}{6} \times 100 = \frac{20}{3}\%$

① $P = 60 \pm 4\,cm$ or $60 \pm \frac{20}{3}\%$

$\frac{1}{A} = \frac{1}{x} + \frac{1}{y}$

$A^{-1} = x^{-1} + y^{-1}$

diffⁿ $\cancel{(-1)} A^{-2} dA = \cancel{(-1)} x^{-2} dx + \cancel{(-1)} y^{-2} dy$

$100 \times \frac{dA}{A \cdot A} = 100 \times \frac{dx}{x \cdot x} + 100 \times \frac{dy}{y \cdot y}$

$\boxed{\frac{\% \Delta A}{A} = \frac{\% \Delta x}{x} + \frac{\% \Delta y}{y}}$

Ques:- ① $R_1 = 20 \pm 1\,\Omega$ $R_2 = 10 \pm 1\,\Omega$ F/o - ① Resistance in series $= R_s = ?$

② $\Delta R_s = ?$

③ $\% \Delta R_s = ?$

Solⁿ:- ∴ $R = R_1 + R_2$

$R = 20 + 10 = 30$

② $\Delta R_s = (\Delta R_1 + \Delta R_2) = 1 + 1 = 2\,\Omega$

①. $R_s = 30 \pm 2\,\Omega$

③ $\% \Delta R_s = \frac{2}{3\cancel{0}} \times 10\cancel{0} = \frac{20\cancel{0}}{3}\%$

② In above question f/o - ① $R_{parallel} = R_p = ?$ ② $\Delta R_p = ?$ ③ $\% \Delta R_p = ?$

Solⁿ:- ∴ $R_p = \frac{R_1 R_2}{R_1 + R_2} = \frac{20 \times 10}{20 + 10} = \frac{20}{3}\,\Omega$

∴ $\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2}$ ⇒ $\frac{\% \Delta R_p}{R_p} = \frac{\% \Delta R_1}{R_1} + \frac{\% \Delta R_2}{R_2}$

$\frac{\% \Delta R_p}{20/3\,\Omega} = \frac{5\%}{20\,\Omega} + \frac{10\%}{10\,\Omega}$

∴ $\% \Delta R_1 = \frac{1}{20} \times 100 = 5\%$

$\% \Delta R_2 = \frac{1}{10} \times 100 = 10\%$

$\% \Delta R_p = \frac{20}{3}\left[\frac{5}{4}\right]\% = \frac{25}{3}\%$

∴ $\Delta \% R_p = \frac{\Delta R_p}{R_p} \times 100$

$\frac{25}{3} = \frac{\Delta R_p}{20/3} \times 100$ ⇒ $\Delta R_p = \frac{\cancel{25}^{5}}{3} \times \frac{\cancel{20}}{3} \times \frac{1}{\cancel{100}} = \frac{5}{9}\,\Omega$

Ques: $V = 10 \pm 1$ cm. $\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$ F/o – ① $f = ?$ ② $\% \Delta f$

$u = 10 \pm 1$ cm. ③ Δf

Solⁿ: $\therefore \frac{1}{f} = \frac{1}{10} + \frac{1}{10} \Rightarrow f = 5$ CM.

$$\Rightarrow \frac{\% \Delta f}{f} = \frac{\% \Delta v}{v} + \frac{\% u}{u}$$

$$\frac{\% \Delta f}{5} = \frac{10\%}{10} + \frac{10\%}{10}$$

$$\% \Delta f = 2 \times 5 = 10\% \quad \text{(Ans).}$$

$$\Rightarrow \frac{\Delta f}{f} \times 10\cancel{0} = 1\cancel{0}$$

$$\Delta f = \frac{5}{10} = 0.5$$

$$\therefore f = 5.0 \pm 0.5 \text{ cm}$$ } (Ans).

UNIT AND DIMENSION

- Fundamental ~~Units~~ Properties

Quantity	Representation	Unit
Mass	M	Kg
length	L	m
time	t	sec
temperature	θ or K	K
Current	I or A	Amp.
Amount of substance	m	mole
Luminous Intensity	Cd	Cd

- Supplementary units

Angle	$\theta = Arc/R$	~~Angle~~ Radian
Solid angle	$\Omega = Area/R^2$	Steradian.

Circle →  $\therefore$ angle at centre of circle $= 2\pi$ rad.

Sphere  $\Omega = \frac{4\pi R^2}{R^2} = 4\pi$ steradian

→ Voltage gradient = E.F.

Ques:- F/o solid angle substended at the corner of a room.

Solⁿ:- $\Omega = \frac{4\pi R^2}{8R^2} = \frac{\pi}{2}$ St.rad. (Ans):-

• Practical Units

1. 1 h.p. = 746 Watt
2. 1 Light year = $365 \times 24 \times 60 \times 60 \times 3 \times 10^8$ m.
3. Parallactic sec. ⇒ Parsec ⇒ distance b/w stars
4. Astronomical unit (measure distance b/w two galaxies)
5. Lunar month ⇒ unit of time (Moon takes time to complete one revolution around Earth)

• Improper Unit

1. Kg-wt → Unit of force
   eg:- 6 Kgwt = 6g Newton.

• Dimension

1. $G = \frac{MLT^{-2} \times L^2}{M^2} = [M^{-1}L^3T^{-2}]$.

2. ~~$\varepsilon_0 = \frac{[MLT^{-2}][L^2]}{[A^2T^2]}$~~ ~~$[ML^3T^{-4}A^{-2}]$~~ $\varepsilon_0 = [M^{-1}L^{-3}A^2T^4]$

3. $\mu_0 = \frac{1}{c^2\varepsilon_0} = \frac{1}{[L^2T^{-2}][M^{-1}L^{-3}A^2T^4]} = [M^1L^1T^{-2}A^{-2}]$

4. Polarisation $= \frac{q}{Area} = \frac{IT}{L^2} = [A^1T^1L^{-2}]$

5. Gradient $= \frac{\text{Physical quantity}}{\text{length}}$

   eg:- (I) gradient of Energy $= \frac{ML^2T^{-2}}{L} = MLT^{-2}$ → Force.

   (II) Voltage gradient $= \frac{W/q}{L} = \frac{\frac{ML^2T^{-2}}{AT}}{L} = MLT^{-3}A^{-1}$

6. E.F. $= \frac{F}{q} = \frac{MLT^{-2}}{AT} = [MLT^{-3}A^{-1}]$

Rules: (1) [L.H.S.] = [R.H.S.]

(2). If $x+y$ or $x-y$ then $[x] = [y]$

Ques: (1) $v = \dfrac{A}{Bt+L}$ $\quad v \Rightarrow$ speed, t = time, L = length

F/o $[A] = ?$ & $[B] = ?$

~~$[A] = [LT^{-1}]$~~

$\therefore [B] = \dfrac{L}{T} = [LT^{-1}]$

$\therefore [A] = v(Bt+L)$

$= LT^{-1} \times L = [L^2T^{-1}]$

(2) $F = \dfrac{A}{Bv+t} + (m)C$. $\quad F$ = force, v = speed, t = time, m = mass.

F/o $[A] = ?$, $[B] = ?$, $[C] = ?$

(1) $[A] = F \times [Bv+t] = [MLT^{-2}] \times [T]$

$= [MLT^{-1}]$.

$\therefore Bv = t$

(2) $[B] = \dfrac{t}{v} = \dfrac{T}{LT^{-1}} = [L^{-1}T^2]$.

(3) $(m)C = F$

$[C] = \dfrac{F}{m} = \dfrac{MLT^{-2}}{M}$

$[C] = [M^0L^1T^{-2}]$.

(3). $\left[P + \dfrac{a}{V^2}\right](V-b) = nRT$ $\quad P$ = Pressure, V = vol^m $\quad$ F/o $\dfrac{[a]}{[b]} = ?$

Sol^n:

$\therefore V = b$

$[b] = m^3 = [L^3]$.

$\therefore \dfrac{a}{V^2} = P \Rightarrow a = P.V^2$

$a = \dfrac{MLT^{-2}}{L^2} \times (L^3)^2 = [ML^5T^{-2}]$

Now, $\dfrac{[b]}{[a]} = \dfrac{L^3}{ML^5T^{-2}} = [M^{-1}L^{-2}T^2]$.

$\Rightarrow \dfrac{[a]}{[b]} = [M^1L^2T^{-2}]$

(3). $A = P2^{(\;)}$ $\rightarrow$ dimensionless $\quad$ (or) $\quad B = Qe^{(xy + \omega z)}$.

$[A] = [P]$.

$[xy] = M^0L^0T^0 = 1$

$[x] = \dfrac{1}{[y]}$ & $[\omega] = \dfrac{1}{[z]}$.

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Page No. 52 → sig. fig.

(4). $C = S\sin(\ )$ → dimensionless
$[C] = [S]$.

OR $D = T\sin^{-1}(\ )$
$[D] = [T]$

OR $X = \omega\log(\ )$
$[X] = [\omega]$.

Ques. ① $q = Ae^{-Bt}$ F/o - $[A] = ?$ $[B] = ?$

$\therefore A = q = It$
$[A] = [A^1T^1]$.

~~$[B] = M^0L^0T^0$~~ $= 1$ $\therefore$ $B \times t = 1$

$B = \frac{1}{t} = [T^{-1}]$.

② $E = A\sin(\omega t + Kx)$ $[A] = ?$ $[\omega] = ?$ $[K] = ?$
E.F.

$\therefore [A] = E = [ML^2T^{-2}]$
$\therefore \omega t = 1$
$\omega = \frac{1}{t} = [T^{-1}] = [M^0L^0T^{-1}]$.

$\therefore Kx = 1$
$K = \frac{1}{x} = [M^0L^{-1}T^0]$

• Finding out Relation b/w more than two physical quantities.

Ques. T = time period of simple pendulum, l = length of simple pendulum, g = gravitational acc^n. F/o dependency of T on l & g.

Sol^n:- $\therefore T \propto l^a g^b$

$M^0L^0T^1 = L^a(LT^{-2})^b$
$\Rightarrow L^0 = L^{a+b}$ & $T^1 = T^{-2b}$
$\Rightarrow 0 = a+b$ $1 = -2b$
$a = -b$ $b = -1/2$
$a = -(-1/2) = 1/2$

Now, $T \propto l^{1/2} \times g^{-1/2} = \sqrt{\frac{l}{g}}$

$\Rightarrow T \propto \sqrt{\frac{l}{g}}$ (Ans)!.

Ques. T = time period of satellite around earth, m = mass of earth, R = radius of earth, G = gravitational constant. Dependency of T on m, R, G.

Sol^n:- $\therefore T \propto m^a R^b G^c$

$$\Rightarrow [M^0L^0T^1] = M^a \times R^b \times G^c$$

$$M^0L^0T^1 = M^a \times L^b \times \left(\frac{MLT^{-2}\times L^2}{M^2}\right)^c$$

$$M^0L^0T^1 = M^a L^b (M^{-1}L^3T^{-2})^c$$

$$M^0L^0T^1 = M^{a-c} \times L^{b+3} \times T^{-2c}.$$

(1) $0 = a - c$, $a = c$, $a = -1/2$

(2) $0 = b + 3c$, $b = -3\times(-1/2)$, $b = 3/2$

(3) $1 = -2c$, $c = -1/2$

Now:-

$$T \propto M^{-1/2} R^{2/3} G^{-1/2}$$

$$T \propto \sqrt{\frac{R^3}{MG}} \quad \text{(Ans)!.} \qquad \boxed{T^2 \propto R^3}$$

Imp.

Ques:- If energy (E) speed (v), time (t) are fundamental quantities then f/d dimensional formula of Pressure.

Soln:-

$$\therefore P \propto E^a v^b t^c$$

$$M^1L^{-1}T^{-2} = \cancel{E^a v^b} (ML^2T^{-2})^a (LT^{-1})^b (T)^c$$

$$M^1L^{-1}T^{-2} = M^a L^{2a+b} T^{-2a-b+c}$$

(1) $1 = a$

(2) $-1 = 2a + b$, $-1 = 2(1) + b$, $b = -3$

(3) $-2 = -2a - b + c$, $-2 = -2(1) - b(-3) + c$, $C = -1+2 = -3$

$$\therefore P \propto E^1 v^{-2} T^{-2} \quad \text{(Ans)}:$$

- Unit Conversions

$$n_1u_1 = n_2u_2$$

$$n_2 = n_1\frac{u_1}{u_2}$$

eg:-

$$J = (n_2)\ \text{ergs.}$$

$$n_2 = (1)\left(\frac{J}{erg}\right)$$

$$n_2 = (1)\left[\frac{M_1L_1^2T_1^{-2}}{M_2L_2^2T_2^{-2}}\right] = (1)\left(\frac{1\,kg}{1\,g}\right)^1\left(\frac{1\,m}{1\,cm}\right)^2\left(\frac{1\,s}{1\,s}\right)^{-2}$$

$$n_2 = \left(\frac{1000\,gm}{1\,gm}\right)^1 \left(\frac{100\,cm}{1\,cm}\right)^2 (1)^{-2}$$

$$n_2 = 1000 \times (100)^2 \times 1$$

$$n_2 = 10^7$$

Now, $1J = (10^7)$ ergs.

Imp

Ques:- F/o value of 1 N in a system of unit u_2 where unit of mass $= 100\,gm$, unit of length $= 20\,cm$ and unit of time is 2 sec.

Soln:- $\therefore$ $1\,N = (n_2)\,u_2$

$$n_2 = (1)\left(\frac{N}{u_2}\right)$$

$$n_2 = (1)\left(\frac{M_1L_1T_1^{-2}}{M_2L_2T_2^{-2}}\right) = (1) \times \left(\frac{1\,kg}{100\,g}\right)^1 \times \left(\frac{1\,m}{20\,cm}\right)^1 \times \left(\frac{1\,s}{2\,s}\right)^{-2}$$

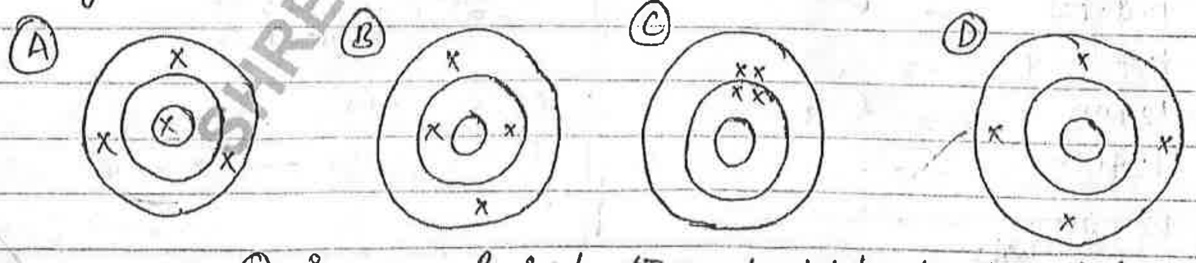
$$n_2 = (1) \times \left(\frac{1000\,g}{100\,g}\right)^1 \times \left(\frac{100\,cm}{20\,cm}\right)^1 \times \left(\frac{1}{2}\right)^{-2}$$

$$n_2 = 1 \times 100 \times 5 \times \frac{1}{4} \times 4 = 200$$

Hence, $1\,N = 200\,u_2$ (Ans)!.

$\rightarrow$ u_2 in terms of Newton $\Rightarrow$ $u_2 = \frac{1}{200}\,N$.

- Accuracy and Precision



$\rightarrow$ C is more Preciside. (Type of mistake should be less)

$\rightarrow$ Order of Accuracy $\Rightarrow$ A > B > C > D

$\rightarrow$ Increasing order of Precision $\Rightarrow$ D < A < B < C

Imp: In experiment of Physics we will always prefer precision over accuracy because after eliminating the errors most precised will become most accurate.

eg.:-

Exp-①	Exp-②
$9.75\ m/s^2$	$10.10\ m/s^2$
$9.85\ m/s^2$	$10.05\ m/s^2$
$9.70\ m/s^2$	$10.10\ m/s^2$
$9.90\ m/s^2$	$10.05\ m/s$

Avg $\Rightarrow g_m = 9.80\ m/s^2$ $\quad g_m = 10.75\ m/s^2$

Here, $g = 9.81\ m/s^2$

→ Exp-① ⇒ more accurate

→ Exp-② ⇒ more precised

• Order of Number

$A \times 10^{x}$ ⇒ $\boxed{0.5 \leq A < 5}$

x → order of No.

eg.:- ① 4982.34
$= 4.98234 \times 10^{3}$

② 501230
$= 0.501230 \times 10^{6}$

• Scientific Method

① 4982.34
$= 4.98234 \times 10^3$

② 501230
$= 5.01230 \times 10^5$

③ .124567
$= 1.24567 \times 10^5$

• Significant figures

Digits	S.F.
124567	→ 6
1240067	→ 7
12400	→ 3
124.00	→ 5
12400 m (1m)	→ 5

Digits	S.F.
236×10^2 m (10^2m)	→ 3
0.00042 m	→ 2
1.00042 m	→ 6

Addition or Substraction

$$\begin{array}{r} 12.634 \\ 1.2 \\ \hline 13.834 \end{array}$$

→ 13.8

→ In addition or substraction the minm no. of significant figures after decimal will be carry on.

Multiplication or Division

$$\frac{1.32 \times 1.6605}{98.243}$$

F/o no. of significant figures in the result of this eqⁿ.

Ans:- No. of significant fig. in Ans = 3.

⇒ We can calculate physical quantity in less accurate measurement.

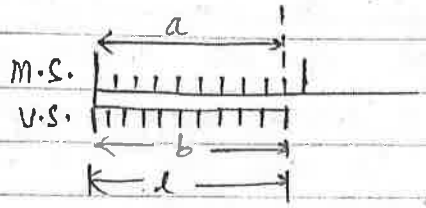
• **Vernier Callipers**

Main scale (M.S.)   Vernier scale (V.S.)

1 cm = 10 division.

$1 \text{ division} = 1 \text{ M.S.D.} = \frac{1}{10} \text{ cm}$

$= 1 \text{ mm}$



let, M.S.D = m & V.S.D = v

then, $l = ma = bv$

$v = \frac{ma}{b}$

Now, least count (L.C.) = 1 M.S.D. − 1 V.S.D

L.C. = 1m − 1v

$L.C. = m - \frac{am}{b}$

$$\boxed{L.C. = \left(\frac{b-a}{b}\right) m}$$

Ques:- 9 division of M.S. = 10 divisions of V.S. 1 cm on main scale = 10 divisions

F/o − ① m = ? ② v = ? ③ L.C = ?

Solⁿ:- ① $m = \frac{1\text{cm}}{10} = 1\text{mm}$

② $v = \frac{am}{b} = \frac{9 \times 1\text{mm}}{10} = .9\text{mm}$

③ L.C. = 1mm − 0.9mm = 0.1mm   (OR) $L.C. = \left(\frac{10-9}{10}\right) 1\text{mm}$ = 0.1mm.

03/10/2019

Ques:- 1 cm of main scale is divided into 20 equal parts. Then f/o least count of Vernier Caliper if ~~19~~ 19 division of main scale is equal to 20 division of Vernier scale.

Solⁿ:- 20 division $= 1\,cm \Rightarrow 1\ MSD = \frac{1\ cm}{20} = \frac{10\ mm}{20} = \frac{1}{2}\ mm$

Now,

19 divisions of M.S. = 20 divisions of V.S.

$\Rightarrow a = 19$ & $b = 20$.

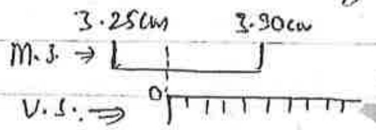
$\therefore L.C. = \left(\frac{b-a}{b}\right) MSD = \left(\frac{20-19}{20}\right) \times \frac{1}{2}$

$\Rightarrow L.C. = 1/40\ mm$ (Ans):-

$$\therefore \boxed{\text{Observation} = \left(\begin{matrix}\text{Reading of}\\ \text{main scale}\end{matrix}\right) + \left(\begin{matrix}\text{No. of Divisions of V.S which is}\\ \text{conciding with any division of M.S.}\end{matrix}\right) \times L.C.}$$

Ques:- In an observation zero of vernier scale lies b/w 3.25 cm and 3.30 cm of main scale. If 16ᵗʰ division of vernier scale concide with any division of main scale. Then f/o observation. It is given that 19 divisions of main scale is equal to 20 divisions of V.S. There is no zero error in the instrument.

Solⁿ:-



$\therefore M.S.D = 0.05\ CM = \frac{1}{20}\ cm = \frac{1}{2}\ mm$

$\therefore L.C. = \left(\frac{b-a}{b}\right) m.s.D.$

$L.C. = \left(\frac{20-19}{20}\right) \times \frac{1}{2} = \frac{1}{40}\ mm.$

3.25 cm

Now, Reading $= 32.5\,mm + 16 \times \frac{1}{40}$

$= 32.9\,mm$ (Ans):-

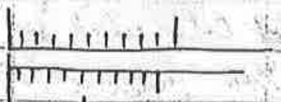
① 32.9 mm ± 0.025 mm

② 32.90 mm ± 0.025 mm

③ 32.900 mm ± 0.025 mm

Zero Error

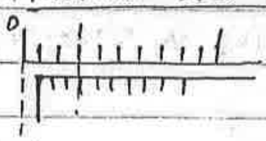
M.S.
V.S.



↳ Without Zero Error

$$\boxed{\text{True Value} = \text{Observation} - \text{Zero error with sign}}$$

(1). Positive Zero Error



Zero Error $\therefore$ $= 0 + 3(0.1)$

$= 0.3\,mm.$

(2). Negative Zero Error

$\therefore$ Zero Error $= -\left(\left[\text{Total no. of division on V.S.}\right] - \left[\text{no of divisions of V.S. coinciding with any division of M.S.}\right]\right) \times L.C.$

$= -(10-3)\,0.1\,mm$

$= -0.7\,mm.$

- Screw Guage

→ Pitch :- Pitch is the distance travelled by the circular scale on main scale in one complete revolution.

→ Generally one MSD. is equal to 1 Pitch.

eg. 4 revolution ⟶ 3 mm

Then, Pitch $= \dfrac{3\,mm}{4}$

→

$$\boxed{L.C. = \frac{\text{Pitch}}{\text{No. of division on Circular Scale}}}$$

Ques :- 1 cm of main scale = 20 division. In 5 revolution : C.S. (circular scale) advanced by 4 mm. No. of divisions on C.S. = 60. Then f/o L.C.

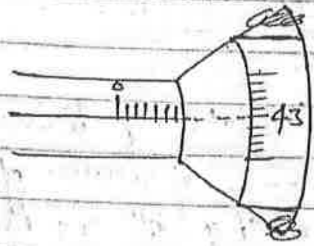
Soln :- $\therefore$ Pitch $= \dfrac{4}{5}\,mm.$

$\therefore L.C. = \dfrac{4/5}{60} = \dfrac{1}{75}\,mm$ (Ans).

Observation

$$\boxed{\text{Observation} = \text{Reading of Main scale} + \begin{pmatrix}\text{no. of division of C.S. which} \\ \text{is conciding with reference} \\ \text{line of main scale}\end{pmatrix} \times L.C.}$$

Ques:- On main scale 1 div = $\frac{1}{2}$ mm. No. of division on C.S. = 50.



F/o observation = ?

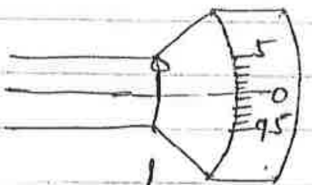
$\therefore$ 1 MSD = $\frac{1}{2}$ mm = Pitch.

$$\therefore L.C. = \frac{1/2}{50} = \frac{1}{100}\,mm.$$

Now, observation $= 6 \times \frac{1}{2}\,mm + 43 \times 0.01\,mm$

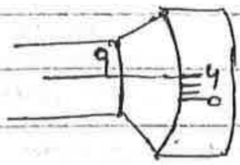
$= 3.43\,mm \pm 0.01\,mm$. (Ans):-

Zero Error



↳ without Zero Error

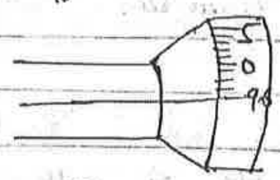
(1). Positive Zero Error



$\therefore$ Zero error $= 0 + 4(0.01)\,mm$

$= 0.04\,mm.$

(2). Negative Zero error



$$\therefore \text{Zero error} = \left[-\begin{pmatrix}\text{total no. of division on C.S.} - \text{no. of divisions conciding with refarence line of M.S.}\end{pmatrix}\right] \times L.C.$$

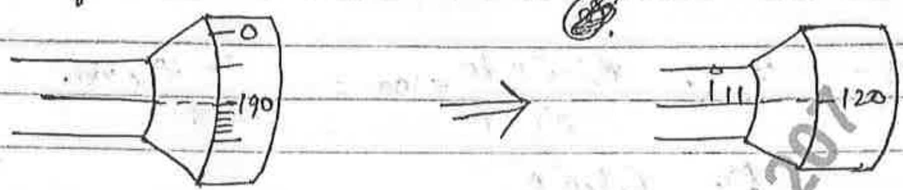
Scanned by CamScanner

$= -[100-96] \times 0.01\,mm$

$= -0.04\,mm.$

Sup. Ques.- 1 cm on M.S. = 5 divisions
no. of divisions on C.S. = 200

~~F/o observation in each case.~~
F/o True value



$\therefore$ 5 div = 1 cm = 10 mm.

$1\text{ div} = \frac{10}{5} = 2\,mm$

$\Rightarrow$ Pitch = 2 mm.

Now, $L.C. = \frac{2}{200} = \frac{1}{100}\,mm.$

(+) Zero error $= -[200-190] \times \frac{1}{100}$

$= -\frac{10}{100} = -0.1\,mm.$

Hence, ~~True value~~ = Observation $= 2 \times 2\,mm + 120 \times \frac{1}{100}\,mm$

$= 5.2\,mm.$

$\therefore$ True value $= (5.2\,mm) - (-0.1\,mm)$

$= 5.2\,mm$

$= 5.30\,mm \pm 0.01\,mm$ (Ans)!.

- Spherometer.

$\rightarrow$ Spherometer works on the same principle as screw guage.

Ques:- Race #1 Que-10 $m = \pi \tan\theta$ If error of θ is constant then f/o value of θ for which $\%\,\Delta m$ will be minm.

Sol^n:- $\% \Delta m = \frac{\Delta m}{m} \times 100$

$= \frac{dm}{m} \times 100$

$\therefore \frac{dm}{d\theta} = \pi \sec^2\theta \Rightarrow dm = \pi \sec^2\theta \, d\theta.$

Now, $\% \Delta m = \frac{\cancel{\pi} \sec^2\theta \, d\theta}{\cancel{\pi} \tan\theta} \times 100 = \frac{\sec^2\theta}{\tan\theta} \, d\theta \times 100 \rightarrow C$

$\% \Delta m = \frac{C \sec^2\theta}{\tan\theta}$

For minima,

$\% \Delta m = \frac{2 \times C}{2 \times \cos^2\theta \cdot \frac{\sin\theta}{\cos\theta}}$

$\% \Delta m = \frac{2C}{2\sin\theta\cos\theta} = \frac{2C}{\sin 2\theta} \rightarrow$ min^n

$\sin 2\theta \rightarrow$ max^m $= 1$

$\Rightarrow 2\theta = \frac{\pi}{2}$

$\theta = \frac{\pi}{4}$ (Ans).

08/08/2019

Vector

→ Vector:- Physical quantity which requires (in addition to magnitude)
(1). A specific direction
(2). Should follow vector's law: — (a). Law of Addition
(b). law of Commutation $(\vec{A}+\vec{B}=\vec{B}+\vec{A})$

Note: ⇒ (1). A physical quantity having dirⁿ may be or may not be vector. eg:- Electric current ⇒ having dirⁿ but do not follow vector law, that's why it is scalar.

(2). Representation of Vector

Tail $\xrightarrow{\vec{A}}$ Head (or) $A \longrightarrow B$, $\vec{AB}$ (or) $A \longleftarrow B$, $\vec{BA}$

(3). Magnitude of Vector, $|\vec{A}|$ or A

$$\vec{A} = \underbrace{|\vec{A}|}_{\text{magnitude}}\ \underbrace{\hat{A}}_{\text{dir}^n \text{ of vector or unit vector of vector}}$$

eg:- (a). $\vec{A} = 3\hat{i}$
$|\vec{A}| = 3$
$\hat{A} = \hat{i}$

(b). $\vec{A} = 3\hat{i} + 4\hat{j}$
$|\vec{A}| = \sqrt{3^2+4^2}$
$|\vec{A}| = \sqrt{25} = 5$

(c). $\vec{A} = 3\hat{i} + 4\hat{j} + 12\hat{k}$
$|\vec{A}| = \sqrt{3^2+4^2+12^2}$
$|\vec{A}| = \sqrt{169} = 13$

(d). $\vec{A} = \hat{a} + 2\hat{b}$ $|\vec{A}|$ = angle required b/w $\hat{a}$ & $\hat{b}$.

(4). When a vector is shifted parallel to itself, it remains same.
(5). When vector is rotated by 'θ' angle (θ other than multiple of 2π), vector gets changed.

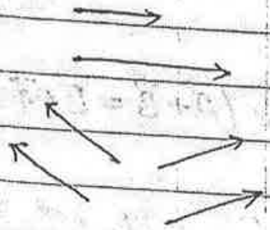
$\xrightarrow{\vec{A}}$ $\xrightarrow{\text{rotated by } \theta}$ $\vec{B}$, θ, $\vec{A}$

$\vec{A} \neq \vec{B}$
$|\vec{A}| = |\vec{B}|$

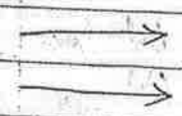
(6). When a frame of reference (FOR) [FOR = observer + co-ordinate system + clock] is rotated or translated then vector remains same but component of vector may or may not be changed.

(a). Parallel vector

$\theta = 0°$



(b). Equal vector



(c). Anti-parallel vector.

$\theta = 180°$

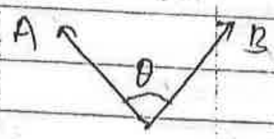
(d). -ve/opposite vector

$A \longleftarrow B$

$A \longrightarrow B$

$\vec{AB} = -\vec{BA}$

(e). Co-initial vector



(f). Co-Planar vector.

* Two vectors always lie in the same plane.

* Three or more vectors may lie out of Plane.

(g). Collinear vector.

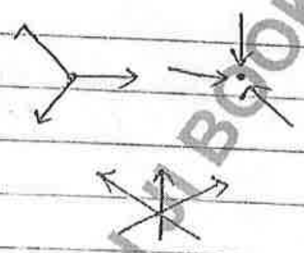
$\longrightarrow \longrightarrow$ 0°

$\longleftarrow \longrightarrow$ 180°

$\longleftarrow \longleftarrow$ 0°

$\longrightarrow \longleftarrow$ 180°

(h) Con-current vector



(i) Null / Zero Vector

→ Dir^n of Null vector can not be defined It is arbitrary.

(j). Unit vector [of $\vec{A}$]

→ Written by $\hat{A}$

→ A vector with unit (1) magnitude.

$\therefore \quad \vec{A} = |\vec{A}| \cdot \hat{A} = 1 \times \hat{A} = \hat{A}$ ⇒ It means unit vector is used to specify dir^n of vector.

$$\because \vec{A} = |\vec{A}|\hat{A}$$

$$\hat{A} = \frac{\vec{A}}{|\vec{A}|} = \frac{\text{vector}}{\text{Magnitude}}$$

eg:- (i). $\vec{A} = 3\hat{i}$

$$\hat{A} = \frac{3\hat{i}}{3}$$

(ii). $\vec{P} = 2\hat{i} + 4\hat{j}$

$$\hat{P} = \frac{2\hat{i} + 4\hat{j}}{\sqrt{4+16}} = \frac{2\hat{i} + 4\hat{j}}{\sqrt{20}}$$

Ques:- If $\vec{A} = \hat{i} - \hat{j}$ and $\vec{B} = 3\hat{i} + 4\hat{j}$. Then write down vector 'c' whose magnitude equals to that of $\vec{A}$ and is in dir^n of $\vec{B}$.

Sol^n:- $\vec{C} = |\vec{C}| \cdot \hat{C}$

$\vec{C} = |\vec{A}| \cdot \hat{B}$

$\vec{C} = \sqrt{1^2 + (-1)^2} \times \dfrac{3\hat{i} + 4\hat{j}}{\sqrt{9+16}}$

$\vec{C} = \dfrac{3\sqrt{2}\,\hat{i} + 4\sqrt{2}\,\hat{j}}{5}$ (Ans):.

(k). <u>Base vector</u>:- In XYZ co-ordinate <u>system/frame</u>, the three unit vectors $\hat{i}$, $\hat{j}$ and $\hat{k}$ (used to indicate dir^n) are k/n as base vectors.

(l). <u>Polar vector</u>:- Vectors having initial point or point of application.

<u>eg</u>:- Displacement, Force, etc.

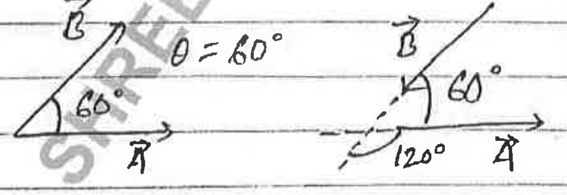
(m). <u>Axial vector</u>:- These are used in rotational motion to define rotational effects.

<u>eg</u>:- $d\vec{\theta}$, $\vec{\omega}$, $\vec{J}$, $\vec{\alpha}$, $\vec{\tau}$

~~Addition of Two vectors~~
~~(A). Law of Triangle:-~~

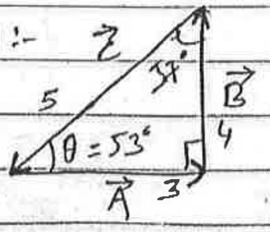
- <u>Angle b/w two vectors</u>

→ Smallest angle b/w head - head or tail - tail of vector.



$\theta = 60°$ $\theta = 120°$

<u>Ques</u>:-

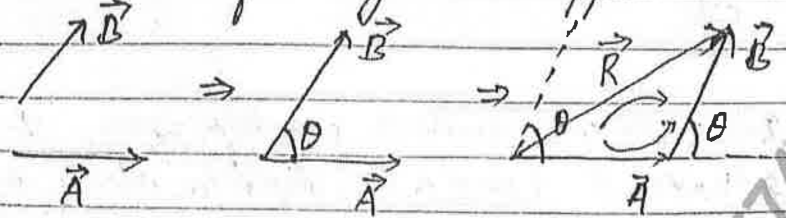


f/o angle b/w -
(i). $\vec{A}$ & $\vec{B} \Rightarrow \theta = 180° - 90° = 90°$
(ii). $\vec{B}$ & $\vec{C} \Rightarrow \theta = 180° - 37° = 143°$
(iii) $\vec{C}$ & $\vec{A} \Rightarrow \theta = 180° - 53° = 127°$

Sol^n:- $\sin\theta = \dfrac{4}{5} \Rightarrow \theta = 53°$

• **Addition of Two Vectors**

(A). Law of Triangle :- If two vectors $\vec{A}$ & $\vec{B}$ (angle b/w them is θ) are represented by two consecutive sides of triangle in same order then their vector sum or resultant is represented by third side of triangle in opposite order.

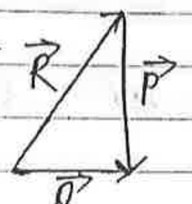


θ = angle

$$\therefore \ \vec{R} = \vec{A} + \vec{B}$$

$$|\vec{R}| = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

Ques :-



F/o correct one :-

(1) $\vec{P} = \vec{Q} + \vec{R}$

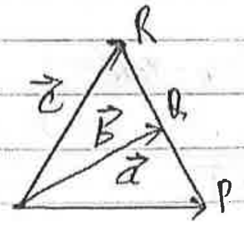
(2). $\vec{Q} = \vec{R} + \vec{P}$

(3). $\vec{R} = \vec{P} + \vec{Q}$

(4). No Idea.

Imp

Ques :-



If $|\vec{PQ}| = 3|\vec{QR}|$ then relation b/w $\vec{a}$, $\vec{b}$ & $\vec{c}$ = ?

Sol^n :-

$$\therefore \ \vec{a} + \vec{PQ} = \vec{b}$$

$$\vec{a} + 3\vec{QR} = \vec{b} \quad - (1)$$

$$\therefore \ \vec{b} + \vec{QR} = \vec{c}$$

$$\vec{QR} = \vec{c} - \vec{b} \quad - (2)$$

Putting in (1)

$$\vec{a} + 3(\vec{c} - \vec{b}) = \vec{b}$$

$$\vec{a} + 3\vec{c} - 3\vec{b} = \vec{b}$$

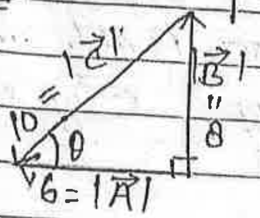
$$\vec{a} + 3\vec{c} = 4\vec{b} \quad \text{(Ans)}$$

09/08/2019

Ques:- If $\vec{B}+\vec{C}=\vec{A}$ and magnitude of $\vec{A}, \vec{B}, \vec{C}$ are respectively 6, 8, 10. Then f/o angle b/w - (I). A & B (II). B & C (III) C & A.

↳ triplet

Sol^n:-



$\therefore \sin\theta = \frac{8}{10} = \frac{4}{5} \Rightarrow \theta = 53^\circ$.

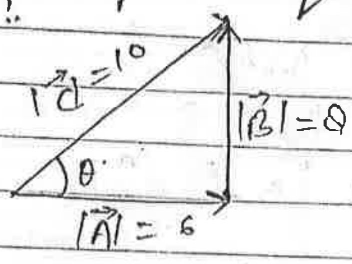
Now, (1) $\vec{A}$ & $\vec{B} = 90^\circ$

(2) $\vec{B}$ & $\vec{C} = 180^\circ - \cancel{53^\circ} = \cancel{127}^\circ$

(3) $\vec{C}$ & $\vec{A} = 53^\circ$ (opposite angles).

Ques:- In previous question if $\vec{A}+\vec{B}=\vec{C}$. Then f/o the angles:-

Sol^n:-



$\therefore \sin\theta = \frac{8}{10} \Rightarrow \theta = 53^\circ$.

Now, (1) $\vec{A}$ & $\vec{B} = 180^\circ - 90^\circ = 90^\circ$

(2) $\vec{B}$ & $\vec{C} = 90^\circ - 53^\circ = 37^\circ$.

(3) $\vec{C}$ & $\vec{A} = 53^\circ$.

Imp

Ques:- If angle b/w $\vec{P}$ and $\vec{Q}$ is 60° and their resultant is $\sqrt{7}Q$. Then f/o value of $P/Q = ?$

Sol^n:- $\vec{P}$ & $\vec{Q} \Rightarrow \theta = 60^\circ$

$|\vec{R}| = \sqrt{7}Q$. $\quad \therefore R = \sqrt{P^2+Q^2+2PQ\cos 60^\circ}$

$$\sqrt{7}Q = \sqrt{P^2+Q^2+PQ}$$

$$7Q^2 = P^2+Q^2+PQ$$

$$7 = \frac{P^2}{Q^2}+1+\frac{P}{Q}$$

Let $x = P/Q$.

$$x^2+x+1-7=0$$

$$x^2+x-6=0$$

$$x^2+3x-2x-6=0$$

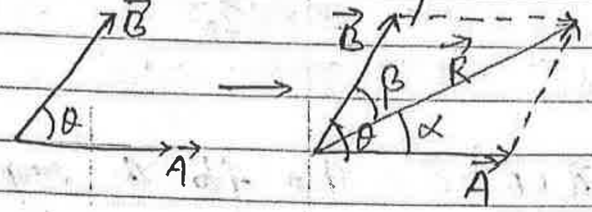
$$x(x+3)-2(x+3)=0$$

$$x=-3 \text{ and } x=2$$

Hence, $\frac{P}{Q} = 2$ (Ans).

(B). Parallelogram law of addition

→ Two vectors $\vec{A}$ & $\vec{B}$ ($\angle$ b/w them is θ) is represented by two adjacent side of parallelogram which are coming out from a common point then their resultant (Vector sum) is represented by diagonal of that Polygon coming out from that common point.



$$\therefore \vec{R} = \vec{A} + \vec{B}$$

$$|\vec{R}| = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

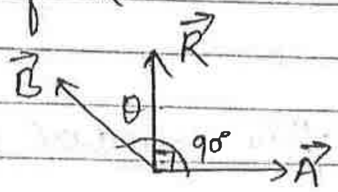
$$\therefore \tan\alpha = \frac{B\sin\theta}{A + B\cos\theta}$$

angle b/w $\vec{R}$ & $\vec{A}$

$$\therefore \tan\beta = \frac{A\sin\theta}{B + A\cos\theta}$$

angle b/w $\vec{R}$ & B

** If $\vec{R}$ is $\perp$ to $\vec{A}$



$$\Rightarrow \frac{1}{0} = \frac{B\sin\theta}{A + B\cos\theta}$$

$$A + B\cos\theta = 0$$

$$\cos\theta = -\frac{A}{B}$$

$$\theta = \cos^{-1}\left(\frac{-A}{B}\right)$$

** If $\vec{R}$ is $\perp$ to $\vec{B}$.

$$\Rightarrow \frac{1}{0} = \frac{A\sin\theta}{B + A\cos\theta}$$

$$B + A\cos\theta = 0$$

$$\cos\theta = -\frac{B}{A}$$

$$\theta = \cos^{-1}\left(-\frac{B}{A}\right)$$

Note: ⇒ (1). Resultant $\vec{R}$ always lie in the plane of $\vec{A}$ & $\vec{B}$.

(2). $R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$

(i). If $\theta = 0°$ ($\vec{A} \parallel \vec{B}$)

$$R = \sqrt{A^2 + B^2 + 2AB}$$

$$R = R_{max} = (A + B)$$

(ii). If $\theta = 180°$ ($\vec{A} \uparrow\downarrow \vec{B}$)

$$R = R_{min} = (A \sim B)$$

Basic Maths

$\sin 2\theta = 2\sin\theta\cos\theta$ $\quad$ $\cos 2\theta = 2\cos^2\theta - 1$

$\sin\theta = 2\sin\frac{\theta}{2}\cos\frac{\theta}{2}$ $\quad$ $1+\cos 2\theta = 2\cos^2\theta$

$\cos 2\theta = \cos^2\theta - \sin^2\theta$ $\quad$ $1-\cos 2\theta = 2\sin^2\theta$

$\cos 2\theta = 1 - 2\sin^2\theta$ $\quad$ $1+\cos\theta = 2\cos^2\frac{\theta}{2}$

(iii). If $\vec{A} \perp \vec{B}$ $(\theta = 90^\circ)$

$$R = \sqrt{A^2+B^2+2AB(0)}$$

$$R = \sqrt{A^2+B^2}$$

(iv). Range of Resultant

$$R_{min} \leq R \leq R_{max}$$

$$(A-B) \leq R \leq (A+B)$$

(3). If $|\vec{A}| = |\vec{B}| = A$

then $R = ?$

$$R = \sqrt{A^2+A^2+2A(A)\cos\theta}$$

$$R = \sqrt{2A^2(1+\cos\theta)}$$

$$R = \sqrt{2A^2\left(2\cos^2\frac{\theta}{2}\right)}$$

$$\boxed{R = 2A\cos\frac{\theta}{2}}$$

Special Points

If $|\vec{A}| = |\vec{B}| = A$ so $R = 2A\cos\left(\frac{\theta}{2}\right)$

(a). If $\theta = 90^\circ$

$\underline{R = \sqrt{2}A}$

(b). If $\theta = 60^\circ$

$R = 2A\cos\frac{60}{2}$

$\underline{R = \sqrt{3}A}$

(c). If $\theta = 120^\circ$

$R = 2A\cos\frac{120^\circ}{2}$

$R = 2A \times \frac{1}{2} \Rightarrow \underline{R = A}$

Note :⇒ Resultant of two unit vector will be unit vector if angle b/w them is 120°.

Ques:- If $|\hat{a}+\hat{b}| = \sqrt{3}$ f/o angle b/w $\hat{a}$ & $\hat{b}$?

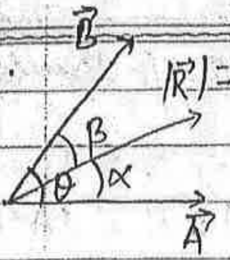
Sol^n:- $R = 2A\cos\frac{\theta}{2}$

$$\sqrt{3} = 2 \times 1 \cdot \cos\frac{\theta}{2}$$

$$\cos\frac{\theta}{2} = \frac{\sqrt{3}}{2}$$

$$\frac{\theta}{2} = 30^\circ \Rightarrow \theta = 60^\circ \text{ (Ans)}.$$

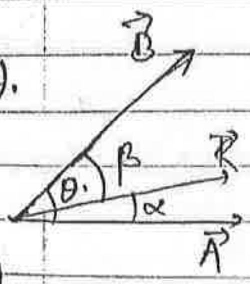
Note :⇒ (1).



$|\vec{R}| = 2A\cos\frac{\theta}{2}$ | If $|\vec{A}| = |\vec{B}| = A$.

$\alpha = \beta = \frac{\theta}{2}$

(2).

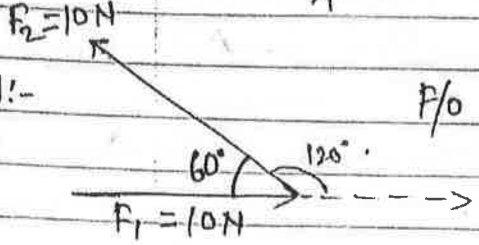


If $|\vec{A}| > |\vec{B}|$

$\Rightarrow [\alpha < \beta]$

* Resultant will shift towards large vector.

Ques :-



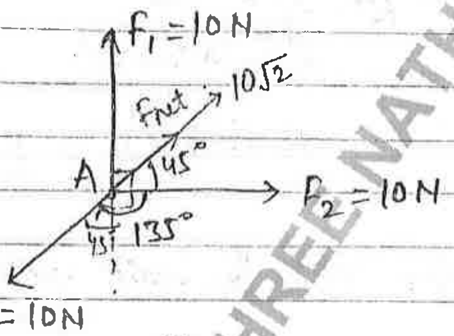
F/o resultant of F_1 & F_2.

Solⁿ :-

$|\vec{R}| = 2A\cos\frac{\theta}{2}$

$|\vec{R}| = 2\times10\cos\frac{120°}{2} = 2\times10\times\frac{1}{2} = 10\text{ N}$ (Ans) :.

Ques :-

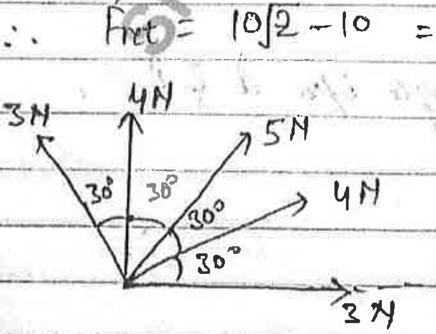


F/o resultant of force on point A.

Solⁿ :-

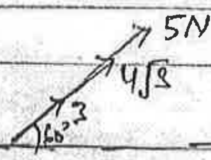
$\therefore\ \vec{F}_{net} = 10\sqrt{2} - 10 = 4.1\text{ N}$ (Ans) :.

Ques :-



F/o resultant of all forces on point 'A' and it's dirⁿ from x-axis

Solⁿ :-



$\therefore R = 5 + 3 + 4\sqrt{3} = (8 + 4\sqrt{3})\text{N}$

at $60°$ from x-axis

Ques:- If $|\hat{a}+\hat{b}| = \sqrt{3}$ Then $|2\hat{a}+3\hat{b}| = ?$

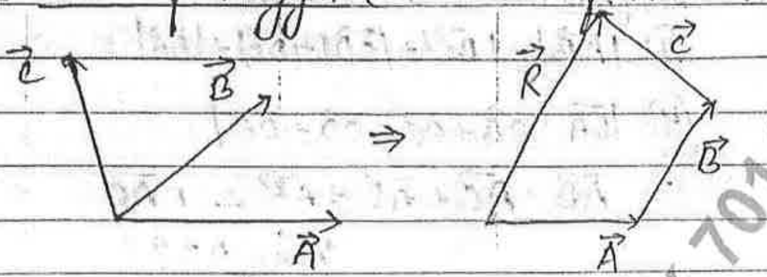
Solⁿ:- $\therefore \sqrt{3} = 2\times 1 \cos\frac{\theta}{2}$

$\theta = 60^\circ$.

Now, $|2\hat{a}+3\hat{b}| = \sqrt{4+9+2\times 2\times 3\cos 60^\circ}$

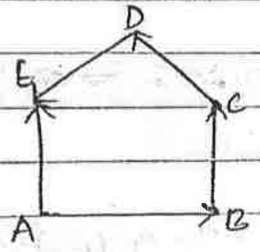
$= \sqrt{19}$ (Ans):

[C] Law of Polygon (Addition of more than 2 vector)



$\therefore \vec{R} = \vec{A}+\vec{B}+\vec{C}$

eg:-

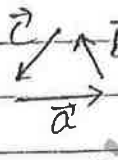


$\therefore \vec{AB}+\vec{BC}+\vec{CD}+\vec{DE} = \vec{AE}$

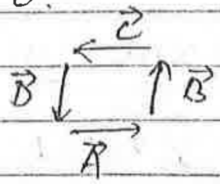
$\vec{AB}+\vec{BC}+\vec{CD}+\vec{DE} - \vec{AE} = \vec{O}$

$\underbrace{\vec{AB}+\vec{BC}+\vec{CD}+\vec{DE}+\vec{EA}}_{\text{same order}} = \vec{O}$

Note: ⇒ (1) If sides of Polygon represented by vectors in same order then their resultant (sum) is $\vec{O}$.



$\therefore \vec{a}+\vec{b}+\vec{c} = \vec{O}$

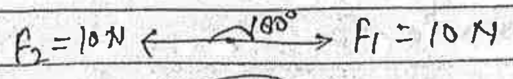


$\therefore \vec{A}+\vec{B}+\vec{C}+\vec{D} = \vec{O}$

(2). If n coplanar vectors of equal magnitude are seperated by equal angle θ (where $\theta = \frac{360^\circ}{n}$) then their resultant will be $\vec{O}$

eg:- (i). 2 vector

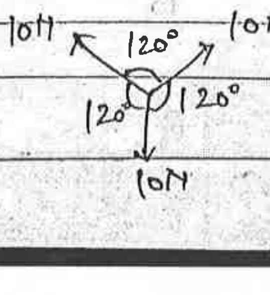
$\theta = \frac{360^\circ}{2} = 180^\circ$



R=0

(ii). 3 vector

$\theta = \frac{360^\circ}{3} = 120^\circ$

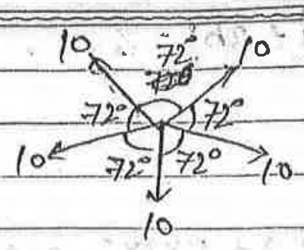


R=0

(iii). 5 vectors

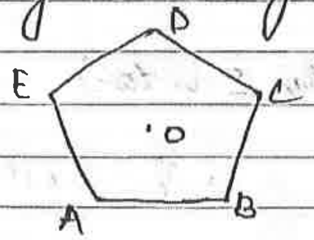
$\theta = \frac{360^\circ}{5}$

$\theta = 72^\circ$.



$R = 0$

Ques:- A regular Pentagon of side 'a'.



① $|\vec{AB} + \vec{BC} + \vec{CD} + \vec{DE}| = |\vec{AE}| = a$

② $|\vec{AB} + \vec{BC} + \vec{CD} + \vec{DE} + \vec{EA}| = 0$

③ $|\vec{AB}| + |\vec{BC}| + |\vec{CD}| + |\vec{DE}| + |\vec{EA}| = 5a$

④ $|\vec{OA} + \vec{OB} + \vec{OC} + \vec{OD} + \vec{OE}| = 0$

Imp. ⑤ $\vec{AB} + \vec{AC} + \vec{AD} + \vec{AE} = n\vec{AO}$

Then $n = ?$

Sol^n:- $\because \vec{AB} + \vec{AC} + \vec{AD} + \vec{AE} = n\vec{AO}$

$\cancel{\vec{AB} + (\vec{AB} + \vec{BC}) + (\vec{AB} + \vec{BC} + \vec{CD}) + (\vec{AB} + \vec{BC} + \vec{CD} + \vec{DE}) = n\vec{AO}}$

$(\vec{AO} + \vec{OB}) + (\vec{AO} + \vec{OC}) + (\vec{AO} + \vec{OD}) + (\vec{AO} + \vec{OE}) = n\vec{AO}$

$4\vec{AO} + (\vec{OB} + \vec{OC} + \vec{OD} + \vec{OE}) = n\vec{AO}$

$4\vec{AO} + \vec{AO} = n\vec{AO}$

$5\vec{AO} = n\vec{AO}$

$\Rightarrow n = 5$ (Ans).

$\left(\because \vec{OA} + \vec{OB} + \vec{OC} + \vec{OD} + \vec{OE} = \vec{0} \quad \vec{OB} + \vec{OC} + \vec{OD} + \vec{OE} = -\vec{OA} = \vec{AO}\right)$

Trick:- $\boxed{\text{no. of sides} = n}$

H.W. = 1 to 24 (Ex-I). , B.B-1

(8). $\vec{A}$, $\vec{B}$, & $\vec{C}$ vectors of different magnitude (co-planar) may have zero resultant if $(A-B) \leq C \leq (A+B)$

eg.:- $|\vec{A}| = 6$, $|\vec{B}| = 8$, $|\vec{C}| = 10$

$2 \leq C \leq 14$, $C = 10$

10/08/19

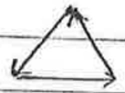
Ques:- W.O.F. combination of forces (Co-planar) may result into eq^n ($F_{net} = 0$).

	F_1	F_2	F_3
(1)	2	1	4
(2)	1	1	3
(3)	5	5	11
✓(4)	2	4	2

$\underset{(2-2)}{0} \leq 4 \leq \underset{(2+2)}{4}$.

Special Points:-

(1). Resultant of two vectors of equal magnitude may be '0' (if $\theta = 180°$)
(2). Resultant of two vectors of different magnitude can never be '0'.
(3). Min^m no. of co-planar vectors of different magnitude to get '0' resultant is 3.

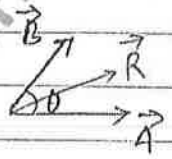


or $(A-B) \leq C \leq (A+B)$

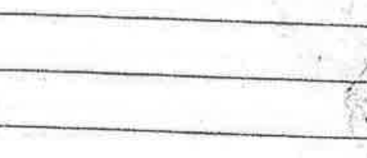
(4). Min^m no. of non-coplanar vectors of different magnitude to get '0' resultant is 4.

- Substraction of Vectors

Vector Sum $\vec{R} = \vec{A} + \vec{B}$



Substraction $\vec{R'} = \vec{A} - \vec{B}$
$\vec{R'} = \vec{A} + (-\vec{B})$



$\therefore R' = \sqrt{A^2 + B^2 + 2AB\cos(180-\theta)}$

$$\boxed{R' = \sqrt{A^2 + B^2 - 2AB\cos\theta}}$$

$\therefore \tan\alpha = \dfrac{B\sin(180-\theta)}{A + B\cos(180-\theta)}$

$$\boxed{\tan\alpha = \frac{B\sin\theta}{A - B\cos\theta}} \qquad \boxed{\tan\beta = \frac{A\sin\theta}{B - A\cos\theta}}$$

Results :- (1). Sum

$$R = \sqrt{A^2+B^2+2AB\cos\theta}$$

Substraction

$$R' = \sqrt{A^2+B^2-2AB\cos\theta}$$

If $\vec{A} \perp \vec{B}$
$\theta = 90^\circ$

$$\Rightarrow R = \sqrt{A^2+B^2} \qquad \Rightarrow R' = \sqrt{A^2+B^2}$$

so at $\theta = 90^\circ$, $R = R'$

If $\vec{A} \parallel \vec{B}$

$$\Rightarrow R = (A+B) \qquad \Rightarrow R' = (A-B)$$

If $\vec{A} \uparrow\downarrow \vec{B}$

$$\Rightarrow R = (A-B) \qquad \Rightarrow R' = (A+B)$$

(2). If $|\vec{A}| = |\vec{B}| = A$

so, $R' = \sqrt{A^2+A^2+2A\cdot A\cos\theta} = \sqrt{2A^2(1-\cos\theta)}$

$R' = \sqrt{2A^2(2\sin\theta/2)}$

Imp.

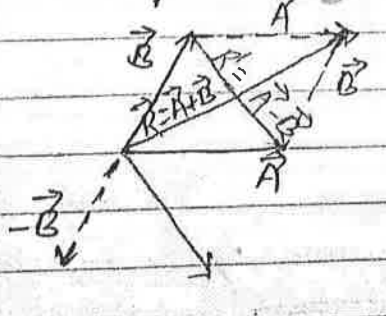
$$\boxed{R' = 2A\sin\frac{\theta}{2}}$$

If $\theta = 60^\circ$ $\quad R' = 2A\sin\frac{60^\circ}{2}$

$$\boxed{R' = A}$$

Imp.
→ Substraction of two unit vector is unit vector if $\theta = 60^\circ$

(3). Graphical Representation



Ques:- If $\vec{A}+\vec{B}=\vec{A}-\vec{B}$ Then -
① $\vec{A}$ is null vector
② $\vec{B}$ is null vector
③ $\vec{A} \perp \vec{B}$
④ No Idea

Ques:- If $\dfrac{|\vec{a}+\vec{b}|}{|\vec{a}-\vec{b}|}=1$ then angle b/w $\vec{a}$ & $\vec{b}$.

Soln:- $\sqrt{a^2+b^2+2ab\cos\theta} = \sqrt{a^2+b^2-2ab\cos\theta}$

$a^2+b^2+2ab\cos\theta = a^2+b^2-2ab\cos\theta$

$4ab\cos\theta = 0$

$\cos\theta = 0$

$\theta = 90^\circ$. (Ans)!.

Ques:. If $|\hat{a}+\hat{b}|=\sqrt{3}$ then $|2\hat{a}-4\hat{b}|=?$

Soln:- $\therefore |\hat{a}+\hat{b}| = \sqrt{1^2+1^2+2\cdot1\cdot1\cos\theta}$

$\sqrt{3} = \sqrt{2+2\cos\theta}$

$3 = \cancel{2(1+\cos\theta)}\ 2+2\cos\theta$

$2\cos\theta = 1$

$\cos\theta = \frac{1}{2} \Rightarrow \theta = 60^\circ$ (Ans)!.

Now, $|2\hat{a}-4\hat{b}| = \sqrt{2^2+4^2-2\times2\times4\cos60}$

$= \sqrt{4+16-\cancel{2}\times2\times4\times\frac{1}{\cancel{2}}}$

$= \sqrt{12} = 2\sqrt{3}$ (Ans)!.

Ques:- If $|\hat{P}+\hat{Q}|=1$ Then $|\hat{P}-\hat{Q}|=?$

Soln:- $\therefore 1 = \sqrt{1^2+1^2+2\cdot1\cdot1\cos\theta}$

$1 = 2+2\cos\theta$

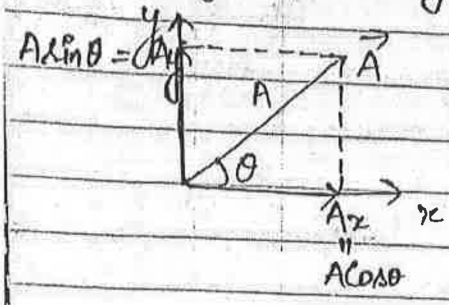
$\cos\theta = -\frac{1}{2}$

Now, $|\hat{P}-\hat{Q}| = \sqrt{1^2+1^2-2\cdot1\cdot1\cdot\cos\theta}$

$= \sqrt{2-2\times\left(-\frac{1}{2}\right)}$

$= \sqrt{3}$ (Ans)!.

- Rectangular / Orthagonal Component (Projection) of Vector in a Plane (2D)

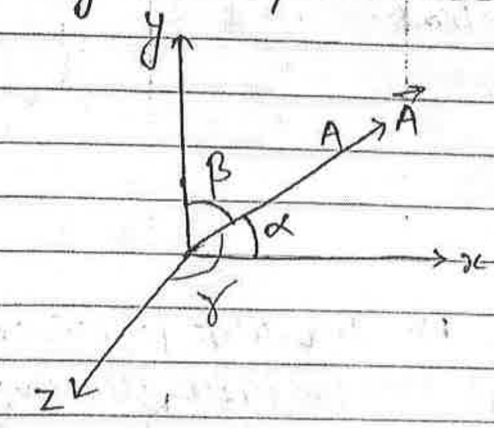


$\therefore \vec{A} = A_x\hat{i} + A_y\hat{j}$

$|\vec{A}| = \sqrt{A_x^2 + A_y^2}$

$\vec{A} = A\cos\theta\,\hat{i} + A\sin\theta\,\hat{j}$

- Rectangular Component in Space (3-D)



$A_x = A\cos\alpha \Rightarrow \frac{A_x}{A} = \cos\alpha$ — ① dirⁿ

$A_y = A\cos\beta \Rightarrow \frac{A_y}{A} = \cos\beta$ — ② cosine.

$A_z = A\cos\gamma \Rightarrow \frac{A_z}{A} = \cos\gamma$ — ③

$\therefore \vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$

$|\vec{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$

$①^2 + ②^2 + ③^2$

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = \frac{A_x^2 + A_y^2 + A_z^2}{A^2}$

$= \frac{A^2}{A^2} = 1$

Imp.

$\Rightarrow \boxed{\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1}$

or

$\boxed{\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2}$

Ques :- If angle made by vector with x & z are 45° & 60° respectively. Then angle made with y-axis will be-

① 60°
② 120°
③ both (1) & (2) ✓
④ 90°

$\therefore \cos^2 45° + \cos^2\beta + \cos^2 60° = 1$

$\frac{1}{2} + \cos^2\beta + \frac{1}{4} = 1$

$\cos^2\beta = 1 - \frac{3}{4} = \frac{1}{4}$

$\cos\beta = \pm 1/2$

$\therefore \cos\beta = +1/2$
$\beta = 60°$
$\cos\beta = -1/2$
$\beta = 120°$

Ques:- x & y component of $\vec{A}$ and resultant of $\vec{A}$ and $\vec{B}$ are resp. 2, 3 & 4, 1. Then

① $\vec{B}$

② dirⁿ of $\vec{B}$

③ dirⁿ cosine of $\vec{B}$

let $A_x = 2$, $A_y = 3$.

$\vec{A} = 2\hat{i} + 3\hat{j}$

$(\vec{A}+\vec{B})_x = 4$

$(\vec{A}+\vec{B})_y = 1$

$\therefore \vec{A}+\vec{B} = 4\hat{i}+\hat{j}$

① $\vec{B} = (\vec{A}+\vec{B}) - \vec{A}$

$= (4\hat{i}+\hat{j}) - (2\hat{i}+3\hat{j})$

$= 2\hat{i} - 2\hat{j}$

② dirⁿ of $\vec{B} \Rightarrow \hat{B} = \frac{\vec{B}}{|B|}$

$= \frac{2\hat{i}-2\hat{j}}{2\sqrt{2}} = \frac{\hat{i}-\hat{j}}{\sqrt{2}}$

③. $\cos\alpha = \frac{B_x}{B} = \frac{2}{2\sqrt{2}} = \frac{1}{\sqrt{2}}$

$\cos\beta = \frac{B_y}{B} = \frac{-2}{2\sqrt{2}} = -1/\sqrt{2}$

Ques:- If $\vec{P} = 2\hat{i} - \hat{j} + 2\hat{k}$ and $\vec{Q} = \hat{i} + 2\hat{j} + \hat{k}$ Then,

① Dirⁿ of $\vec{P}$ & $\vec{Q}$

② Dirⁿ cosine of $\vec{P}$ & $\vec{Q}$.

③ Dirⁿ of resultant of $\vec{P}$ & $\vec{Q}$.

④ Dirⁿ cosine of resultant of $\vec{P}$ & $\vec{Q}$.

⑤ Angle made by resultant of $\vec{P}$ & $\vec{Q}$ with x-axis.

Solⁿ: ① $\hat{P} = \frac{2\hat{i} - \hat{j} + 2\hat{k}}{3}$

$\hat{Q} = \frac{\hat{i}+2\hat{j}+\hat{k}}{\sqrt{6}}$

② $\vec{P}$

$\cos\alpha = \frac{2}{3}$

$\cos\beta = -1/3$

$\cos\gamma = 2/3$

$\vec{Q}$

$\cos\alpha = 1/\sqrt{6}$

$\cos\beta = 2/\sqrt{6}$

$\cos\gamma = 1/\sqrt{6}$

③ $\vec{R} = \vec{P}+\vec{Q} = 3\hat{i} + \hat{j} + 3\hat{k}$

dirⁿ $\Rightarrow \hat{R} = \frac{\vec{R}}{|\vec{R}|} = \frac{3\hat{i}+\hat{j}+3\hat{k}}{\sqrt{19}}$

④ $\cos\alpha = 3/\sqrt{19}$

$\cos\beta = 1/\sqrt{19}$

$\cos\gamma = 3/\sqrt{19}$

⑤ $\because \cos\alpha = 3/\sqrt{19}$

$\alpha = \cos^{-1}\left(\frac{3}{\sqrt{19}}\right)$

- DOT Product (Scalar Product)

DOT product of $\vec{A}$ & $\vec{B}$ is given by-

$\boxed{\vec{A}\cdot\vec{B} = AB\cos\theta}$ where, θ = angle b/w $\vec{A}$ & $\vec{B}$.

(1). If $\theta = 90°$

$\vec{A}\cdot\vec{B} = AB\cos 90° = 0$

eg:- $\hat{i}\cdot\hat{j} = 1\times1\times\cos 90° = 0$

$\hat{j}\cdot\hat{k} = 0$

$\hat{k}\cdot\hat{i} = 0$

(2). If $\theta = 0°$ $\Rightarrow [\vec{A}\cdot\vec{B} = AB\cos 0° = AB]_{max}$

(3). If $\theta = 180°$ $\Rightarrow [\vec{A}\cdot\vec{B} = AB\cos 180° = -AB]_{min}$

eg:- $\hat{i}\cdot\hat{i} = 1\times1\times\cos 0° = 1$

$\hat{j}\cdot\hat{j} = 1$

$\hat{k}\cdot\hat{k} = 1$

$\vec{A}\cdot\vec{A} = A\times A\times\cos 0° = A^2$

$\vec{a}\cdot\vec{a} = a^2$

Self-dot Product

(4). $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$

$\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$

$\boxed{\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y + A_zB_z}$

(5). Dot product is Commutative.

$\vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A}$

$AB\cos\theta = BA\cos\theta$.

(6). Dot Product is distributive

$\vec{A}\cdot(\vec{B}+\vec{C}) = \vec{A}\cdot\vec{B} + \vec{A}\cdot\vec{C}$

$= AB\cos\theta_1 + AC\cos\theta_2$

eg:- $(\vec{a}+\vec{b})\cdot(\vec{a}-\vec{b}) = ?$

$= \vec{a}\cdot\vec{a} - \cancel{\vec{a}\cdot\vec{b}} + \cancel{\vec{a}\cdot\vec{b}} - \vec{b}\cdot\vec{b}$

$= a^2 - b^2$

12/08/2019

Imp:

Ques:- If angle b/w $\hat{a}$ and $\hat{b}$ is 60° then f/o value of
$(\hat{a}+3\hat{b})\cdot(\hat{a}-2\hat{b}) = ?$

Sol^n:-

$\cancel{= \hat{a}\cdot\hat{a} - \hat{a}\cdot 2\hat{b} + 3\hat{b}\cdot\hat{a} - 3\hat{b}\cdot 2\hat{b}}$

$\cancel{= 1 - 2 + 3 - 6}$

$\cancel{= -4}$

$\because (\hat{a}+3\hat{b})\cdot(\hat{a}-2\hat{b}) = ?$

$= \hat{a}\cdot\hat{a} - \hat{a}\cdot 2\hat{b} + 3\hat{b}\cdot\hat{a} - 3\hat{b}\cdot 2\hat{b}$

$= 1\times1\times\cos 0^\circ - 1\times2\cos 60^\circ + 3\times1\times\cos 60^\circ - 3\times2\times\cos 0^\circ$

$= \cancel{1} - \cancel{1} + 1.5 - 6$

$= -4.5$ (Ans):.

Ques:- If $|\hat{a}+\hat{b}| = \sqrt{3}$ Then $(3\hat{a}+2\hat{b})\cdot(\hat{a}-\hat{b}) = ?$

Sol^n:-

$\therefore \sqrt{3} = 2\times1\cos\frac{\theta}{2}$

$\cos\frac{\theta}{2} = \frac{\sqrt{3}}{2}$

$\frac{\theta}{2} = 30^\circ$

$\theta = 60^\circ$.

Now, $(3\hat{a}+2\hat{b})\cdot(\hat{a}-\hat{b}) = ?$

$= 3\hat{a}\cdot\hat{a} - 3\hat{a}\hat{b} + 2\hat{b}\hat{a} - 2\hat{b}\hat{b}$

$= 3\times1\times\cos 0^\circ - 3\times1\times\cos 60^\circ + 2\times1\times\cos 60^\circ - 2\times1\times\cos 0^\circ$

$= 3 - \frac{3}{2} + \frac{2}{2} - 2$

$= 2 - \frac{3}{2} = \frac{1}{2}$ (Ans):.

- Application of DOT Product

(1). If two vectors are ⊥ to eachother then their dot product will be zero.

Ques:- If $\vec{A} = 2\hat{i} - \alpha\hat{j} + 3\hat{k}$
$\vec{B} = \hat{i} + \alpha\hat{j} + 2\hat{k}$
If $\vec{A} \perp \vec{B}$ then $\alpha = ?$

Sol^n:-

$\therefore \vec{A}\cdot\vec{B} = AB\cos 90^\circ = 0$

$(2\hat{i} - \alpha\hat{j} + 3\hat{k})\cdot(\hat{i} + \alpha\hat{j} + 2\hat{k}) = 0$

$2 - \alpha^2 + 6 = 0$

$\alpha^2 = 8$

$\alpha = 2\sqrt{2}$ (Ans):.

or $\alpha = -2\sqrt{2}$

Ques:- $\vec{A} = a\hat{i} + a\hat{j} + 3\hat{k}$
$\vec{B} = a\hat{i} - 2\hat{j} - \hat{k}$
F/o value of 'a' for which $\vec{A}$ is orthogonal to $\vec{B}$.

Sol^n:- For Orthogonal ($\perp$),

$$\vec{A}\cdot\vec{B} = 0$$

$$(a\hat{i}+a\hat{j}+3\hat{k})\cdot(a\hat{i}-2\hat{j}-\hat{k}) = 0$$

$$a^2-2a-3=0$$

$$\therefore a = \frac{-(-2)\pm\sqrt{4-4\times1\times(-3)}}{2\times1}$$

$$a = \frac{2\pm4}{2}$$

$$\Rightarrow a = \frac{2+4}{2} = 3, \quad a = \frac{2-4}{2} = -1$$

$$\Rightarrow a = 3 \text{ or } a = -1 \text{ (Ans):-}$$

Imp

Ques:- If $(\hat{a}+\hat{b})$ and $(2\hat{a}-3\hat{b})$ are $\perp$ to eachother then angle b/w $\hat{a}$ & $\hat{b}$ will be?

Sol^n:-

$$\therefore (\hat{a}+\hat{b})\cdot(2\hat{a}-3\hat{b}) = 0$$

$$\Rightarrow \hat{a}\cdot2\hat{a} - 3\hat{a}\cdot\hat{b} + 2\hat{a}\cdot\hat{b} - 3\hat{b}\cdot\hat{b} = 0$$

$$2 - (1)(1\times1\times\cos\theta) - 3(1) = 0$$

$$\cos\theta = -1$$

$$\theta = 180^\circ \text{ (Ans):-}$$

(2). To find angle b/w two vectors.

$$\vec{A}\cdot\vec{B} = AB\cos\theta$$

$$\cos\theta = \frac{\vec{A}\cdot\vec{B}}{AB}$$

$$\theta = \cos^{-1}\left(\frac{\vec{A}\cdot\vec{B}}{AB}\right)$$

Imp

Ques:- F/o angle b/w $(\hat{i}+2\hat{j})$ & $(\hat{k}-\hat{i})$ vector.

Sol^n:-

$$\therefore (\hat{i}+2\hat{j})\cdot(\hat{k}-\hat{i}) = \sqrt{5}\cdot\sqrt{2}\cos\theta$$

$$-1 = \sqrt{10}\times\cos\theta$$

$$\theta = \cos^{-1}\left(\frac{-1}{\sqrt{10}}\right) \text{ (Ans):-}$$

Ques:- $\vec{P} = \hat{i} + 2\hat{j} + 3\hat{k}$
$\vec{Q} = \hat{i} + \hat{j} + \hat{k}$
$\vec{S} = \hat{k} + 2\hat{j} - \hat{i}$

f/o angle b/w $\vec{P}$ and resultant of $\vec{Q}$ and $\vec{S}$.

Sol^n:- $\vec{R}_{QS} = 2\hat{k} + 3\hat{j}$

Now, $\vec{P}\cdot\vec{R} = PR\cos\theta$.

$(\hat{i} + 2\hat{j} + 3\hat{k})\cdot(2\hat{k} + 3\hat{j}) = \sqrt{14}\cdot\sqrt{13}\cos\theta$

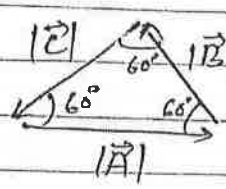
$6 + 6 = \sqrt{182}\cos\theta$

$\cos\theta = \dfrac{12}{\sqrt{182}} \Rightarrow \theta = \cos^{-1}\left(\dfrac{12}{\sqrt{182}}\right)$ (Ans):

Imp.

Ques:- If $\vec{A} + \vec{B} + \vec{C} = \vec{0}$ & $|\vec{A}| = |\vec{B}| = |\vec{C}| = 1$ Then f/o value of - $\vec{A}\cdot\vec{B} + \vec{B}\cdot\vec{C} + \vec{C}\cdot\vec{A}$

Sol^n:-



$\therefore \vec{A}\cdot\vec{B} + \vec{B}\cdot\vec{C} + \vec{C}\cdot\vec{A} = AB\cos\theta_1 + BC\cos\theta_2 + CA\cos\theta_3$

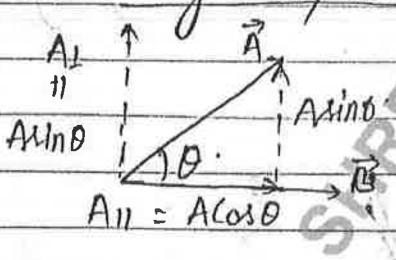
$= 1\times1\times\cos120° + 1\times1\times\cos120° + 1\times1\times\cos120°$

$= \left(\frac{-1}{2}\right) + \left(\frac{-1}{2}\right) + \left(\frac{-1}{2}\right)$

$= -\frac{3}{2}$ (Ans):

$\therefore \theta_1 = \theta_2 = \theta_3 = 180° - 60°$
$= 120°$

③. Projection / Components of $\vec{A}$ along $\vec{B}$



Ⓐ Component of $\vec{A}$ along $\vec{B}$

(i). Scalar form $= A\cos\theta$.

$\because \vec{A}\cdot\vec{B} = AB\cos\theta$

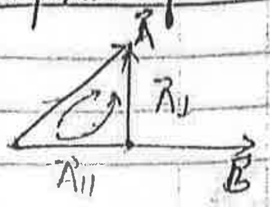
$A\cos\theta = \dfrac{\vec{A}\cdot\vec{B}}{B}$

$$\boxed{\text{Projection} = \frac{\vec{A}\cdot\vec{B}}{B} \text{ or } \vec{A}\cdot\hat{B}}$$

(ii). Vector form $= (A\cos\theta)\hat{B}$

$= \left(\dfrac{\vec{A}\cdot\vec{B}}{B}\right)\hat{B} = (\vec{A}\cdot\hat{B})\hat{B}$

(B) Component of $\vec{A}$ ⊥ to $\vec{B}$ $(\vec{A}_\perp)$



Δ Law,

$$\vec{A} = \vec{A}_{\parallel} + \vec{A}_{\perp}$$

$$\boxed{\vec{A}_{\perp} = \vec{A} - \vec{A}_{\parallel}}$$

Imp.

Ques:- $\vec{A} = \hat{i} + 2\hat{j} + 3\hat{k}$ f/o Projection of $(\hat{i}+2\hat{j}+3\hat{k})$

$\vec{B} = \hat{i} + \hat{j} + \hat{k}$

(i) Parallel to $(\hat{i}+\hat{j}+\hat{k})$

(ii) ⊥ to $(\hat{i}+\hat{j}+\hat{k})$

Solⁿ:- (1) Projection $(\vec{A}_{\parallel}) = \dfrac{\vec{A}\cdot\vec{B}}{B}\hat{B}$

$$= \frac{1+2+3}{\sqrt{3}} \times \left[\frac{(\hat{i}+\hat{j}+\hat{k})}{\sqrt{3}}\right] = 2\hat{i} + 2\hat{j} + 2\hat{k} \text{ (Ans)}$$

(2) Projection $(\vec{A}_{\perp}) = \vec{A} - \vec{A}_{\parallel}$

$$= (\hat{i}+2\hat{j}+3\hat{k}) - (2\hat{i}+2\hat{j}+2\hat{k})$$

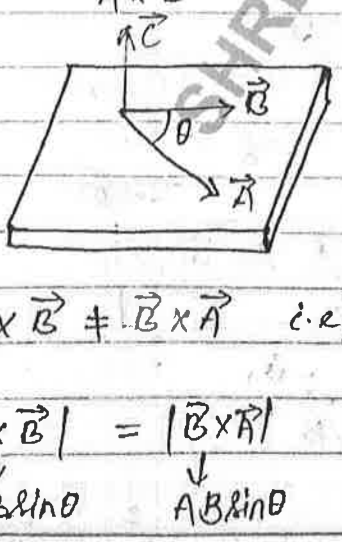
$$= -\hat{i} + \hat{k} = \hat{k} - \hat{i} \text{ (Ans)}$$

- Cross Product (Vector Product)

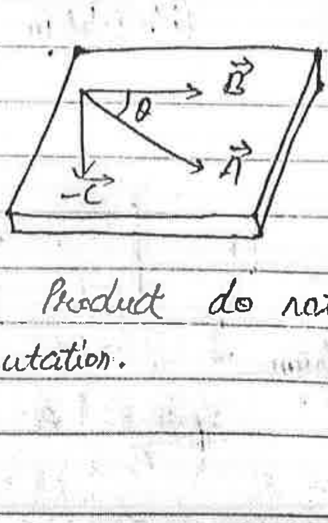
$$\vec{A}\times\vec{B} = \underbrace{AB\sin\theta}_{\text{magnitude}}\ \underbrace{\hat{n}}_{\text{dirⁿ}}$$

Here, $\hat{n}$ = unit vector of $\vec{A}\times\vec{B}$ (dirⁿ of $\vec{A}\times\vec{B}$) which is ⊥ to plane of $\vec{A}$ & $\vec{B}$ and dirⁿ can be determined by Right hand Thumb rule.

$\vec{A}\times\vec{B}$



$\vec{B}\times\vec{A}$



→ $\vec{A}\times\vec{B} \neq \vec{B}\times\vec{A}$ i.e; Cross Product do not follow Law of Commutation.

→ $|\vec{A}\times\vec{B}| = |\vec{B}\times\vec{A}|$

$\downarrow$ $AB\sin\theta$ $\qquad$ $\downarrow$ $AB\sin\theta$

→ $\vec{A}\times\vec{B} = -(\vec{B}\times\vec{A})$

→ unit vector ⊥ to $\vec{A}$ & $\vec{B}$, $\hat{n} = \dfrac{\vec{A}\times\vec{B}}{AB\sin\theta}$

→ Cross Product follows distributive law.

$$\vec{A}\times(\vec{B}+\vec{C}) = (\vec{A}\times\vec{B})+(\vec{A}\times\vec{C})$$

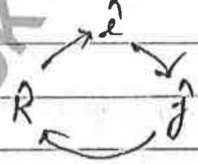
Note :⇒ (1). $\theta = 0^\circ \Rightarrow |\vec{A}\times\vec{B}| = AB\sin 0^\circ = \vec{0}$

$$\left\{\begin{array}{l} \hat{i}\times\hat{i} = 1\times1\times\sin\theta\,\hat{n} = \vec{0} \\ \hat{j}\times\hat{j} = \vec{0} \\ \hat{k}\times\hat{k} = \vec{0} \\ \vec{A}\times\vec{A} = \vec{0} \end{array}\right\}$$

* Self-cross Product is always $\vec{0}$.

(2). If $\theta = 90^\circ$ $(\vec{A}\perp\vec{B}) \Rightarrow \vec{A}\times\vec{B} = AB\sin 90^\circ = (AB)_{max}$

Dirn



CW	ACW
$\hat{i}\times\hat{j} = \hat{k}$	$\hat{j}\times\hat{i} = -\hat{k}$
$\hat{j}\times\hat{k} = \hat{i}$	$\hat{k}\times\hat{j} = -\hat{i}$
$\hat{k}\times\hat{i} = \hat{j}$	$\hat{i}\times\hat{k} = -\hat{j}$

(3). $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$

$\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$

$$\vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = \hat{i}(A_yB_z - A_zB_y) - \hat{j}(A_xB_z - A_zB_x) + \hat{k}(A_xB_y - A_yB_x)$$

Ques:- $\vec{A} = 3\hat{i}$ $B = -2\hat{j}$

① $\vec{A}\times\vec{B} = -6(\hat{i}\times\hat{j}) = -6\hat{k}$

② $\vec{B}\times\vec{A} = -6(\hat{j}\times\hat{i}) = -6(-\hat{k}) = 6\hat{k}$ } (Ans):

~~Ques:- Angular velocity, $\vec{\omega} = [\hat{i}+\hat{j}]$; Radial vector, $\vec{r} = (\hat{j}+\hat{k})$. F/o~~

v

(4). $\vec{C} = \vec{A} \times \vec{B}$

$\vec{C} \perp \vec{A} \Rightarrow \vec{C} \cdot \vec{A} = 0$

$\vec{C} \perp \vec{B} \Rightarrow \vec{C} \cdot \vec{B} = 0$

eg:- Use of Cross Product in Mechanics.

(a) Torque, $\vec{\tau} = \vec{r} \times \vec{F}$

$\vec{\tau} \perp \vec{r} \Rightarrow \vec{\tau} \cdot \vec{r} = 0$

$\vec{\tau} \perp \vec{F} \Rightarrow \vec{\tau} \cdot \vec{F} = 0$

(b) $\vec{v} = \vec{\omega} \times \vec{r}$

$\vec{v} \perp \vec{\omega} \Rightarrow \vec{v} \cdot \vec{\omega} = 0$

$\vec{v} \perp \vec{r} \Rightarrow \vec{v} \cdot \vec{r} = 0$

Ques:- If $\underset{(\text{radial})}{\vec{r}} = (\hat{j} + \hat{k})$ and $\vec{\omega} = (\hat{i} + \hat{j})$ Then f/o linear velocity of Particle.

Solⁿ:- $\therefore \vec{v} = \vec{\omega} \times \vec{r}$

$\vec{v} = (\hat{i} + \hat{j}) \times (\hat{j} + \hat{k})$.

$\vec{v} = \hat{k} - \hat{j} + \hat{i}$

$\vec{v} = \hat{i} - \hat{j} + \hat{k}$

$|\vec{v}| = \sqrt{1^2 + (-1)^2 + 1^2}$

$|\vec{v}| = \sqrt{3}$ m/s. (Ans):-

Ques:- $\vec{A} \cdot \vec{B} = |\vec{A} \times \vec{B}|$ Then angle b/w $\vec{A}$ & $\vec{B}$.

Solⁿ:- $AB\cos\theta = AB\sin\theta$.

$\frac{\sin\theta}{\cos\theta} = 1$

$\tan\theta = 1$

$\theta = 45°$ (Ans):-

Imp.

Ques:- F/o unit vector $\perp$ to vector $(\hat{i} + 2\hat{j} + \hat{k})$ and $(\hat{k} - \hat{i})$ ...

Solⁿ:- $\therefore \hat{A} = \frac{\vec{A} \times \vec{B}}{|\vec{A} \times \vec{B}|}$

13/09/2019

$\hat{n} = \frac{2(\hat{i}-\hat{j}+\hat{k})}{2\sqrt{3}}$

$\hat{n} = \hat{i}-\hat{j}+\hat{k}$ (Ans):-

$$\vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 1 \\ -1 & 0 & 1 \end{vmatrix}$$

$= \hat{i}(2) - \hat{j}(2) + \hat{k}(2)$

$= 2\hat{i} - 2\hat{j} + 2\hat{k}$.

$|\vec{A}\times\vec{B}| = \sqrt{2^2+2^2+2^2} = \sqrt{12} = 2\sqrt{3}$

Imp.

Ques:- $\vec{P} = 2\hat{i} - 4\hat{j} + \hat{k}$
$\vec{Q} = \hat{i} + 2\hat{j} - 3\hat{k}$

Then f/o value of-
$(\vec{P}+\vec{Q})\cdot(\vec{P}\times\vec{Q}) = ?$

Solⁿ:-

$(\vec{P}+\vec{Q})$ → always $\perp$ to in plane of P & Q

$(\vec{P}\times\vec{Q})$ → always $\perp$ to plane of P & Q.

$\perp \Rightarrow$ dot Product $= 0$ (Ans).

Imp.

- Condition of Co-planarity of 3 vectors.

→ Condition of co-planarity of $\vec{A}, \vec{B}$ & $\vec{C}$.

$[\vec{A}\cdot(\vec{B}\times\vec{C}) = 0]$

eg.:- Check that $\vec{P} = \hat{i}+\hat{j}+\hat{k}$, $\vec{Q} = -\hat{i}+\hat{k}$, $\vec{R} = \hat{i}+2\hat{j}+3\hat{k}$ are co-planar or not.

$\therefore [\vec{P}\cdot(\vec{Q}\times\vec{R}) = 0]$.

$\Rightarrow (\hat{i}+\hat{j}+\hat{k})\cdot[(-\hat{i}+\hat{k})\times(\hat{i}+2\hat{j}+3\hat{k})] = (\hat{i}+\hat{j}+\hat{k})\cdot(-2\hat{k}+3\hat{j}+\hat{j}-2\hat{i})$

$= (\hat{i}+\hat{j}+\hat{k})\cdot(-2\hat{i}+4\hat{j}-2\hat{k})$

$= -2+4-2$

$= 0$

⇒ Hence, $\vec{P}, \vec{Q}$ & $\vec{R}$ are co-planars.

→ Condition of $\vec{A} \parallel \vec{B}$

$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$

$\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$

for $\vec{A}\parallel\vec{B}$

$$\frac{A_x}{B_x} = \frac{A_y}{B_y} = \frac{A_z}{B_z}$$

Ques:- $\vec{A} = 2\hat{i} - \hat{j} - 8\hat{k}$ If $\vec{A} \parallel \vec{B}$ Then f/o α & β.
$\vec{B} = 4\hat{i} + \alpha\hat{j} - \beta\hat{k}$

Sol^n:- $\therefore \frac{2}{4} = \frac{-1}{\alpha}$ $\therefore \frac{2}{4} = \frac{+3}{+\beta}$

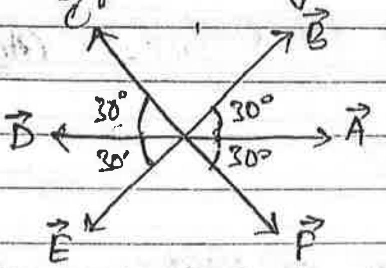
$\alpha = -2$ (Ans). $\beta = 2 \times 3 = 6$ (Ans).

Note ⇒ (1). If $\vec{A}$ and $\vec{B}$ represent two sides of Δ then area of $\Delta = \frac{1}{2}|\vec{A} \times \vec{B}|$.

(2). If $\vec{A}$ & $\vec{B}$ are two sides of Parallelogram then area of Parallelogram $= |\vec{A} \times \vec{B}|$

(3). If $\vec{d_1}$ & $\vec{d_2}$ are two diagonal of Parallelogram then area of Parallelogram $= \frac{1}{2}|\vec{A} \times \vec{B}|$

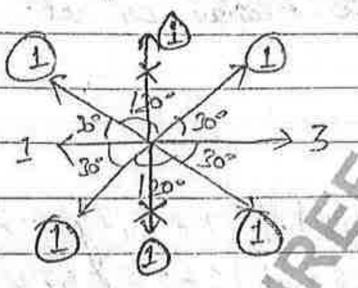
Ques:-



If magnitude of $\vec{A}, \vec{B}, \vec{C}, \vec{D}, \vec{E}$ & $\vec{F}$ are respectively 3, 2, 3, 6, 5 & 4. Then f/o value of $\left[\vec{A} + \frac{\vec{B}}{2} + \frac{\vec{C}}{3} + \frac{\vec{D}}{6} + \frac{\vec{E}}{5} + \frac{\vec{F}}{4}\right]$.

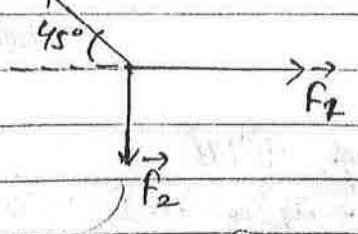
3 1 1 1 1 1

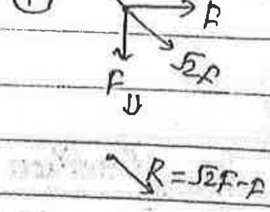
Sol^n:-



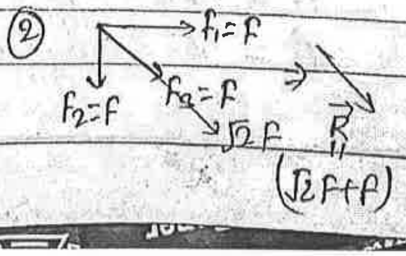
$\therefore |\vec{R}_{Net}| = 3 - 1 = 2$ (Ans).

Ques:- If $|\vec{F_1}| = |\vec{F_2}| = |\vec{F_3}|$

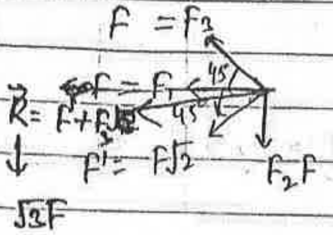




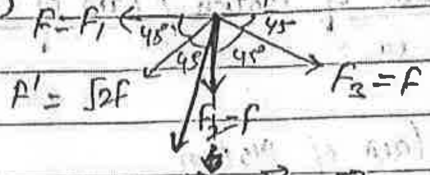
(A)	dir^n (B)
① $\vec{F_1} + \vec{F_2} + \vec{F_3}$	
② $\vec{F_1} + \vec{F_2} - \vec{F_3}$	
③ $\vec{F_2} - \vec{F_1} + \vec{F_3}$	
④ $\vec{F_2} - \vec{F_1} - \vec{F_3}$	



(3)



(4)



$\vec{R} = \vec{F}' + \vec{F}_3$

$|\vec{R}| = \sqrt{(\sqrt{2}F)^2 + F^2}$

$= \sqrt{3}F$

Kinematics

PAGE NO.
DATE : 28/08/19

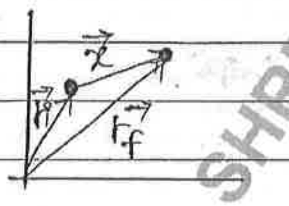
Rest - The posit^n of particle does not change with time.
Motion - posit^n changes with time.
NOTE: - Rest & Mot^n are relative term i.e., they depend on the frame of reference.

1-D Motion / Straight Line Motion :-

Distance - The actual path length travelled by the object.
- → S.I. unit → meter (scalar quantity).
- → distance travelled is always +ve.
- → For a moving object, distance always ↑es.
- → If distance travelled is 0, then the body must be @ rest.

Displacement - It is the shortest possible path b/w initial & final posit^n.
- → S.I. unit → meter (vector quantity). dir^n → from initial to final ".
- → displacement may be +ve, -ve or zero.
- → For a moving object, displacement may ↑ or ↓.
- → If displacement is 0, then the body is either at rest or crossing the initial position.

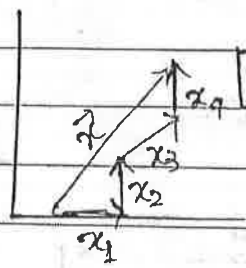
*



$\vec{r_f} = \vec{r_i} + \vec{x}$

$\boxed{\vec{x} = \vec{r_f} - \vec{r_i}}$

*



$\boxed{\vec{x} = \vec{x_1} + \vec{x_2} + \vec{x_3} + \vec{x_4}}$

Q. An object move 30 m towards north, 20 m towards east and $20\sqrt{2}$ m south west, find net displacement.

→ Net displacement ⇒ $\vec{x} = 30\hat{j} + 20\hat{i} + (-20\hat{i} - 20\hat{j})$

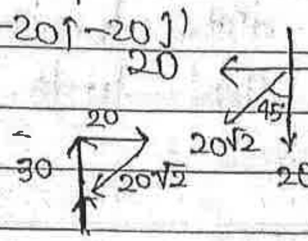
$= 10\hat{j}$ m.

$x_1 = 30\hat{j}$

$x_2 = 20\hat{i}$

$x_3 = 20\sqrt{2} = -20\hat{i} - 20\hat{j}$

$\vec{x} = 10\hat{j}$ m.



Q. A monkey moves 40 mt. towards north, take a right turn & moves 30 mt. & then climbs a tree 50 mt ht. Find magnitude of net displacement.

→ $x_1 = 40\hat{j}$ $\quad \vec{x} = 30\hat{i} + 40\hat{j} + 50\hat{k}$ $\quad |\vec{x}| = \sqrt{x_1^2 + x_2^2 + x_3^2}$

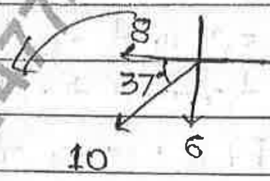
$x_2 = 30\hat{i}$. $\quad |\vec{x}| = 50\sqrt{2}$ m.

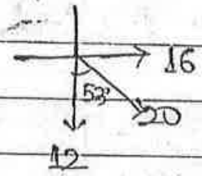
$x_3 = 50\hat{k}$.

Q. $x_1 \rightarrow 5$ m North $\quad \vec{x_1} \Rightarrow 5\hat{j}$

$x_2 \rightarrow 10$ m $37°$ S of W. $\quad \vec{x_2} = -8\hat{i} - 6\hat{j}$

$x_3 \rightarrow 20$ m $53°$ E of S. $\quad \vec{x_3} = 16\hat{i} - 12\hat{j}$

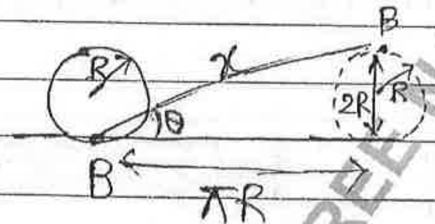




$\boxed{\vec{x} = 8\hat{i} - 13\hat{j}}$

Q. A wheel of radius 'R' is given half revolutⁿ on a rough H/z surface. Find magnitude of displacement of pt. 'B'.

→



$x^2 = (\pi R)^2 + (2R)^2$

$x = R\sqrt{\pi^2 + 4}$.

$\theta = \tan^{-1}\left(\frac{2}{\pi}\right)$. (dirⁿ).

*

A → C.

dist. = $l_1 + l_2$. $\quad \boxed{\text{dist.} \geq |\text{displ.}|}$

|displ.| = $l_1 - l_2$.

1 2 3 4 5 6

BB-1

EX-1 - Q1 to 7.

1 2 3 4 5 6 7

Done

Speed :– It is distance travelled per unit time
- → S.I. unit → m/s (scalar quantity)
- → For a moving obj., speed is always +ve.
- → " " " " " may ↑ or ↓.

Uniform speed – Body covers equal distances in equal intervals of time.

~~Variable~~ / Non-uniform speed – Unequal distances in equal intervals of time.

Average speed
$$V_{avg} = \frac{\text{Total distance}}{\text{Total time}} = \frac{\Delta s}{\Delta t}$$

Instantaneous speed
$$V_{inst.} = \frac{ds}{dt}$$

velocity – It is displacement per unit time.
- → S.I. unit → m/s (vector quantity) dirn is same as displacement
- → For a moving obj, velocity can be +ve, –ve or zero → applicable only for avg. velocity

Uniform velocity – The magnitude & dirn of velocity remain constant.

Non-uniform velocity – Either directn or magnitude or both changed.

Average velocity
$$\vec{V}_{avg} = \frac{\text{Total displacement}}{\text{Total time}} = \frac{d\vec{x}}{dt}$$

Instantaneous velocity
$$\vec{v} = \frac{d\vec{x}}{dt}$$

* $\frac{\text{dist.}}{\Delta t} \geq \frac{|\text{disp.}|}{\Delta t}$

$$\boxed{V_{avg} \geq |\vec{V}_{avg}|}$$

For 'dt'

$$\frac{\text{dist.}}{dt} = \frac{|\text{disp.}|}{dt}$$

$$\boxed{V_{inst.} = |\vec{v}|}$$

Q. A person travells 1st half time of its mot^n @ speed v_1 & the other half @ v_2. Find avg. speed.

→ $t \quad 2t \quad t$; $v_1 \mid v_2$.

$$V_{avg.} = \frac{v_1 t + v_2 t}{2t} = \boxed{\frac{v_1 + v_2}{2}}.$$

Q. A person covers 1st half dist. @ v_1 & other half @ v_2, find avg. speed.

→ $x \quad 2x \quad x$; $v_1 \quad v_2$

$$V_{avg} = \frac{2x}{\frac{x}{v_1} + \frac{x}{v_2}} = \boxed{\frac{2 v_1 v_2}{v_1 + v_2}}.$$

Q. A person moves 1st half time at 36 km/hr. and in the other half he covers 1st half distance at 20 km/hr. & other half @ 30 km/hr, Find avg. speed.

→ $t \mid t$; $x \mid x$; 36 km/hr, 20 km/h, 30 km/. ; ← V_1 → ← V_2 →

$$V_2 = \frac{2 \times 20 \times 30}{20 + 30} = 24 \text{ km/hr.}$$

$$V_1 = 36 \text{ km/hr.}$$

$$V_{avg} = \frac{36 + 24}{2} = 30 \text{ km/hr.}$$

Q. A person covers 1st half dist. @ 4 m/s. & in other half he covers 1st half time at 3 m/s & other half time at 5 m/s. Find avg. spe.

→ $x \mid t \mid t$; 4 m/s. 3 m/s 5 m/s ; ← V_2 →

$$V_2 = \frac{3+5}{2} = 4 \text{ m/s.}$$

$$V_1 = 4 \text{ m/s.}$$

$$V_{avg.} = \frac{2 \times 4 \times 4}{4+4} = \frac{32}{8} = 4 \text{ m/s.}$$

*

Q. posit^n of a particle is given as, $x = t^2 - 4t$. Find dist. travelled from $t = 0$ to $t = 3$.

→ dir^n change → [vel. = 0].

$x = t^2 - 4t$

$t = 0 \quad t = 3$

$t = 2$

$V = 2t - 4 = 0$

$\boxed{t = 2 \text{ sec}}$

$x(0) = 0.$
$x(2) = -4\,m.$
$x(3) = -3\,m$

4 m, 1 m → 5 mt

Time Average velocity :-

$V = f(t)$

$$V_{avg.} = \frac{\int_{t_1}^{t_2} V\,dt}{t_2 - t_1}$$

Space Average velocity :-

$V = f(x)$

$$V_{avg.} = \frac{\int_{x_1}^{x_2} V\,dx}{x_2 - x_1}$$

calculated when velocity is function of 'x'..

Q. Find avg. velocity from 0 to 3 sec. $0 \leq t \leq 3$

$V = (2t + 3)\ m/s.$

→ $$V_{avg.} = \frac{\int_0^3 (2t+3)\,dt}{3-0} = \frac{\left[t^2 + 3t\right]_0^3}{3-0} = \frac{18}{3}$$

$= 6\ m/s.$

Q. Find avg. velocity. If x is from 0 to $\pi/2$.

$V = \cos x\ m/s.$

$0 \leq x \leq \frac{\pi}{2}.$

BB → 2.
EX-1 → 8-22
EX-II → 1,5,10,21
III → 1,2,3,4

PAGE NO.
DATE : / /

$$\rightarrow V_{avg.} = \frac{\int_0^{\pi/2} (\cos x)\, dx}{\pi/2 - 0} = \frac{\sin x \Big|_0^{\pi/2}}{\pi/2} = \frac{2}{\pi}\ m/s.$$

02-09-2019

Acceleration :- It is the rate of change of velocity.
→ unit → m/s^2 (Vector quantity) dirⁿ → along change in velocity.
→ accⁿ can be +ve, -ve or zero.

Uniform accⁿ → The dirⁿ & magnitude of accⁿ remain constant.
Non-uniform accⁿ → Either dirⁿ or magnitude or both change.

$$\vec{a}_{avg.} = \frac{\text{change in velocity}}{\text{change in time}} = \frac{\Delta\vec{v}}{\Delta t}.$$

$$\boxed{\vec{a} = \frac{d\vec{v}}{dt}} \times \frac{dx}{dx}.$$

$$\boxed{\vec{a} = \vec{v}\,\frac{dv}{dx}}$$

* If velocity of a body is constant, then accⁿ must be zero.
* If speed is constant, then accⁿ may or may not be zero.
↳ a is zero, when body moves in straight line and non-zero when motⁿ is on a curved path.
Eg. Uniform Circular Motion.
* Accⁿ is +ve, when its dirⁿ is along the assumed +ve dirⁿ, & vice-versa.
* If magnitude of velocity (speed) ↓es then the accⁿ is c/d retardation or deceleration.
* If velocity & accⁿ are in the same dirⁿ, then magnitude of velocity ↑es.

$\longrightarrow$ $\qquad$ $\longleftarrow$

$\vec{v} = $ '+' ve $\qquad$ $\vec{v} \Rightarrow$ '−' ve

$\longrightarrow$ $\qquad$ $\longleftarrow$

$\vec{a} \Rightarrow$ '+' ve $\qquad$ $\vec{a} \Rightarrow$ '−' ve .

* If vel. & acc^n are in opp. dir^n then mag. of velocity dec.

+ve acc$^n \Rightarrow$ speed may ↑ or ↓

$\longrightarrow$ $\qquad$ $\longleftarrow$

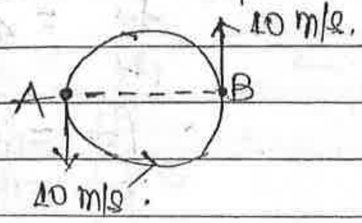
$\vec{v} \Rightarrow$ '+' ve. $\qquad$ $\vec{v} =$ '−' ve.

$\longleftarrow$ $\qquad$ $\longrightarrow$

$\vec{a} \Rightarrow$ '−' ve. $\qquad$ $\vec{a} \Rightarrow$ '+' ve.

[Retardation]

Q. A particle moves on a circular path of radius 10 mtr at const. speed of 10 m/s. Find mag. of avg. acc^n when the particle completes half revolution.



$\rightarrow$ $\qquad \vec{a}_{avg.} = \dfrac{\Delta \vec{V}}{\Delta t}$

$\Delta \vec{V} = \vec{V}_f - \vec{V}_i$

$= 10\hat{j} - (-10\hat{j})$

$= 20\hat{j}$.

$|\Delta \vec{V}| = 20$ m/s.

$\Delta t = \dfrac{\pi R}{v}$

$\boxed{a = \dfrac{20}{\pi} \text{ m/s}^2}$

Q. Find the distance travelled before coming to rest.

$x = 20 + 12t - t^3$.

$\rightarrow$ $\quad v = 12 - 3t^2 = 0$ $\qquad x(0) = 20$ m.

$\boxed{t = 2s}$ $\qquad x(2) = 36$ m.

No dir^n change,

so, $\boxed{x = 16 \text{ m.}}$

Q. Find the position when velocity becomes max^m.

$x = 6t^2 - t^3$.

→ $v = 12t - 3t^2$

$\frac{dv}{dt} = 12 - 6t = 0$.

$\boxed{t = 2}$

double differentⁿ = -6 ↳ maxima.

$x = 6t^2 - t^3$
$= 6 \times 4 - 8$
$= 24 - 8 = \boxed{16\ m.}$

Q. Initial velocity of a particle is 5 m/s and its accⁿ is given as - $a = 4(t-1)\ m/s^2$. Find velocity after 2 sec.

→ $\frac{dv}{dt} = 4t - 4$.

$\Rightarrow \int_5^v dv = \int_0^2 (4t-4)dt$

$\boxed{v = 5\ m/s}$

Retardation

Q. Retardatⁿ of a particle is given as - $a = -\sqrt{v}$. If initial velocity is 25 m/s then find the time after which body comes to rest.

→ $a = -\sqrt{v}$

$\frac{dv}{dt} = -\sqrt{v}$.

$-\frac{dv}{\sqrt{v}} = dt$

$\Rightarrow -v^{1/2}\,dv = dt$

$-\int_{25}^{0} v^{-1/2}\,dv = \int_0^t dt$

$= -\left[2v^{1/2}\right]_{25}^{0} = t$

$\boxed{t = 10\ s}$

BB → 3
Ex-I → ~~23~~ ~~40~~
II → 2, 6, 14, 19, 32, 35, 36
III → 1 – 4, 19, 23

SRG 8.

PAGE NO.
DATE : / /

8. A particle starts from rest from origin. Its velocity is given as $v = \alpha\sqrt{x}$. ($\alpha \to$ const.)
Find – (i) $x - t$.
(ii) $v - t$.
(iii) $a - t$.

→ $v = \alpha\sqrt{x}$.

$\frac{dx}{dt} = \alpha\sqrt{x}$.

$\int_0^x x^{-1/2}\,dx = \alpha\int_0^t dt$

$2x^{1/2} = \alpha t$

$\Rightarrow \boxed{x = \frac{\alpha^2 t^2}{4}}$

$\frac{dx}{dt} = v = \boxed{\frac{\alpha^2 t}{2}}$

$\frac{dv}{dt} = \boxed{a = \frac{\alpha^2}{2}}$

Equations of Motions :-

These are applicable only when $\vec{a}$ is constant.

i) $\vec{v} = \vec{u} + \vec{a}t$. (time interval)

ii) $\vec{s} = \vec{u}t + \frac{1}{2}\vec{a}t^2$. (displacement)

iii) $v^2 - u^2 = 2\vec{a}\cdot\vec{s}$.

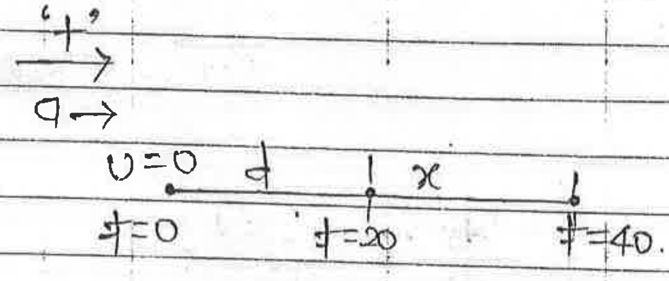
iv) $\vec{s}^{\,n\text{th}} = \vec{u} + \frac{\vec{a}}{2}(2n-1)$. → Vector eqns.

displacement in nth second.

The above eqns are vector eqns therefore, they are always use with proper sign convention.

PAGE NO.
DATE : 4/9/19

Q. A particle starts from rest with const a. If it covers dist. 'd' in 1st 20 sec. then find dist. covered in next 20 sec.

→ 

$$d = \frac{1}{2} a (20)^2 \quad —(1)$$

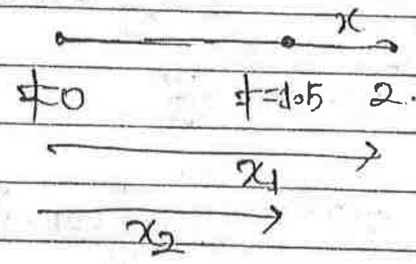
$$d + x = \frac{1}{2} a (40)^2 \quad —(2)$$

dividing (1) ÷ (2)

$$\boxed{x = 3d}$$

Q. A particle starts from rest with uniform acc^n 2 m/s². Find dist. travelled in 4th half second. → 1.5 से 2.

→ $\cancel{x = \frac{1}{2} \times 2 (0.5)^2}$ $\quad a = 2\ m/s^2$. $\quad u = 0$



$$x = x_1 - x_2$$

$$= \frac{1}{2} 2 \times 4 - \frac{1}{2} \times 2 \times \frac{9}{4}$$

$$= \frac{7}{4}\ m.$$

H.W. BB-4 Ex-1 (41) 47 52
II 8, 9 (13)
III 20

P500 C-50 B-150 45 Time

PAGE NO.
DATE : / /

Q. A particle having const. acc^n covers 2 mt. in 1^{st} 2 sec & 14 mt. in next 4 sec. Find its velocity after 7 sec.

→ $\vec{a}$ u 2 m 14 m

t=0 t=2 t=6.

$$2 = 2u + \frac{1}{2}a \times 4 \quad -(1)$$

$$16 = 6u + \frac{1}{2}a \times 36 \quad -(2)$$

$$6 = 6u + 6a \quad -(1)$$

$$16 = 6u + 18a \quad -(2)$$

$$u = \frac{1}{6}\ m/s^2$$

$$a = \frac{5}{6}\ m/s^2$$

$$V = u + at$$

$$= \frac{1}{6} + \left(\frac{5}{6}\right)7 \Rightarrow \boxed{V = 6\ m/s}$$

Q. A car moving at 10 m/s can be stopped by applying brakes in 2 mtrs. If the same car moves @ 40 m/s Find the min^m stopping dist. (Assume const. $retardat^n$).

$$\rightarrow \quad 0 - (10)^2 = -2a(2) \quad -(1)$$

divide.

$$0 - (40)^2 = -2a(x) \quad -(2)$$

$$\boxed{x = 32\ m}$$

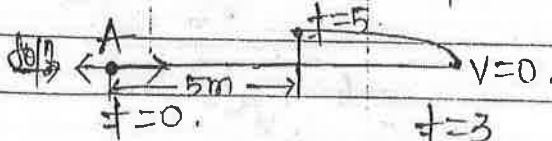
Result

$$0 - u^2 = 2a(x)$$

$$0 - n^2u^2 = 2a(x_1)$$

$$\boxed{x_1 = n^2 x}$$

Q. A body moves with a const. accⁿ $-2\,m/s^2$. It passes a pt. 'A' with velocity 6 m/s. Find the <u>distance</u> travelled from pt. A in next 5 sec.

Sol:


$V = u + at$

$0 = 6 - 2 \times t$

$\boxed{t = 3}$

$t = 0 \longrightarrow t = 3$ sec,

$x_1 = 6 \times 3 - \frac{1}{2} \times 2 \times 9$

$= 9$ m.

$t = 3 \longrightarrow t = 5$ sec

$x_2 = -\frac{1}{2} \times 2 \times 4$

$x_2 = -4$ m.

total distance travelled $= \boxed{13\ m}$

Motion under gravity / Free fall Motion :-

→ If the only force acting on a body is its weight i.e. accⁿ of body is 'g' downward, it is said to be in free fall motⁿ.

$a = g \downarrow$

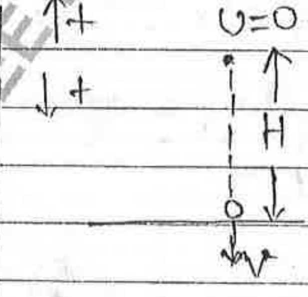
$g = 9.8\ m/s^2$.

$\approx 10\ m/s^2$,

$\approx \pi^2\ m/s^2$.

$= 980\ cm/s^2$.

$= 32\ ft/s^2$.

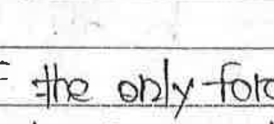


$H = \frac{1}{2} g t^2$

$\boxed{t = \sqrt{\frac{2H}{g}}}$

$V^2 = 2gH$

$\boxed{V = \sqrt{2gH}}$



$0 - u^2 = -2gH$.

$\boxed{H = \frac{u^2}{2g}}$

$v = u + at$

$0 = u - gt$

$\boxed{t_1 = \frac{u}{g}}$

* $t_2 = \sqrt{\frac{2H}{g}} = \sqrt{\frac{2u^2}{2g^2}}$

$$\boxed{t_2 = \frac{u}{g}}$$

* $T = t_1 + t_2$

$$\boxed{T = \frac{2u}{g}}$$

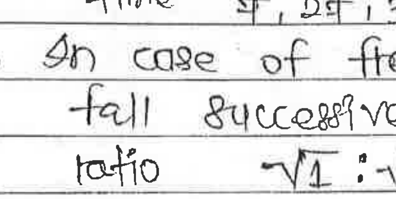
* $v^2 - u^2 = -2g \times 0$

$$\boxed{v = u}$$

NOTE –

1. speed at any point on the path is same whether the body is moving is upwards or downwards.
2. Time of Ascent = Time of descent.
3. In case of free fall from rest, distances covered in time $t, 2t, 3t$ ---- will be in the ratio $1^2 : 2^2 : 3^2$ ..
4. In case of free fall from rest, the time taken to fall successive equal distances will be in the ratio $\sqrt{1} : \sqrt{2} - \sqrt{1} : \sqrt{3} - \sqrt{2}$ ----
5. In case of free fall from rest, distance travelled in successive equal time interval will be in the ratio 1 : 3 : 5 : 7 ... so on.

Q. Water drops are falling from a tap of ht. 8 m in regular time interval. The 3rd drop leaves the tap when the 1st drop reaches the ground. Find hts of 2nd drop from ground.



(t) (2t)

2nd & 1st

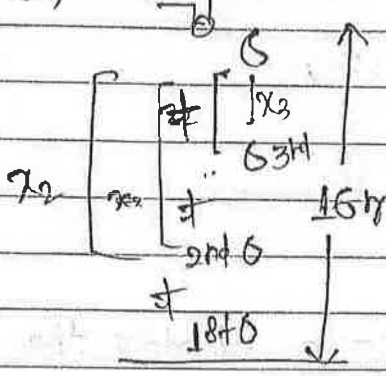
$\frac{x_2}{8} = \frac{1}{4}$ (3rd pt. Note)

$\Rightarrow x_2 = 2m.$

$h_2 = 8 - 2$
$= 6 m.$

VAIBHAV

Q. Ht of tap = 16 mt.
4th drop leaves the tap when 1st drop reaches the ground. Find ht of 2nd & 3rd drop from ground.

→ 

(2t) (3t)
2nd & 1st

$\frac{x_2}{16} = \frac{4}{9}$

$x_2 = \frac{16 \times 4}{9} = \frac{64}{9}$

$h_2 = 16 - \frac{64}{9} = \frac{80}{9}$ m.

(t) (3t)
3rd & 1st

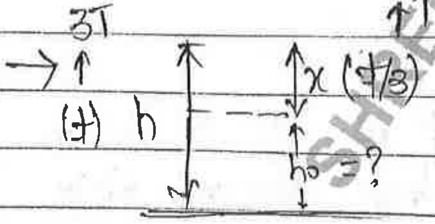
$\frac{x_3}{16} = \frac{1}{9} \Rightarrow x_3 = \frac{16}{9}$ m.

$9x = 16$
$x = \frac{16}{9}$ m

$h_3 = 16 - \frac{16}{9} = \frac{128}{9}$ mt.

9.9.19.

Q. A body drop from ht. 'h' reaches ground in time 't'. Find the ht. after time t/3.

→ 

$\frac{1}{9} = \frac{x}{h}$

$x = \frac{h}{9}$.

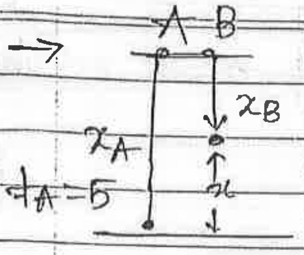
$h = \frac{1}{2}gt^2$

$x = \frac{1}{2}g\frac{t^2}{9}$.

$x = \frac{h}{9}$

$h_0 = h - \frac{h}{9} \Rightarrow \boxed{h_0 = \frac{8h}{9}}$

Q. Two balls A & B are dropped from the same ht. B is dropped 2 sec. after 'A' Find sep. b/w them 3 sec after 'B' is dropped.

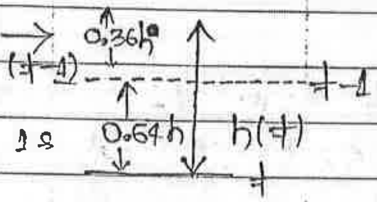


$x = x_A - x_B$

$= \frac{1}{2} \times 10 \times 25 - \frac{1}{2} \times 10 \times 9$

$\boxed{x = 80\ m}$

Q 3. A body dropped from ht. 'h' covers 64 % distance in the last second of motion. Find value of 'h'.



$h = \frac{1}{2} g t^2$ --- (1)

$0.36 h = \frac{1}{2} g (t-1)^2$ --- (2)

IInd $0.64\ h = \frac{g}{2} [2t-1]$ --- (3)

from (1) ÷ (2)

$\frac{1}{0.36} = \frac{t^2}{(t-1)^2}$

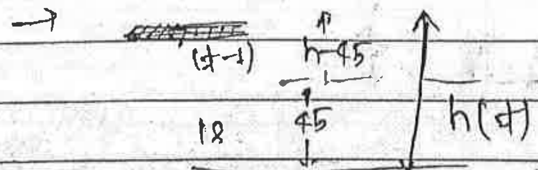
$\frac{1}{0.6} = \frac{t}{t-1}$

$0.6 t = t - 1$

$\boxed{t = 2.5\ s}$ ⇒ $\boxed{h = 31.25\ m}$

$\cancel{h = \frac{1}{2} \times 10 \times 2.5 \times 2.5}$

Q 4. A body dropped from ht. 'h' covers 45 mt in the last 1 sec. Find h. $h = \frac{1}{2} g t^2$



$h - 45 = \frac{1}{2} g (t-1)^2$ (I) ⇒ $\cancel{\frac{h}{45} = \frac{(t-1)^2}{(2t-1)}}$

$45 = \frac{g}{2} [2t - 1]$ (II)

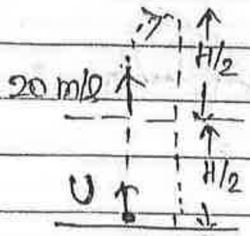
IV

$t = 5$ $h = \frac{1}{2} g t^2 = \boxed{125\ m}$

$\frac{v^2}{40} = \frac{40(2m)}{...}$ 40

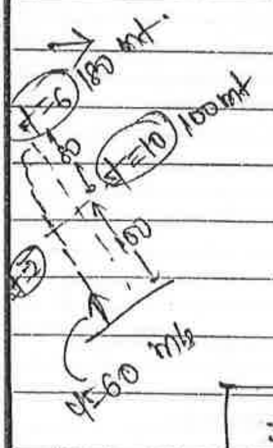
PAGE NO.
DATE : / /

Q. A body projected vertically upward has speed 20 m/s at half the max^m ht. Find max^m ht.



$$\frac{H}{2} = \frac{(20)^2}{2\times10} \Rightarrow \boxed{H = 40\ m}$$

Q. A body is thrown upward such that its max^m ht is 180 mt. Find the time @ which its ht. is 100 mt.



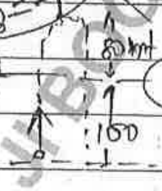
$v^2 = 2gH.$

$v^2 = 2\times10\times180$

$v^2 = 3600.$

$v = 60\ m/s.$

max h $= \frac{v}{g} = \frac{60}{10} = 6$ sec.



$6s - t = \frac{v}{g} = \sqrt{\frac{2H}{g}} = \sqrt{\frac{2\times180}{10}}$

$t = 2s$ 80 mt. 100 mt. 100

$= \sqrt{\ }$ $6+4$

$t = 10$ sec

$80 = \frac{1}{2}\times g\times(\Delta t)^2$

$\Delta t = 4$ s.

$$\boxed{t = 2 \text{ sec or } 10 \text{ sec.}}$$

Q. A body thrown upwards crosses a point at time t_1 & t_2. Find —

(i) Time of flight

(ii) Initial vel.

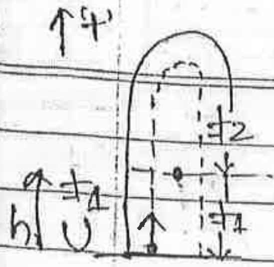
(iii) Ht of the point.

(i) $T = t_1 + t_2.$

$\rightarrow$ $u = \sqrt{2gH}.$

Acceleration

PAGE NO.
DATE : / /



(i) $T = t_1 + t_2$.

(ii) $\frac{2u}{g} = t_1 + t_2$

$u = \frac{g}{2}(t_1 + t_2)$.

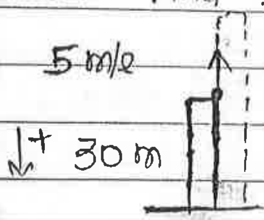
(iii) $h = ut_1 - \frac{1}{2} g t_1^2$

$= \frac{g}{2}(t_1 + t_2) t_1 - \frac{1}{2} g t_1^2$.

$= \cancel{\frac{g}{2} t_1^2} + \frac{g t_1 t_2}{2} - \cancel{\frac{1}{2} g t_1^2}$

$\boxed{h = \frac{1}{2} g t_1 t_2}$

Q. Find the time after which body reaches the ground.



$\cancel{H = \frac{u^2}{2g} = \frac{25}{20} = \frac{5}{4}}$

$\cancel{\text{Total } H = 30 + 1.25 = 31.25}$

(I)

$30 = -5t + 5t^2$.

divided by 5

$t^2 - t - 6 = 0$.

$t^2 - 3t + 2t - 6 = 0$.

$t(t-3) + 2(t-3) = 0$

$(t-3)(t+2) = 0$

$\boxed{t = 3}$, $\cancel{t = -2}$

(II)

$t_1 = 0.5 \left(\frac{u}{g}\right)$

$h = \frac{(5)^2}{2 \times 10} \left(\frac{u^2}{2g}\right)$

$= 1.25$ m.

$t_2 = \sqrt{\frac{2 \times 31.25}{10}}$

$= \sqrt{6.25}$

$t_2 = 2.5$.

$t = 3$ s

BB → 6 EX-1

53-83.

II → 15, 18, 26, 27.

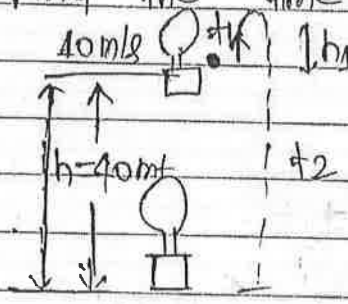
III – 6, 7, 9, 21, 22, 24.

Net accelerat... ✓✓✓✓✓✓✓

PAGE NO.
DATE : / /

Q. A baloon starts rising from ground with const. accⁿ 1.25 m/s². After 8 sec. A stone is dropped from it Find the time after which it hits the ground.

→



$a = 1.25$ m/s²

$t_0 = 8$ sec.

$h = \frac{1}{2} g \times a \times t_0^2$.

$h = 40$ mt.

$V = \cancel{u}^{0} + at_0$

$V = 10$ m/s.

अब 40 mt. ht. से obj. को 10 m/s से फेंका .

NOW, $t_1 = 1$ sec $\frac{u}{g}$

$h_1 = \frac{10^2}{2 \times 10} = 5$ m

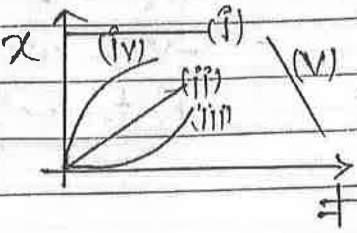
$t_2 = \sqrt{\frac{2 \times 45}{10}}$ ⇒ $t_1 + t_2 = 1 + 3 = 4$ sec.

$t_2 = 3$ sec.

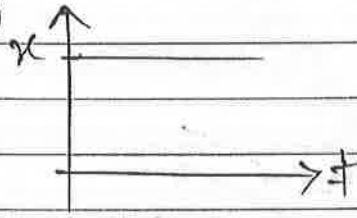
——X——

Graphical Analysis :-

1. x – t graph –

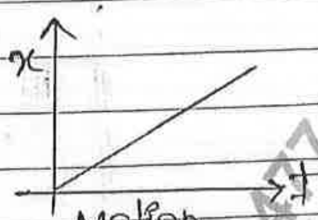


(i)



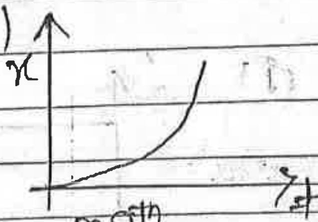
Rest.
v = 0.
a = 0

(ii)



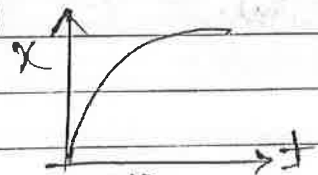
Motion
→ v = +ve & const.
uniform motn.
a = 0

(iii)



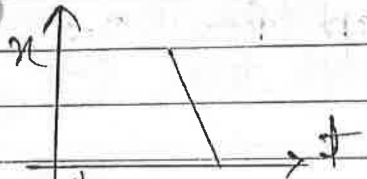
motn
→ v = +ve, ↑ing.
a = +ve, Non-uniform motn.

(iv)



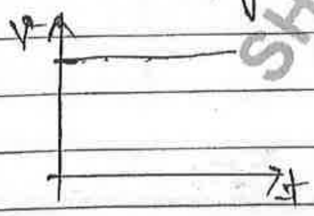
→ motn
→ v = +ve, ↓ing
→ a = –ve (Retardatn).

(v)

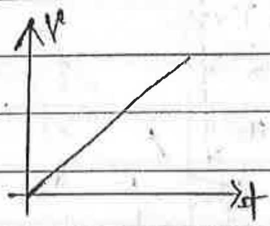


→ motn
→ v = –ve & const.
→ uniform motn. → a = 0.

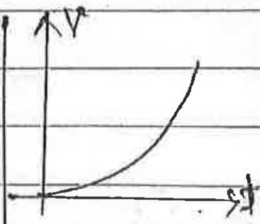
2. v – t graph –



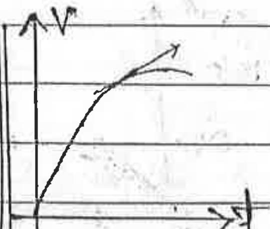
v = +ve, const.
uniform motn
a = 0



v = +ve, ↑ing.
a = +ve, const.



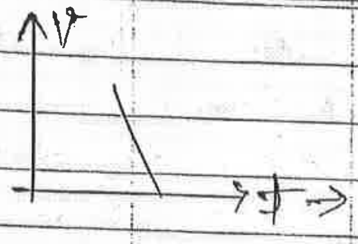
v = +ve, ↑ing
a = +ve, ↑ing



v = +ve, ↑.
a = +ve ↓
NO retardn

PAGE NO.
DATE : / /

(V)



v = +ve, ↓

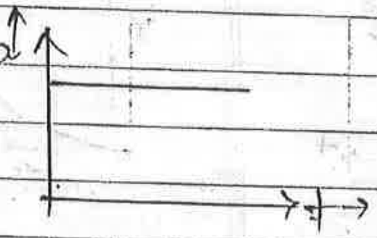
a = '-ve', const

Retardation.

Motion under gravity

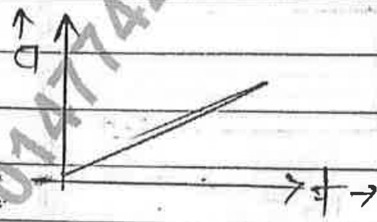
3. a-t graph -

(i)



a = +ve, const.

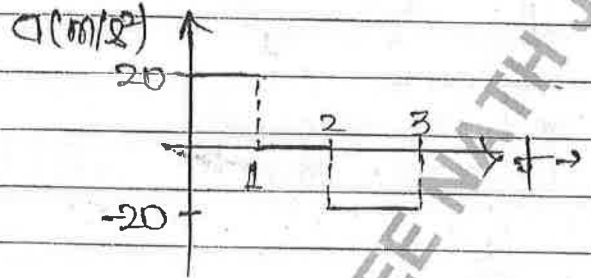
(ii)



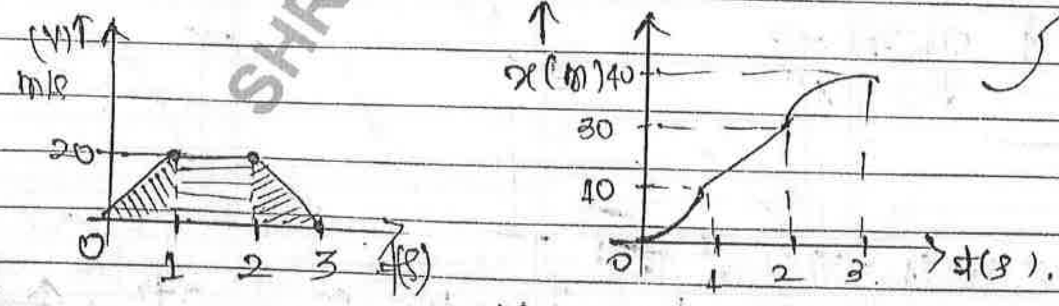
a = +ve, ↑ing

13.9.19

Q. A body starts from rest from origin. Draw its v-t & x-t graph from a-t graph given below -



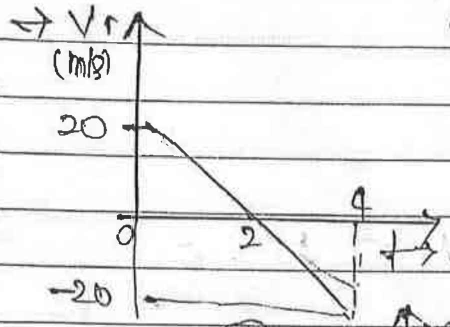
→



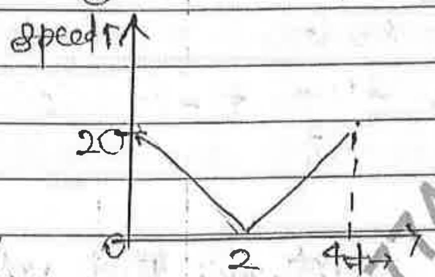
Q. A ball is projected upward at 20 m/s. Taking upward dirn as +ve. Draw the following graph.

(i) v-t (ii) speed-time (iii) displacement-time (iv) distance time (v) a-t graph.

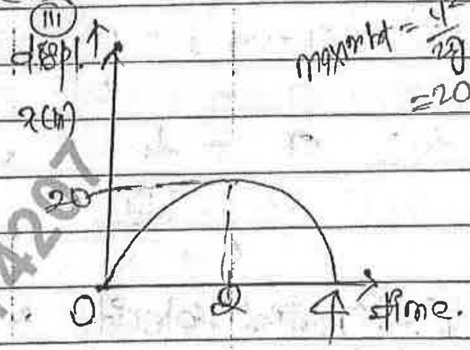
(i)



(ii)

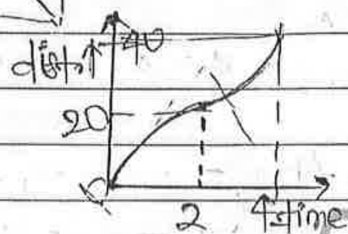


(iii)

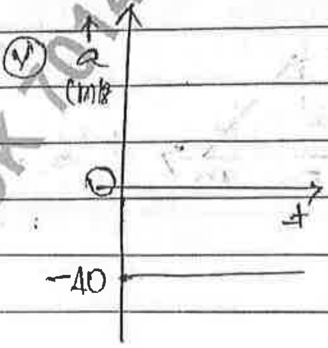


$$max^{m} ht = \frac{u^2}{2g} = 20.$$

(iv)

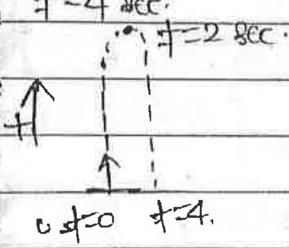


(v)



u = 20 m/s.

t = 4 sec.



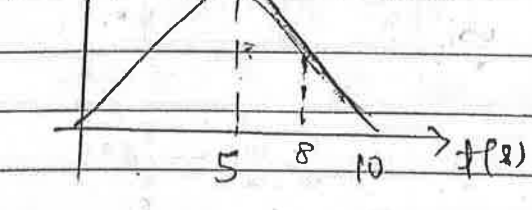
Q. From the v-t graph given below calculate –

(i) displacement in 10 sec.

(ii) avg. velocity in 10 sec.

(iii) avg. accn in 1st 5 sec.

(iv) accn @ t = 8 sec.



(i) ~~...~~

(ii)

(iii) ~~...~~

(iv)

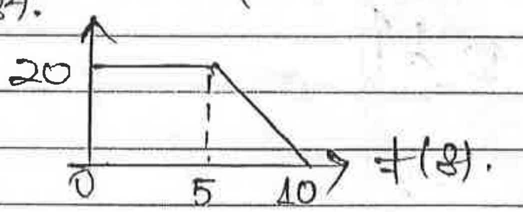
(i) Area $(\Delta x) = \frac{1}{2} \times 10 \times 100 = 500$ m.

(ii) $V_{avg.} = \frac{\Delta x}{\Delta t} = \frac{500}{10} = 50$ m/s.

(iii) $a_{avg} = \frac{\Delta V}{\Delta t} = \frac{\cancel{50}\ 100}{\cancel{4}\ 5} = 20$ m/s^2

(iv) $a = \frac{dv}{dt} = \frac{-100}{5} = -20$ m/s^2.

Q. Initial velocity is 10 m/s. then find vel. at 10 sec.



~~$\Delta x = 20 \times 5 + \frac{1}{2} \times 5 \times 20$~~

Area $= \Delta V = V_f - V_i$

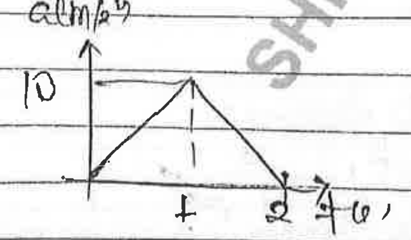
$V_f - 10 = \frac{1}{2}[5 + 10]\,20$

$V_f - 10 = 150$

$\boxed{V_f = 160 \text{ m/s}}$

Q. If a particle start from rest. then find its max^m velocity from given a-t graph.

→

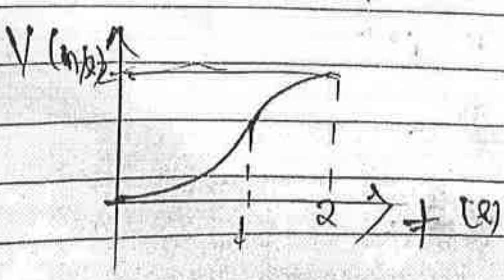


Area $= \Delta V = V_f - V_i$

$V_f - 0 = \frac{1}{2} \times 2 \times 10$

$\boxed{V_f = 10 \text{ m/s}}$

max^m vol. @ $t = 2$ sec



→ when solve by graph → [Q-4 II] graph Batao.

Vaibhav

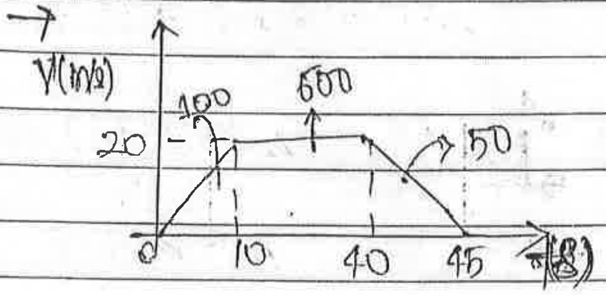
BB-5. Exer — 84 – 111.

Ex – 3 – 8

PAGE NO.

DATE : / /

Q. A body starting from rest has const. accⁿ 2 m/s². for 10 sec. It then moves with const. velocity for next 30 sec. & then retards uniformally at 4 m/s². till it stops. Find total distance travelled.



$$\text{Area} = \frac{1}{2} \times 10 \times 20 + 30 \times 20 + \frac{1}{2} \times 5 \times 20$$

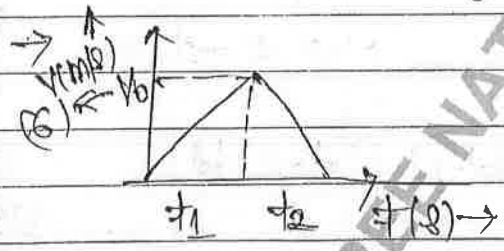
$$= 100 + 600 + 50$$

$$= 750 \text{ m}$$

Q. A body starting from rest has accⁿ 2 m/s² for some time and then retardatⁿ of 3 m/s² till it stops again. If total motⁿ is for 5 sec. then calculate —

i) total distance travelled.

ii) maxᵐ velocity.



$$2 = \frac{v_0}{t_1} \qquad 3 = \frac{v_0}{t_2}$$

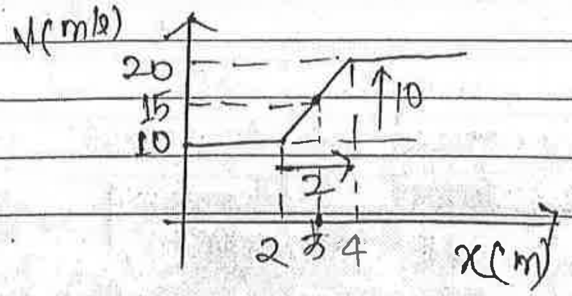
$$t_1 + t_2 = 5 \text{ sec.}$$

$$\frac{v_0}{2} + \frac{v_0}{3} = 5.$$

$$\boxed{v_0 = 6 \text{ m/s}}$$

$$\Delta x = \frac{1}{2} \times 5 \times 6 = 15 \text{ m.}$$

Q. Find accⁿ at $x = 3$ m.



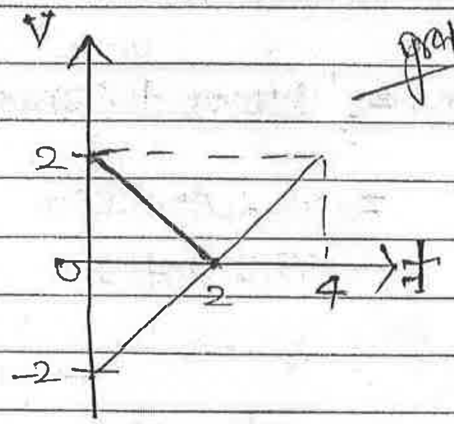
$$a = v\frac{dv}{dx} = 15\left(\frac{10}{2}\right)$$

$$= 75 \text{ m/s}^2.$$

PAGE NO.
DATE : 16/9/19

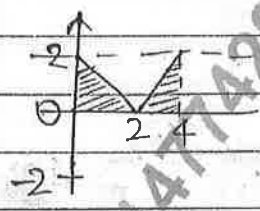
Q. velocity of a particle is given as :—
$v = |t-2|$ m/s.
Find its displacement in the 1st 4 sec.

→ graphically,



↘ → After mod $|t-2| = v$
↗ → Without mod $t-2 = v$

⟹



$\Delta x = 2\left[\frac{1}{2} \times 2 \times 2\right]$

$\Delta x = 4$ m.

by eqⁿ →

$|2| = 2.$

$v = |t-2|$ m/s.

$\boxed{\text{case I}}$: if $t-2 > 0$
$t > 2$
$v = t-2.$

$\boxed{\text{case II}}$: if $t-2 < 0.$
$t < 2$
$v = 2-t.$

$$\Delta x = \int_0^2 (2-t)\,dt + \int_2^4 (t-2)\,dt$$

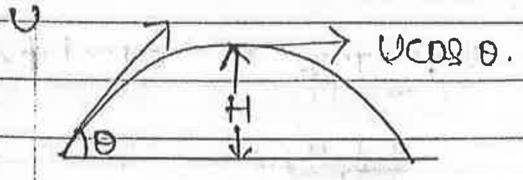
$\Delta x = 2+2$

$\Delta x = 4$ m.

—x— — —x—

PROJECTILE MOTION :-

θ को 2 part में Break.
x-axis → 1-D
&
y-axis → 1-D.



$U_x = U\cos\theta.$ $U_y = U\sin\theta.$

$a_x = 0.$ $a_y = -g$

I. Time of flight [T] (y-axis).

$$0 = U\sin\theta\, T - \frac{1}{2} g T^2.$$

$$\boxed{T = \frac{2U\sin\theta}{g}} \quad \text{or} \quad T = \frac{2U_y}{g}$$

II. Range [R] (x-axis)

$$R = U\cos\theta \times \frac{2U\sin\theta}{g}$$

$$\boxed{R = \frac{U^2\sin 2\theta}{g}} \quad \text{or} \quad R = \frac{2U_x U_y}{g}.$$

III. Max^m Height [H] (y-axis).

$$0 - U^2\sin^2\theta = -2gH.$$

$$\boxed{H = \frac{U^2\sin^2\theta}{2g}} \quad \text{or} \quad H = \frac{U_y^2}{2g}.$$

NOTE :-

1. In Projectile motⁿ, the horizontal component of velocity ($U\cos\theta$) always remains constant.
2. The velocity becomes minᵐ @ the highest point of projectile. Therefore, momentum & KE are also minᵐ at the highest point.

PAGE NO.
DATE : / /

3. For a given value of u, Range is max^m @ $\theta = 45°$.

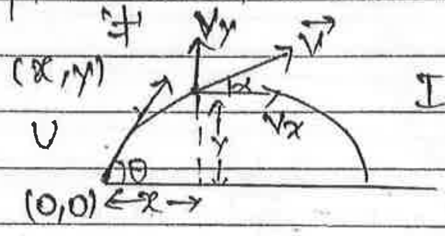
4. For a given value of u, Range is equal for complimentary angles.

θ, $(90-\theta)$

T & H not

However, time of flight & max^m Height will be different.

Displacement & velocity at time 't'.



I. $x = u\cos\theta\, t$

$y = u\sin\theta\, t - \frac{1}{2}gt^2$.

$$\boxed{\vec{r} = x\hat{i} + y\hat{j}}$$

II. $V_x = u\cos\theta$

$V_y = u\sin\theta - gt$.

$$\boxed{\vec{V} = V_x\hat{i} - V_y\hat{j}}$$

III. $\tan\alpha = \frac{V_y}{V_x}$

$\alpha = \tan^{-1}\left(\frac{V_y}{V_x}\right)$

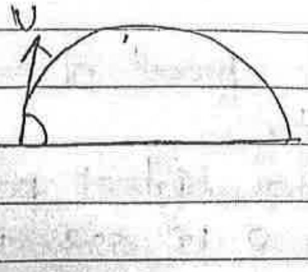
$\alpha = \tan^{-1}\left(\frac{u\sin\theta - gt}{u\cos\theta}\right)$

Q. A particle is projected at 50 m/s at an $\angle 53°$ with vertical. After two sec. find -

(I) displacement vector

(II) velocity vector.

(III) $\angle$ b/w horizontal & velocity vector.



$u \sin\theta = 30$ m/s

$u = 50$ m/s, $\theta = 37°$, $u\cos\theta = 40$ m/s.

$x = u\cos\theta\, t$ & $y = u\sin\theta\, t - \frac{1}{2}gt^2$.

$x = 40 \times 2$ $\quad = 30\times 2 + \frac{1}{2}\times 10\times 1$

$= 80$ m. $\quad = 40$ m.

$$\boxed{\vec{r} = (80\hat{i} + 40\hat{j})\,m.}$$

icon

PAGE NO.

DATE : / /

(ii) $V_x = 40$ m/s.

$V_y = 30 - 10 \times 2$

$V_y = 10$ m/s.

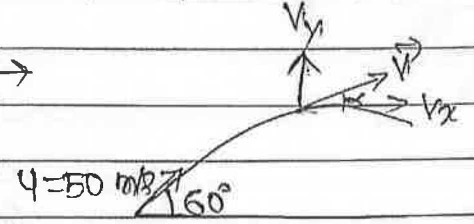
$$\boxed{\vec{V} = (40\hat{i} + 10\hat{j})\ m/s}$$

(iii) $\alpha = \tan^{-1}\left(\frac{1}{4}\right)$.

18.9.19.

Q. A particle is projected at 50 m/s at ∠ 60° with Hz. find the time after which velocity makes an ∠ 30° with Hz.

→



$\tan\alpha = \frac{V_y}{V_x}$.

$u\cos\theta = 25$ m/s

$u\sin\theta = 25\sqrt{3}$ m/s

$\alpha = \tan^{-1}\left[\frac{u\sin\theta - gt}{u\cos\theta}\right]$

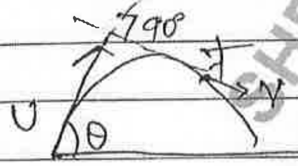
$\tan 30° = \cancel{\tan^{-1}}\left(\frac{25\sqrt{3} - 10t}{25}\right)$

$\frac{1}{\sqrt{3}} = \frac{25\sqrt{3} - 10t}{25} \Rightarrow 25 = 75 - 10\sqrt{3}t$

$\Rightarrow \boxed{t = 5/\sqrt{3}}$.

Q. A particle is projected at u & ∠ θ with Hz. find the time after which its vel. becomes ⊥ to the initial velocity.

→



• product of initial vel. and time 't' = 0.

$u = u\cos\theta\,\hat{i} + u\sin\theta\,\hat{j}$.

~~$v = v\sin\theta\,\hat{i}$~~

$v = u\cos\theta\,\hat{i} + (u\sin\theta - gt)\hat{j}$.

$u \cdot v = 0$.

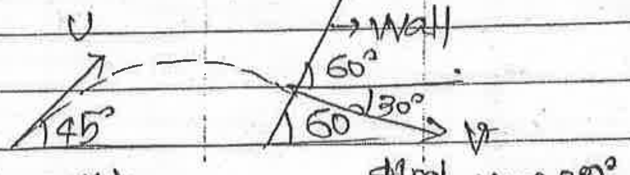
$[u\cos\theta\,\hat{i} + u\sin\theta\,\hat{j}] \cdot [u\cos\theta\,\hat{i} + (u\sin\theta - gt)\hat{j}] = 0$

$\Rightarrow u^2\cos^2\theta + u^2\sin^2\theta - u\sin\theta\, gt = 0$.

$\Rightarrow u^2 - u\sin\theta\, gt = 0$.

$$\boxed{t = \frac{u}{g\sin\theta}}$$

Q. If the particle strikes the wall perpendicularly then find the speed with which it strikes the ball.

→ 

initial $u\cos 45°$ final $v\cos 30°$

$$\frac{u}{\sqrt{2}} = \frac{v\sqrt{3}}{2}$$

$$\boxed{v = \sqrt{\frac{2}{3}}\,u}$$

Eqn of Trajectory :-

$$x = u\cos\theta\, t \longrightarrow t = \frac{x}{u\cos\theta}$$

$$y = u\sin\theta\, t - \frac{1}{2}gt^2$$

$$y = u\sin\theta \cdot \frac{x}{u\cos\theta} - \frac{1}{2}g \times \frac{x^2}{u^2\cos^2\theta}$$

$$\boxed{y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}}$$

$$y = x\tan\theta\left[1 - \frac{gx}{2u^2\cos^2\theta\tan\theta}\right] \rightarrow \frac{\sin\theta}{\cos\theta}$$

$$y = x\tan\theta\left[1 - \frac{x}{\frac{2u^2\sin\theta\cos\theta}{g}}\right]$$

$$\boxed{y = x\tan\theta\left[1 - \frac{x}{R}\right]}$$

Q. From given eq^n of trajectory the ∠ of project^n & Range of projectile.

$y = ax - bx^2$.

convert & compare

→ $y = ax\left[1 - \frac{bx}{a}\right]$.

$y = ax\left[1 - \frac{x}{a/b}\right]$ & $R = a/b$.

$\tan\theta = a$

$\theta = \tan^{-1}(a)$.

Q. From the given eq^n of trajectory. Find —

1) θ

II) Range

III) u.

IV) max^m ht.

V) time of flight

$y = \sqrt{3}\,x - 5x^2$.

$y = \sqrt{3}x\left[1 - \frac{x}{\sqrt{3}/5}\right]$.

$y = x\tan\theta\left[1 - \frac{x}{R}\right]$

on compareing.

I) $\theta = 60°$.

II) $R = \sqrt{3}/5$.

$\cancel{u^2 = \frac{Rg}{\sin 2(60)}} = \cancel{\frac{\sqrt{3}/5 \times 10^2}{\sin 120 = \omega 3}} = \cancel{\frac{2\sqrt{3}}{\sqrt{3}}}$ $\cancel{4}$

III) $R = \frac{u^2 \sin 2\theta}{g}$

→ $\frac{\sqrt{3}}{5} = \frac{u^2}{10_2} \times \frac{\sqrt{3}}{2}$ → $\boxed{u = 2\ m/s}$

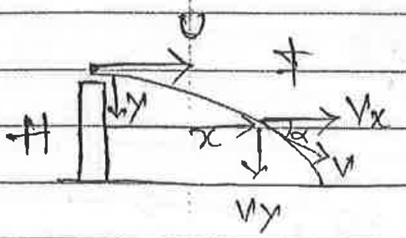
PAGE NO.

DATE : / /

iv) $H = \frac{u^2 \sin^2\theta}{2g} = \frac{3}{20}$ m

v) $T = \frac{2u\sin\theta}{g} = \frac{\sqrt{3}}{5}$ sec.

Horizontal Projectile :-



i) Time of flight -

y-axis

$-H = -\frac{1}{2} g T^2$

$$\boxed{T = \sqrt{\frac{2H}{g}}}$$

$u_x = u$. $u_y = 0$.

$a_x = 0$ $a_y = -g$.

ii) Range

x-axis

$$\boxed{R = uT}$$

iii) $x = ut$

$y = -\frac{1}{2} g t^2$

$$\boxed{\vec{r} = ut\hat{i} - \frac{1}{2} g t^2 \hat{j}}$$

iv) $v_x = u$

$v_y = -gt$.

$$\boxed{\vec{v} = u\hat{i} - gt\hat{j}}$$

v) $\tan\alpha = \frac{v_y}{v_x}$.

$$\boxed{\alpha = \tan^{-1}\left(\frac{v_y}{v_x}\right)}$$

Q. A particle is projected horizontally from a ht. with speed 30 m/s. after 2 sec. find its displ. & velocity.

H.W. BB → 7.
EX → 111 to 157.

PAGE NO.
DATE : / /

$\vec{r} = ut\,\hat{i} - \frac{1}{2}gt^2\,\hat{j}$

→ ① $\vec{r} = 30 \times 2\,\hat{i} - \frac{1}{2} \times 10 \times 4\,\hat{j}$

$\boxed{\vec{r} = 60\,\hat{i} - 20\,\hat{j}}$

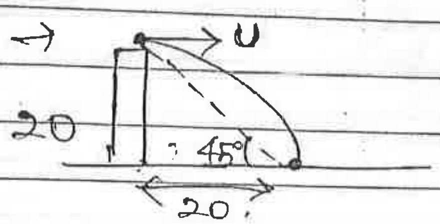
displacement.

② v.

$\vec{v} = u\,\hat{i} - gt\,\hat{j}$

$\vec{v} = \boxed{30\,\hat{i} - 20\,\hat{j}}$

Q. A body is thrown horizontally from ht. 20 mt. of the line joining pt. of projectn & striking pt. on ground makes 45° with Hz. then find initial speed.



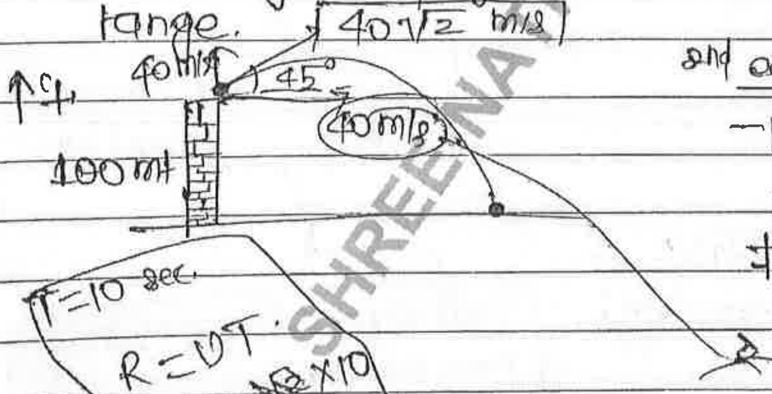
$T = \sqrt{\frac{2H}{g}} = \sqrt{\frac{2 \times 20}{10}} \quad \sqrt{\frac{40}{10}}\,4$

$= 2.$

$R = UT$

$U = \frac{R}{T} = \frac{20}{2} = 10$ m/s.

Q. For the given projectile motn, calculate time of flight & range.



$T = 10$ sec.

$R = UT$

$= 40\sqrt{2} \times 10$

$= 400$ mt

2nd eqn of mot. (y-axis)

$-100 = 40t - 5t^2$

$t^2 - 8t - 20 = 0.$

$t^2 - 10t + 2t - 20 = 0.$

$\therefore \boxed{t = 10 \text{ sec}}$

$R = UT$

$= 40 \times 10$

$= 400$ mt

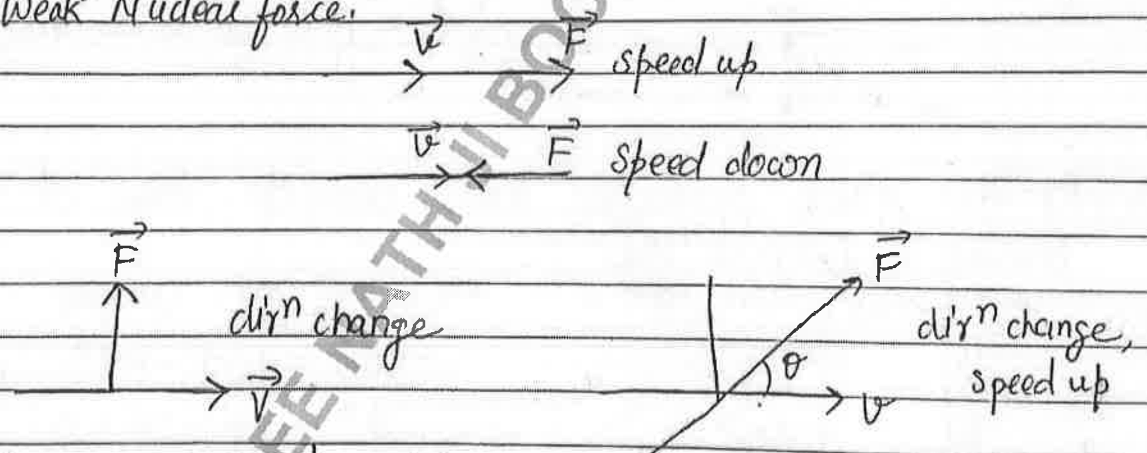
NLM "Newton's Laws of Motion"

* Force (If $\vec{v}$ = const, speed const, there is no change in direction)
  ↳ It is a pull or push which changes the state of body. rest to motion, speed up, speed down, dir^n change.

→ It also changes the configuration of body.

① Gravitational Force
② Electromagnetic → ① Coulomb force
② contact → Friction, Normal
③ Tension
④ Spring Force
⑤ Viscous Force

③ Strong Nuclear force.
④ Weak Nuclear force.



unit	MKS	CGS
	$kg\text{-}\frac{m}{s^2}$	$gm\text{-}\frac{cm}{s^2}$
SI →	newton N	dyne

$1N = 10^5$ dyne

$\therefore$ 1 dyne $= 10^{-5} N$

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NLM

Gravitational or Practical unit

$1\ kg\text{-}wt = 9.8\ N \simeq 10\ N$

$1\ gm\text{-}wt = 980\ dyne \simeq 1000\ dyne$

$1\ kg\ Force = 9.8\ N \simeq 10\ N$

$1\ gm\ Force = 980\ dyne \simeq 1000\ dyne$

* Inertia (Resistance to change)

→ It is the property of body due to which it opposes change in it's state.

→ In Ənce of net External force a body continue state of rest or uniform motion.

→ Mass is measurement of inertia

$$\boxed{\text{Inertia} \propto \text{mass}}$$

Types – Inertia of rest
Inertia of Motion
Inertia of dir^n.

(1) Newton's 1st law of Motion

Every body continues to be in its state of rest or uniform motion in a straight line unless compelled by some external unbalanced force.

→ This law gives quality of force that force changes state, body it self can't change it's state.

$$\left.\begin{array}{l}\vec{F}_{ext} = 0 \\ \vec{a} = 0\end{array}\right\} \quad \left.\begin{array}{l}\vec{v} = 0 \\ \vec{v} = \text{constant}\end{array}\right\} \text{equilibrium}$$

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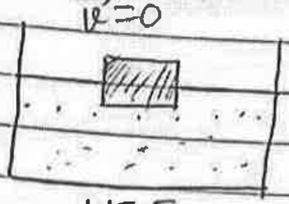
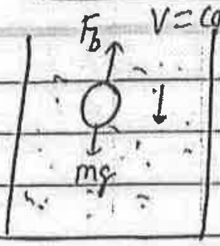
$\boxed{\text{Static Eqb}^m} \rightarrow \vec{F}_{net} = 0 \leftarrow \boxed{\text{dynamic Eqb}^m}$

$\vec{v} = 0$ $\qquad$ $V = \text{constant.}$





$V_T = \text{const}$

$W = F_b$ $\qquad$ $F_b = W_{eff}$

★ Momentum $[\vec{P}]$

Quantity of Motion,

$\vec{P} = m\vec{v}$ $\quad [MLT^{-1}]$

$= kg\,ms^{-1}$

$\Rightarrow$ vector

★ Newton 2nd law of Motion

The Rate of change of momentum of a body is directly porportional to the applied external Force and this change in momentum takes place in the dir^n of applied Force.

$$\vec{F} \propto \frac{\vec{\Delta P}}{\Delta t} \qquad \vec{F} = K\frac{\vec{\Delta P}}{\Delta t}$$

Time interval is very small $\Delta t \rightarrow 0$

$$\vec{F} = K\frac{\vec{\Delta P}}{dt} \quad \because [k=1] \qquad \boxed{F = \frac{dp}{dt}}$$

Force

Avg. Force

$F_{av} = \frac{\vec{\Delta P}}{\Delta t}$ ✓

Instantaneous Force

$\vec{F} = \frac{dp}{dt} = \frac{d\,m\vec{v}}{dt}$ ✓

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$$\left[a = \frac{v-u}{t}\right] \qquad s = \left[\frac{v+u}{2}\right]t$$

$$\boxed{\vec{F} = m\frac{d\vec{v}}{dt} + v\frac{dm}{dt}}$$

m = constant

$$\boxed{\vec{F} = m\frac{d\vec{v}}{dt} = m\vec{a}}$$

v = const

$$\boxed{\vec{F} = \vec{v}\frac{dm}{dt}}$$

→ water jet
→ conveyer belt
→ Rocket propulsion

Q A particle is moving along straight line under const force it's velocity changes from 2 m/s to 3.5 m/s in 3 sec. If mass of the particle is 2 kg. Find Force on it.

$3.5 = 2 + 3a \qquad \left[a = \frac{v-u}{t}\right]$

$1.5 = 3a$

$a = \frac{1}{2}$

$F = ma = 2 \times \frac{1}{2} = 1\,N$

Q A particle of mass 10 kg is moving along straight line with velocity 10 m/s now a constant force acts on a body for 4 sec by giving speed of 2 m/s in opposite dir^n. Find accl^n?

Sol^n $u = 10\,m/s$ $\qquad \left[a = \frac{v-u}{t}\right]$

$t = 4\,sec$

$v = -2\,m/s \qquad \therefore a = \frac{-2-10}{4} = -3\,m/s^2$.

Q Two mutually ⊥ Force of magnitude 5N and 12N acting on a body in space of mass 26 kg. Find magnitude of accl^n.

$F_{net} = \sqrt{5^2 + 12^2} = 13\,N \qquad a = \frac{F}{m} = \frac{13}{26} = \frac{1}{2} = 0.5\,m/s^2$

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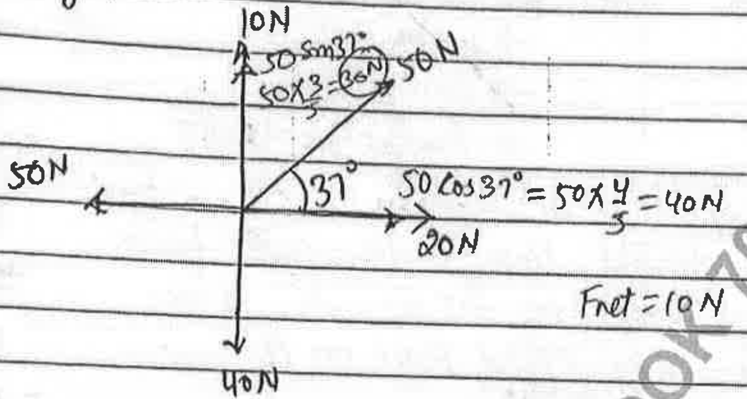
Q. Three forces $(2\hat{i}+\hat{j}+5\hat{k})N$, $(\hat{i}+\hat{j}+5\hat{k})N$ & $(\hat{i}+\hat{j}+2\hat{k})N$ acts on a particle of mass 2kg. Find magnitude of accl^n.

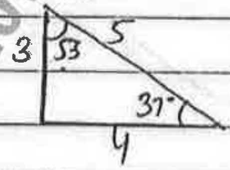
$\vec{F}_{net} = \vec{F_1}+\vec{F_2}+\vec{F_3} = 4\hat{i}+3\hat{j}+12\hat{k}$

$|\vec{F}_{net}| = \sqrt{4^2+3^2+12^2} = 13$

$a = \frac{F}{m} = \frac{13}{2} = 6.5\ m/s^2$

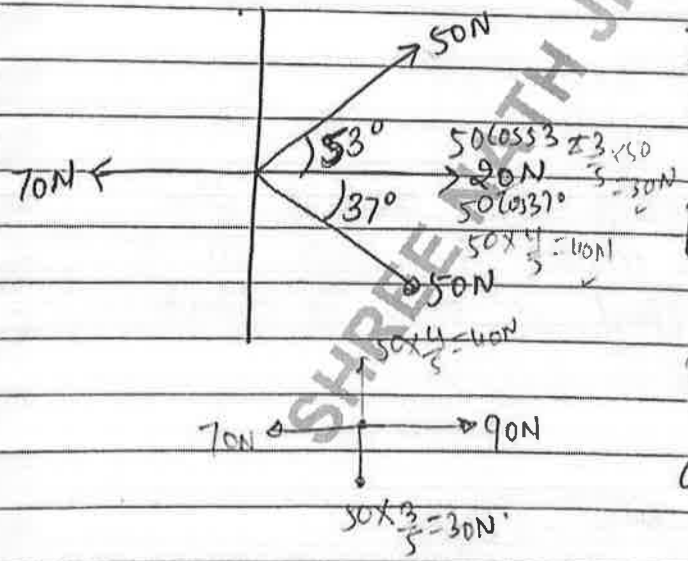
Q. In the given situation find magnitude of accl^n of particle it's mass 5 kg?





$F_{net} = 10N$ $\quad a = \frac{10}{5} = 2\ m/s^2$

Q. Find mag accl^n of particle mass 5kg.



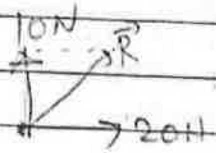
$\vec{F_y} = 10\hat{j}$

$\vec{F_x} = 20\hat{i}\ N$

$|\vec{F}| = \sqrt{F_x^2+F_y^2}$

$= \sqrt{400+100}$

$|\vec{F}| = 10\sqrt{5}$



$a = \frac{F}{m} = \frac{10\sqrt{5}}{5} = 2\sqrt{5}\ m/s^2$

Q. linear momentum of a particle is $P = at - bt^2$, where a & b are const, and $t \rightarrow$ time. Find the time at which Fnet on particle is zero.

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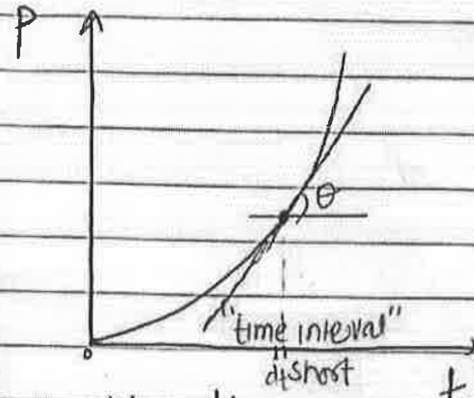
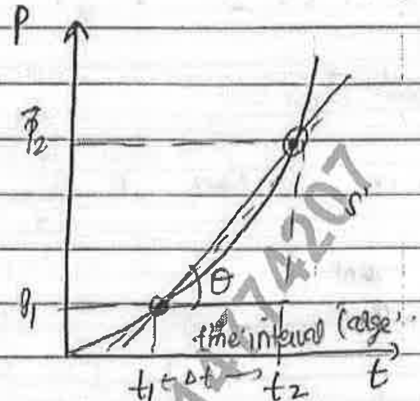
$P = at - bt^2$

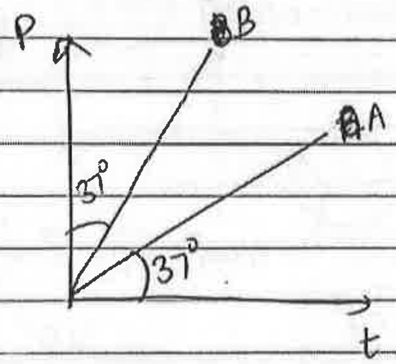
$F = \frac{dp}{dt} = a - 2bt = 0$ $\quad \boxed{t = \frac{a}{2b}}$



$\text{slope} = \frac{dp}{dt} = \tan\theta = F_{in}$



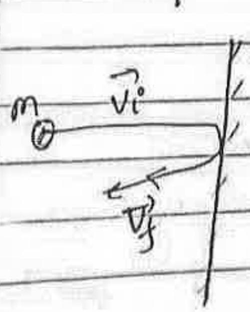
$\text{slope} = \tan\theta = \frac{\Delta\vec{P}}{\Delta t} = \vec{F}_{av}$



$\frac{F_A}{F_B} = ? \quad \frac{\tan 37^\circ}{\tan 53^\circ} = \frac{3/4}{4/3} = \frac{9}{16}$

Impulse $[\vec{I}]$ ⇒ A large force in short interval of time, which changes the momentum.

Impulse = Force × time interval



$d\vec{I} = \vec{F}dt$

$\vec{I} = \int_{t_1}^{t_2} \vec{F}dt = \int_{\vec{P}_1}^{\vec{P}_2} d\vec{p}$

$\boxed{I = \vec{P}_2 - \vec{P}_1 = \Delta\vec{P}}$ Impulse, momentum theorem.

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$$\vec{F} = \frac{d\vec{p}}{dt}$$

$$\int_{t_1}^{t_2} \vec{F}dt = \int_{\vec{p}_1}^{\vec{p}_2} d\vec{p}$$

** $\boxed{\vec{I} = m[\vec{V}_f - \vec{V}_i]}$

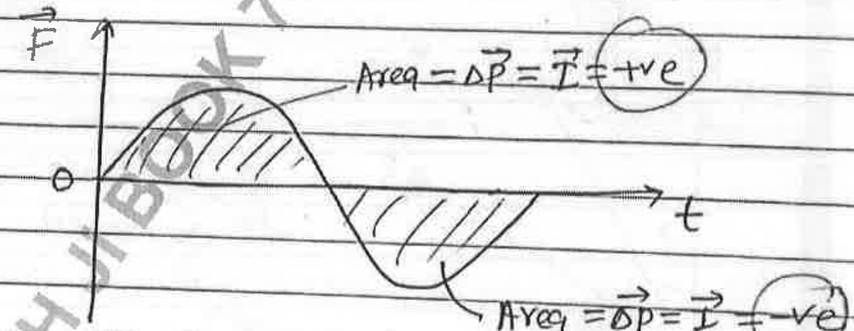
$\vec{F} \rightarrow$ variable $\boxed{\vec{I} = \int_{t_1}^{t_2} \vec{F}dt}$

$\vec{F} \Rightarrow$ const = average Force

$$\vec{I} = \vec{F}_{av}\int_{t_1}^{t_2} dt = \vec{F}_{av}(t_2 - t_1)$$

** $\boxed{\vec{I} = \vec{F}_{av}(\Delta t)}$

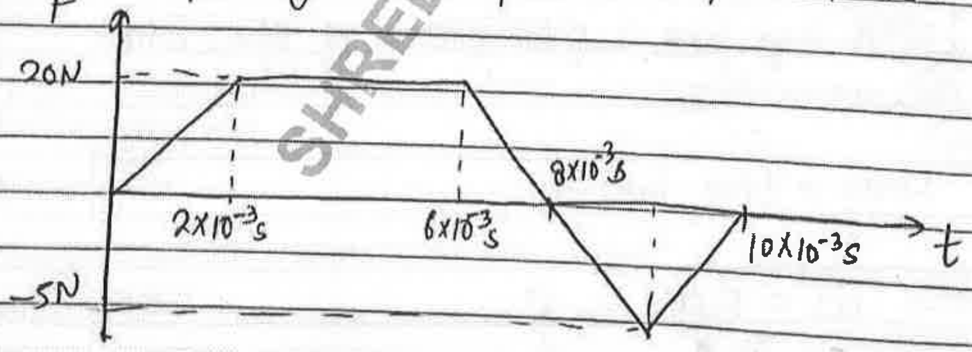
$$\boxed{\begin{gathered} {}^*\vec{I} = m(\vec{V}_f - \vec{V}_i) \\ \vec{I} = \vec{F}_{av}\Delta t \\ \vec{I} = \int_{t_1}^{t_2}\vec{F}dt \\ {}^*\vec{F}_{av} = \frac{m(\vec{V}_f - \vec{V}_i)}{t} \end{gathered}}$$



$\vec{I} = \int \vec{F}dt = $ Area*

~~Slope = $\frac{dF}{dt}$~~

Q Find Impulse given to particle in first 10ms.



$$\vec{I} = \frac{1}{2}(4+8)\times10^{-3}\times20 - \frac{1}{2}\times2\times10^{-3}\times5$$

$$= 10^{-3}[120-5]$$

$$\underline{I = 0.115}$$

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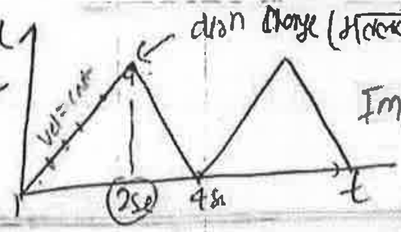
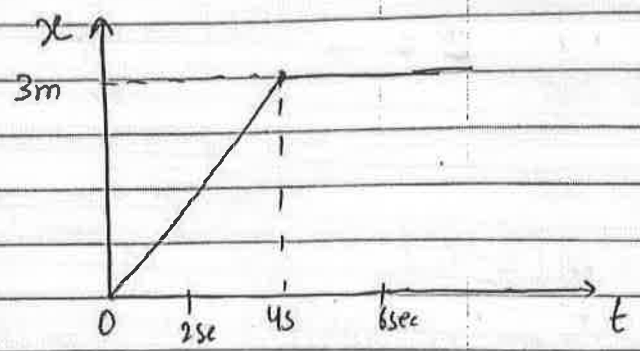
(1) NCERT



Impulse time = 2sec

NCERT-Exercise

Q A particle is moving along straight line and its mass 4kg. Find Impulse at time $t = 4sec$?



$t = 4sec$

$\vec{V_f} = 0$

$\vec{V_i} = \frac{3}{4}$ m/s (Slope)

$\vec{I} = m[\vec{V_f} - \vec{V_i}]$

$= 4[0 - \frac{3}{4}]$

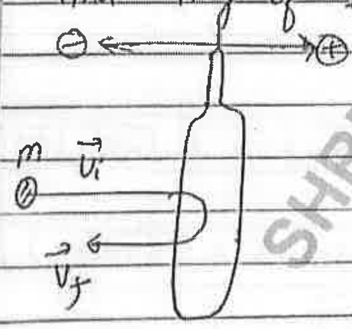
$= -3$ kg m/s

$t = 2sec$ $\quad V_i = \frac{3}{4}$ m/s

$V_f = 3/4$ m/s

$\vec{I} = 4\left(\frac{3}{4} - \frac{3}{4}\right) = 0$

Q A batsman hits back a ball straight towards the bowler giving it the same initial speed 12 m/s. (assume motion is st. line) Find mag of Impulse given to the ball mass 0.15 kg.



$\vec{I} = m[\vec{V_f} - \vec{V_i}]$

$= 0.15[-12\hat{i} - 12\hat{i}]$

$= 0.15 \times 24 (-\hat{i})$

$= -3.60\hat{i}$ N-S

$|\vec{I}| = 3.6$ N-S, towards the bowler

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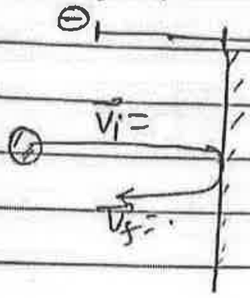
B·B-1 [5, 8, 9, 11]
Ex-1 [24-26, 29-31, 34, 40, 42, 44, 46]
Ex-2 [3, 7, 10, 26, 32]
Ex-3 [11]

Race (P-30) 1-5, 8, 10, 11, 14, 17, 18, 20, 21

Q A Ball of mass 40gm collides Normally with a wall with speed 16 m/s & rebound with speed 12m/s. The time of contact b/w the wall and ball is 0.04 seconds. Find

① mag. of Impulse given to the ball.

② mag. of Average force on the ball.



$$I = m[\vec{v}_f - \vec{v}_i]$$
$$= 40\times10^{-3}[-12\hat{i} - 16\hat{i}]$$
$$= 40\times10^{-3}\times 28(-\hat{i})$$
$$= 112\times10^{-2}$$
$$\vec{I} = 1.12(-\hat{i})$$

② $F_{av} = \frac{dp}{dt}$

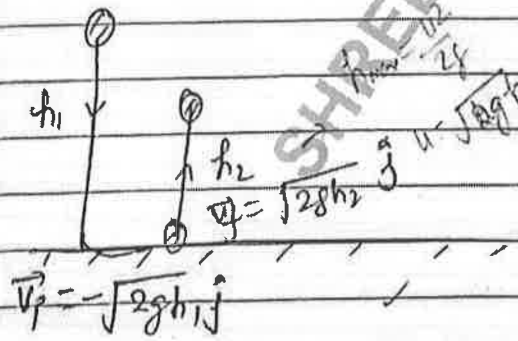
$$= \frac{1.12}{0.04}$$

$F_{av} = 28\ N$

Q A Ball of mass 10gm is droped from height 5m above floor. after bouncing it attains height 0.8m. if contact time is 5msecond. Find

① magnitude of Impulse of the ball.

② " " Average force on the ball.



$$\vec{I} = m[\vec{v}_f - \vec{v}_i]$$
$$= 10\times10^{-3}[\sqrt{2gh_2}\,\hat{j} - (-\sqrt{2gh_1})\hat{j}]$$
$$= 10\times10^{-3}[\sqrt{2\times10\times0.8} + \sqrt{2\times10\times5}]\hat{j}$$
$$= 10\times10^{-3}\times 14\hat{j}$$
$$\vec{I} = 0.14\hat{j}\ N\text{-}s$$
$$|\vec{I}| = 0.14\ N\text{-}s$$

② $F_{avg} = \frac{\Delta p}{\Delta t}$

$$= \frac{0.14\times10^{3}}{5}$$

$F_{av} = 28\ N$

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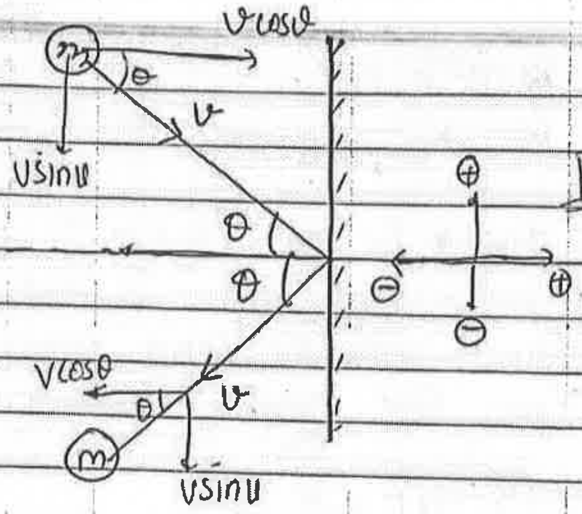
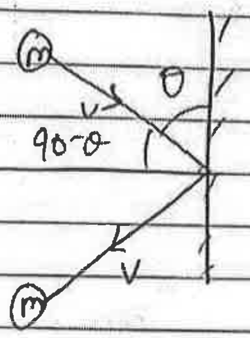
$\Delta \vec{P}_y = 0 \qquad |\Delta\vec{P}| = \sqrt{P_x^2 + P_y^2}$



$$\Delta\vec{P}_x = -mv\cos\theta\,\hat{i} - mv\cos\theta\,\hat{i} = -2mv\cos\theta\,\hat{i}$$

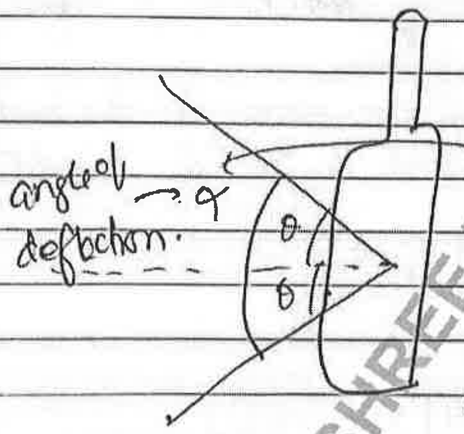
D.P. $\boxed{|\Delta\vec{P}_x| = 2mv\cos\theta}$ $\qquad |\vec{P}_y| = 0$

$$|\vec{F}_{av}| = \frac{2mv\cos\theta}{\Delta t}$$



$$|\Delta\vec{P}| = 2mv\cos(90-\theta)$$

$$|\Delta\vec{P}| = 2mv\sin\theta$$ ~~Aieee~~ AIPM 2K06



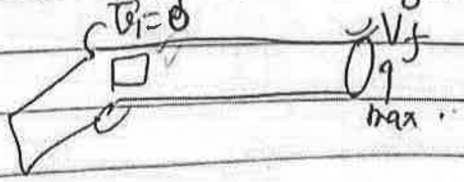
angle of deflection → α

Ball deflected by bat through angle α.

$\alpha = 2\theta$

$\boxed{\theta = \alpha/2}$

Que. A machine gun fires 360 bullets per minute, each of mass 20 gm with speed 400 m/s. Find magnitude of average force acting on a gun?



$$\vec{F}_{av} = \frac{nm[\vec{v}_f - \vec{v}_i]}{\Delta t} \quad (\vec{v}_i \to 0)$$

$$\vec{F}_{av} = \frac{nmv\,\hat{i}}{\Delta t}$$

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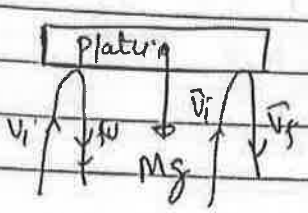
$$|\vec{F}_{av}| = \frac{n\,mv}{\Delta t} = \frac{\overset{12}{\cancel{360}} \times 2\cancel{0}\times 10^{-3} \times 4\cancel{00}}{\cancel{360}} \Rightarrow \frac{n}{\Delta t} \rightarrow \text{bullet/sec.}$$

$$= 48\ N$$

Que. A plate of mass 500gm has to be kept floating in air for this purpose 20 marbles per a second are projected which strike plate Normally and rebounds with the same speed. mass of each marble is 5gm. Find speed of marble.



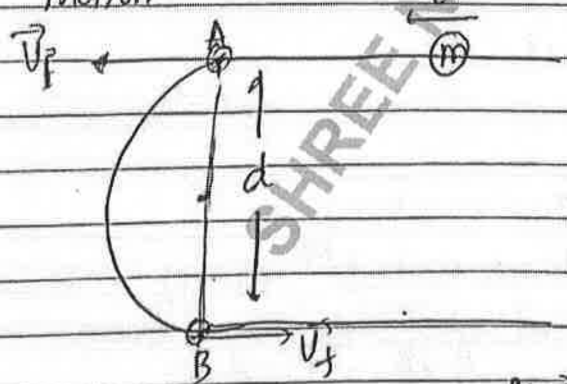
$$|\vec{F}_{av}| = n\left(\frac{m[\vec{v}_f - \vec{v}_i]}{t}\right)$$

$$|\vec{F}_{av}| = \frac{-2n\,mv\hat{j}}{\Delta t} = Mg$$

$$2\times 20\times 5\times 10^{-3}\,v = 500\times 10^{-3}\times 10$$

$$\underline{v = 25\ m/s}$$

Que A U-shaped frame is kept horizontally in which a bead of mass 'm' is moving a/c to the figure.
Find mag. of Average Force given by the wire to the bead in semicircular part AB. speed of bead is 'v' in compelet motion.



$$\vec{F}_{av} = \frac{m[\vec{v}_f - \vec{v}_i]}{\Delta t}$$

$$= \frac{m[v\hat{i} - (-v\hat{i})]}{\Delta t}$$

$$= \frac{2mv\,\hat{i}}{\Delta t}$$

$$t = \frac{\text{dist}}{\text{spee}} = \frac{\pi r}{v}$$

$$|\vec{F}_{av}| = \frac{2mv}{\pi r/v} = \frac{2mv^2}{\pi r} = \boxed{\frac{4mv^2}{\pi d}}$$

$$r = \frac{d}{2}$$

Ans.

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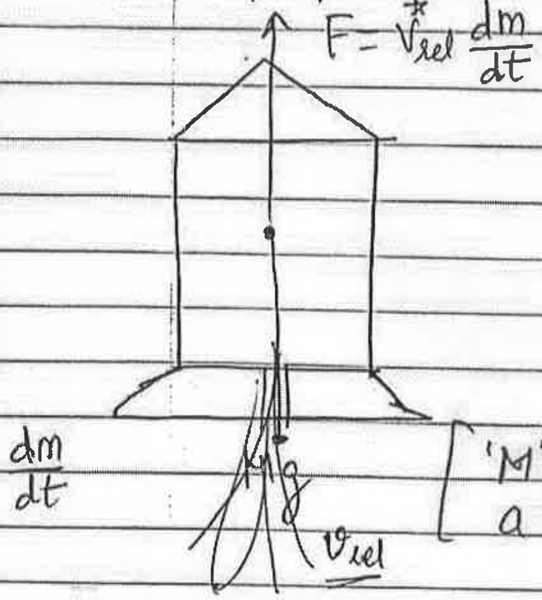
* Variable mass.

① Rocket propulsion:



M = mass of rocket (Fuel included) ← variable

v_{rel} = velocity of gas exhaust w.r.t rocket

$\frac{dm}{dt}$ = rate of fuel consumption (const)

['M' reduces, a ↑ses]

① on earth surface

$$V_{rel}\frac{dm}{dt} < Mg$$

Rocket will not lift

*② $V_{rel}\frac{dm}{dt} > Mg \Rightarrow$ Rocket will lift

$$\boxed{V_{rel}\frac{dm}{dt} - Mg = Ma}$$

$a \Rightarrow$ variable
($a \Rightarrow$ when mass M) → initial.

when mass of rocket is 'M'

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③ Gravity free space

$$\boxed{V_{rel}\frac{dm}{dt} = Ma}$$

$$\boxed{a = \frac{V_{rel}}{M}\frac{dm}{dt}}$$ when mass 'M'

④ To just lift the rocket or const velocity.

$$\boxed{V_{rel}\frac{dm}{dt} = Mg}$$

⑤ Mass after time 't'

$$= M - \frac{dm}{dt}t$$

Que A rocket of mass 1000 kg is in gravity free space. Now fuel starts burning at the rate of 50 kg/sec and speed of gas exhaust Relative to rocket is 400 m/s. Find accl^n & accl^n after 15 sec.

$$V\frac{dm}{dt} = ma$$

$$400 \times 50 = 1000\, a$$

$$20\ m/s^2 = a$$

* mass after 15 sec

$$= 1000 - 50\times 15$$

$$= 1000 - 750$$

$$= 250\ kg.$$

$$V\frac{dm}{dt} = ma$$

$$400 \times 50 = 250\, a$$

$$a = 80\ m/s^2$$

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Ex-1 (Q-32,35-38,43)
Ex-2 (Q-1,3,7,10,17)

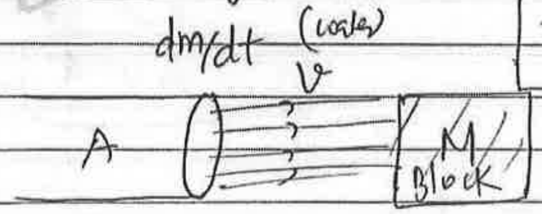
Que. A rocket of mass 2000 kg has to be launched from a earth surface with accl^n 20 m/s². Find the Rate at which the fuel has to be consumed. ~~vel. of~~ speed of gas exhaust relative to rocket is 1 km/s.

$$v\frac{dm}{dt} - mg = ma$$

$$1000\frac{dm}{dt} - 2000g = 2000 \times 20$$

$$\frac{dm}{dt} = 40 + 20 = 60 \text{ kg/s}$$

② Water jet.



$\boxed{F = v\frac{dm}{dt}}$ ⇒ also equal to Force required to hold the pipe

$$F = v\frac{dm}{dt}$$

$$= v\left(\frac{dV}{dt}\right)\rho \qquad \frac{dV}{dt} = Av$$

rate of flow

$$= v(Av)\rho$$

** D.P.

$$\boxed{F = Av^2\rho}$$

(i) water does not rebound.

$$F = u\frac{dm}{dt}$$

$$Ma = u\frac{dm}{dt}$$

$$\left[a = \frac{u}{M}\frac{dm}{dt}\right]$$

$$\boxed{F = Av^2\rho}$$

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$$\boxed{F = AV^2 \rho}$$

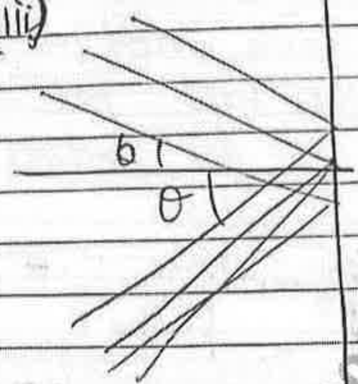
(ii) rebounds

$$F = 2v\frac{dm}{dt}$$

$$a = \frac{2v}{M}\frac{dm}{dt}$$

$$\boxed{F = 2AV^2\rho}$$

(iii)



$$\Delta P = 2mv\cos\theta$$

$$F = \frac{2dm}{dt}v\cos\theta$$

$$F = 2\left(\frac{v\,dm}{dt}\right)\cos\theta$$

$$\boxed{F = 2(AV^2\rho)\cos\theta}$$

Q Ring Similar Qu Ex? Water stream emerges out from a house pipe of diameter 2cm with speed 20cm/sec and incident normally on a block of 5kg placed on a frictionless surface. Find acc^n of block at this instant. (water collides inelastically)

$$a = \frac{F}{m} = \frac{AV^2\rho}{m} = \frac{\pi r^2 V^2 \rho}{m}$$

$$\boxed{F = AV^2\rho}$$

$$a = \frac{3.14 \times (10^{-2})^2 \times (0.2)^2 \times 10^6}{5} = \frac{3.14 \times 10^{-4} \times 0.04 \times 10^3}{5}$$

$$a = 2.5 \times 10^{-3} \text{ m/s}^2$$

(3) Conveyor belt

Sand is dropped @ $\frac{dm}{dt}$.

$$\boxed{F = v\frac{dm}{dt}}$$

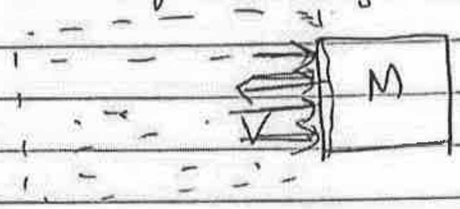
✓ Additional force required to maintain the same speed of the belt

$$\boxed{F = (u-v)\frac{dm}{dt}}$$

$V_{relative}$

Module (M)

Q. A sattelite in Force free space scops interplanatary dust @ $\frac{dm}{dt} = \alpha V$ (where $\alpha \to$ const & $V \to$ velocity). Find acc^n of sattelite?



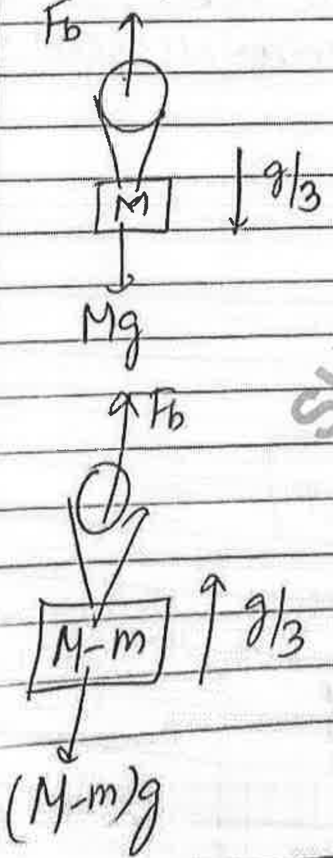
$$0 - v\frac{dm}{dt} = Ma$$

$$a = -\frac{v}{M}\frac{dM}{dt}$$

$$a = -\frac{v}{M} \times \alpha v = \boxed{-\frac{\alpha v^2}{M}} \text{ Ans}$$

Neet

Q. A ballon of mass 'M' is descending with acc^n $g/3$. Now a mass 'm' is detached from it so that ballon starts ascending with acc^n $g/3$. Volume remain unchange, then find 'm'?



$$Mg - F_b = \frac{Mg}{3} \quad \text{(I)}$$

$$F_b - (M-m)g = \frac{(M-m)g}{3} \quad \text{(II)}$$

$$\cancel{Mg} - \cancel{Mg} + mg = \frac{g}{3}[M + M - m]$$

$$m\cancel{g} = \frac{\cancel{g}}{3}[2M - m]$$

$$3m = 2M - m$$

$$2\cancel{4}m = \cancel{2}M$$

$$m = \frac{M}{2}$$

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II^{nd} law of Motion is consistent with I^{st} law.

$$\vec{F}_{ext} = \frac{d\vec{p}}{dt}$$

$$\vec{F}_{ext} = 0 \quad \frac{d\vec{p}}{dt} = 0$$

$$d\vec{p} = 0$$

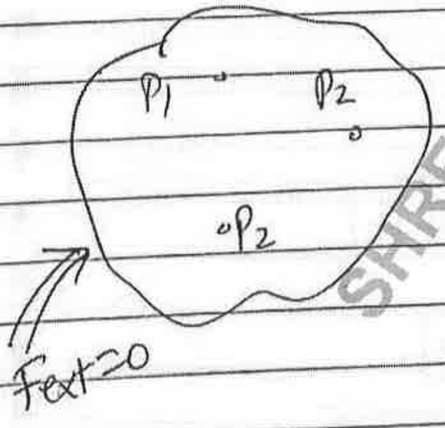
$$\vec{p} = const$$

$$m\vec{v} = const$$

$$\boxed{\vec{v} = const}$$

Conservation of linear momentum:

If Net External Force on System of particle is zero. then Net Momentum of the system remains Const.



$$\vec{F}_{ext} = 0$$

$$d\vec{p} = 0$$

$$\vec{P}_{system} = Const$$

$$\boxed{P_1 + P_2 + P_3 + \cdots + P_n = Const}$$

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Newtons IIIrd law of Motion:

⊖ To Every Action there is Equal & opposite Rxn.

Force etc

⇒ (i) Force always exist in pair (Action & Reaction)

↳ Here A/c & Rx stands for Force

⇒ (ii) Action & Rxn is not Cause effect Both acts at Same Instant.

⇒ (iii) Action Rxn act on two diff bodies do not cancelled each other.

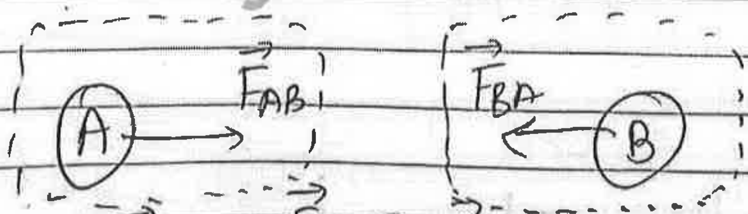
⇒ If we consider the motion of one body then one Force out of two force is releavant. $a_A = ?$ $a_B = ?$

If we take Both bodies as a whole system then Resultant of Action - Rxn is $\boxed{\text{Zero.}}$
which implies Action - Rxn is internal Force of System.

$$\vec{F}_{AB} = -\vec{F}_{BA}$$

$$\vec{F}_{AB} + \vec{F}_{BA} = 0$$

$$\vec{P}_A + \vec{P}_B = \text{Const}$$



→ System

$$\vec{F}_{BA} = -\vec{F}_{AB}$$

$$a_A = \frac{\vec{F}_{AB}}{m_A} \qquad a_B = \frac{\vec{F}_{BA}}{m_B}$$

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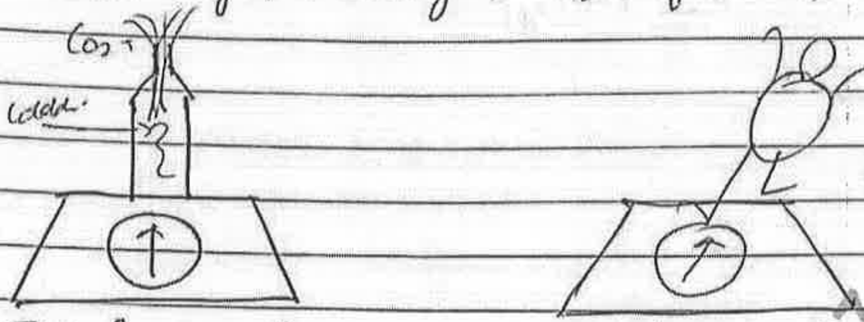
Ex-1 (6, 9, 10, 11, 13, ...)
21-46 (39, 41, 45 ...)
Ex-2 (11), 24

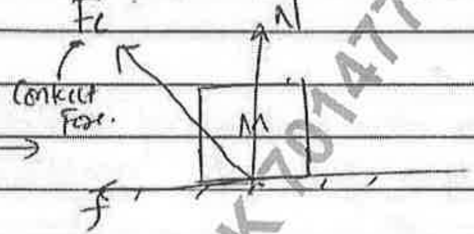
Ex → Swimming, walking, Flight of Birds, Rocket propulsion etc.



First increases than become same as initial

First increases then becomes zero.

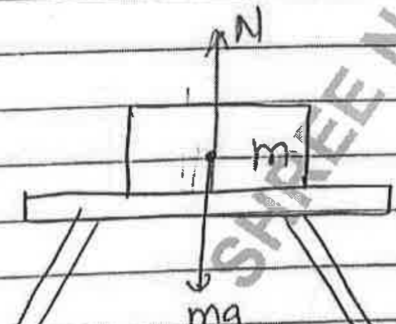
Normal Rxn Force: →



When a body is placed on a surface then a contact force acts which is ⊥ to the contact surface & towards the Body.

Or

NCERT — Component of Contact Force which is ⊥ to contact surface and directed towards the body.



$\vec{N} + \vec{W} = 0$

$\vec{N} = -\vec{W}$

$\boxed{N = W}$

$\boxed{N = mg}$

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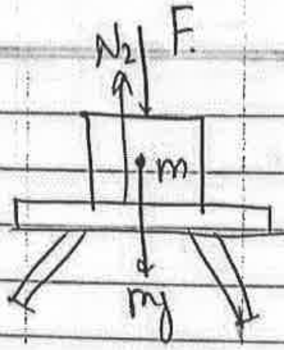
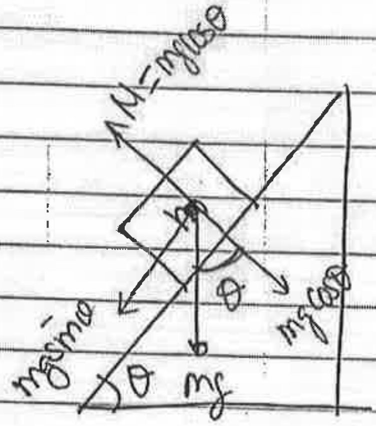
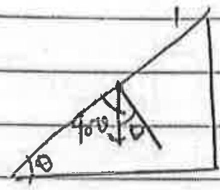
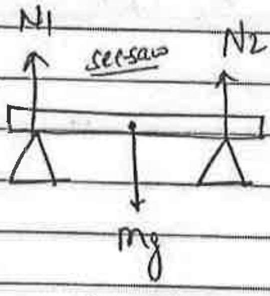
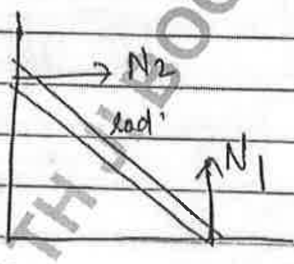


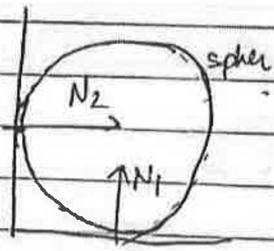
$N_2 = F + mg$



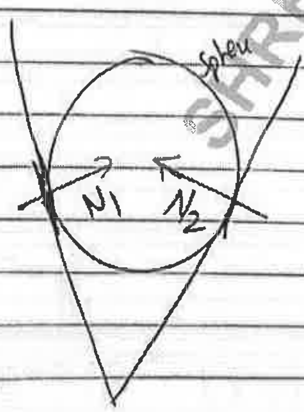


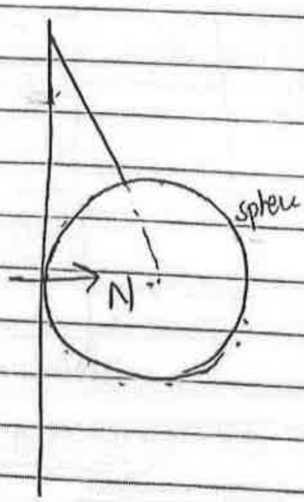






$N_1 + N_2 = mg$





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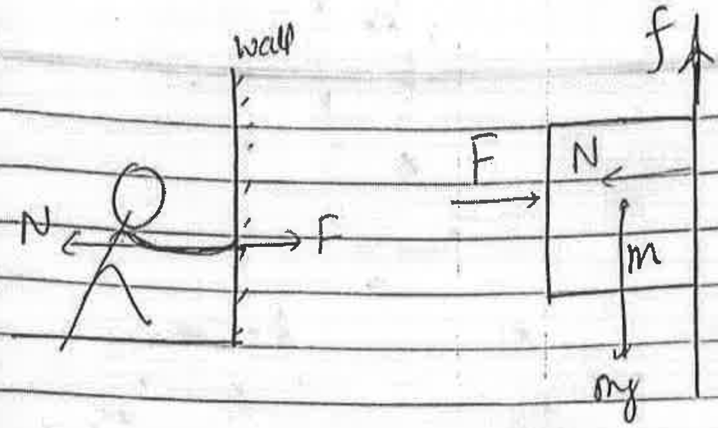
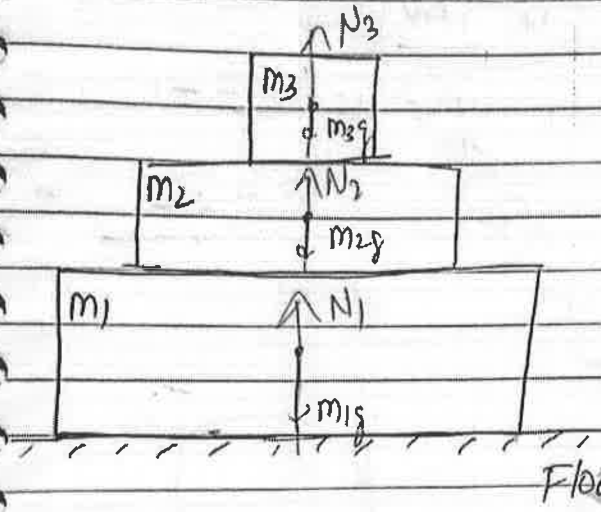




① Normal force by Floor on m_1

$$N_1 = (m_1+m_2+m_2)g$$

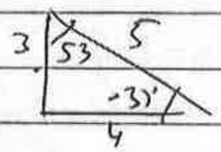
② Normal Force by surface of m_1 on m_3

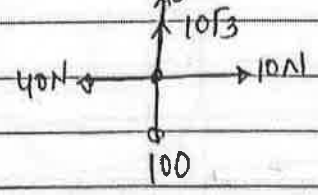
$$N_2 = (m_2+m_3)g$$

③ Normal force by surface of m_2 on m_3

$$N_3 = m_3 g$$

Q. Find accⁿ & Normal Force on block.





$a = \frac{F_{net}}{m}$

$a = \frac{30}{10}$

$\boxed{a = 3\ m/s^2}$

अगर N=0, है
मतलब contact नहीं रहा है।

$N = 100 - [30 + 17.3]$

$= 100 - 47.3$

$N = 52.7$ Newton

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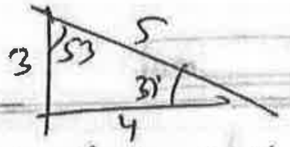
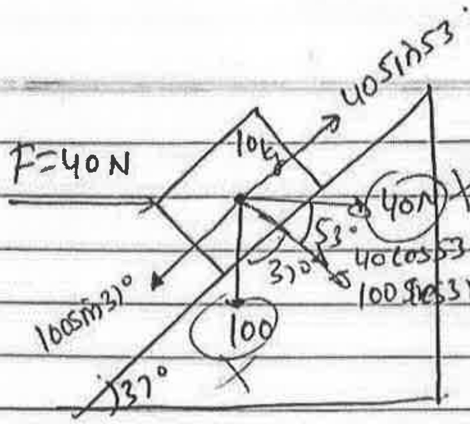
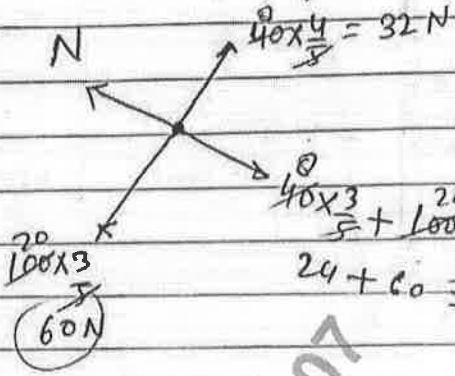






$24 + 60 = 74N$

$N = 74N$

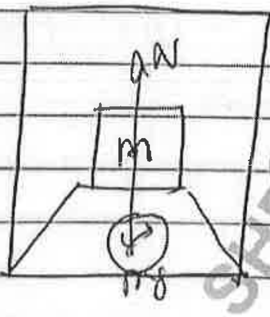
$a = \frac{F_{net}}{m} = \frac{60-32}{10} = \frac{28}{10}$

$= 2.8\ m/s^2$

Weigh machine always Measures Normal Force:

Lift or Elevator:

Case-I lift at rest or moving with uniform velocity up or down.

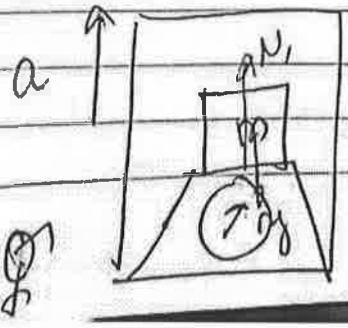


$\vec{v} = 0$ $\quad N = mg$

$\vec{v} = const$ $\quad W = actual$

up, down

Case-II lift ascending with uniform accn.



$N_1 - mg = ma$

$N_1 = m(g+a)$ $\quad \boxed{W_a > W}$

$\boxed{g_{eff} = g + a}$

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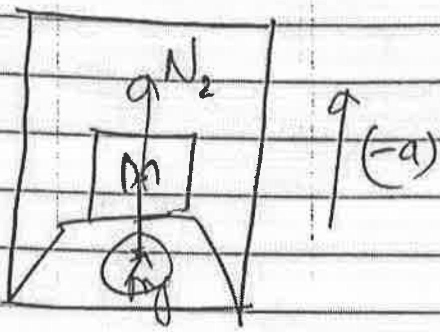
Case-III Lift Ascending with uniform Retardation.



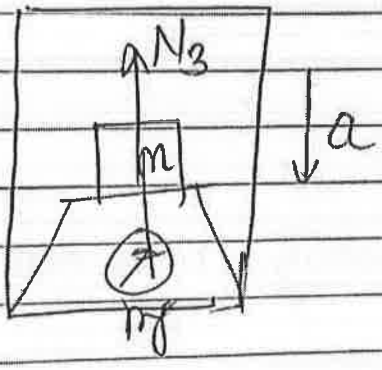
$N_2 - mg = m(-a)$

$N_2 = m(g-a)$

$W_{a_2} < W_{actual}$

$\boxed{g_{eff} = g-a}$

Case-IV Lift descending with uniform acc^n



(a) $a < g$

$mg - N_3 = ma$

$N_3 = m(g-a)$

$W_3 < W$

$\boxed{g_{eff} = g-a}$

(b) $a = g$

$N_4 = m(g-g)$

$N_4 = 0$

Weightlessness

or Free fall

(c) $a > g$ leaves the floor and hits the ceiling

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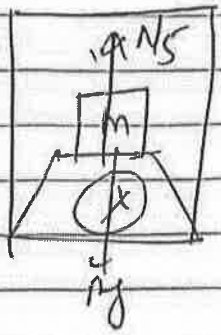
Case-V Lift descending with uniform Retardation



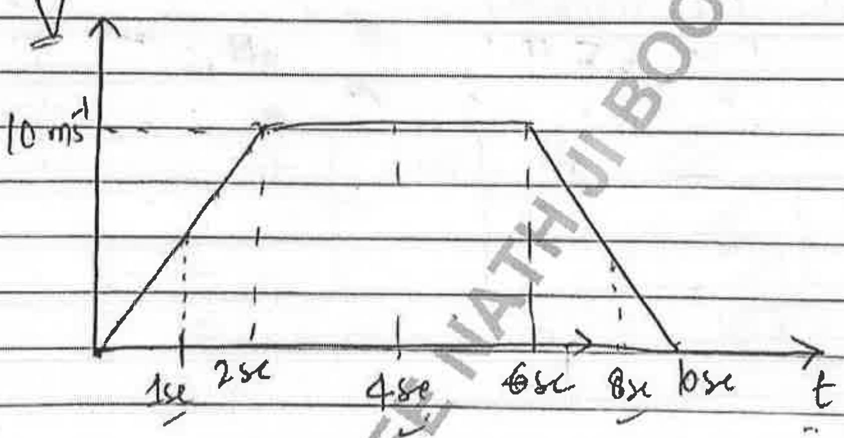
$\downarrow (-a)$

$$mg - N_5 = m(-a)$$

$$N_5 = m(g+a)$$

$$\underline{W_5 > W}$$

Que. A person of mass 50 kg is standing inside a lift, Now lift starts ascending it's velocity-time graph is given, Find the weight of person at time $t = 1$ sec, $t = 4$ sec, $t = 8$ sec.



① $t = 1$ sec

$$\text{slope} = \tan\theta = \frac{10}{2} = 5\,m/s^2$$

$$N = m(g+a)$$

$$= 50(10+5)$$

$$= 50 \times 15 = 750 \text{ Newton}$$

$$N = 75 \text{ kg-wt}$$ (spring balance reading)

② $t = 4$ sec

$a = 0$

$V =$ const

$N_1 = mg$

$= 500$ N

$= 50$ kg-wt

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③ *

$t = 8$ sec

(Retardation) → But dirᶰ not change.
Speed down)

Slope $= \tan\theta = -\frac{10}{4} = -2.5\ m/s^2$

$\uparrow a = -2.5\ m/s^2$ $\quad N = m(g-a)$

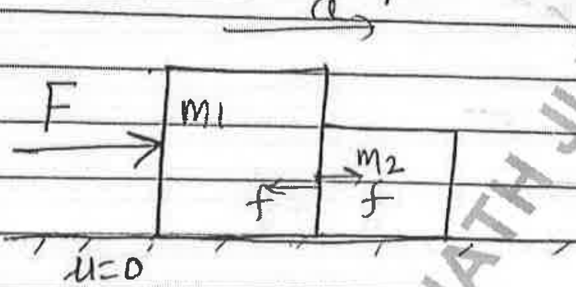
$= 50(10-2.5)$

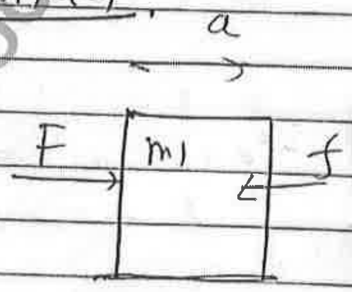
$= 50 \times 7.5$

$= 375\ N$

Reading of spring balance $= \underline{37.5\ kg\text{-}wt}$

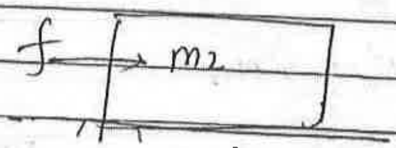
* Blocks Kept in contact.





$$\boxed{a = \frac{\text{Net Fext}}{\text{Total mass}}}$$

$F - f = m_1 a$ — ①



$f - 0 = m_2 a$

$f = m_2 a$ — ②

② in ①

$F - m_2 a = m_1 a$

$$\boxed{a = \frac{F}{m_1 + m_2}}$$

$$f = \frac{m_2 F}{m_1 + m_2}$$

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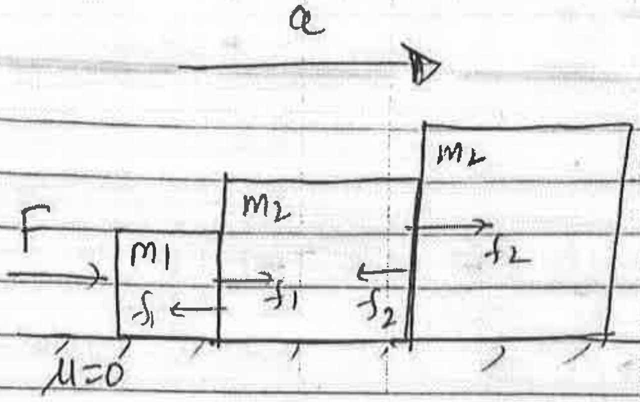
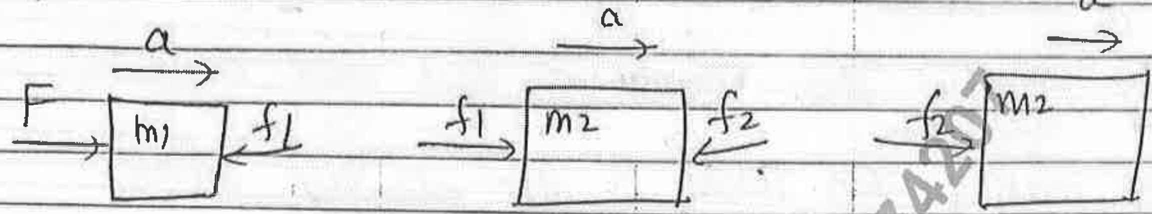
65, 67, 69, 6
-Ex.2 (4, 6, 11, 13, 31)
RQ-2 (1-6)



$$a = \frac{F}{m_1 + m_2 + m_2}$$



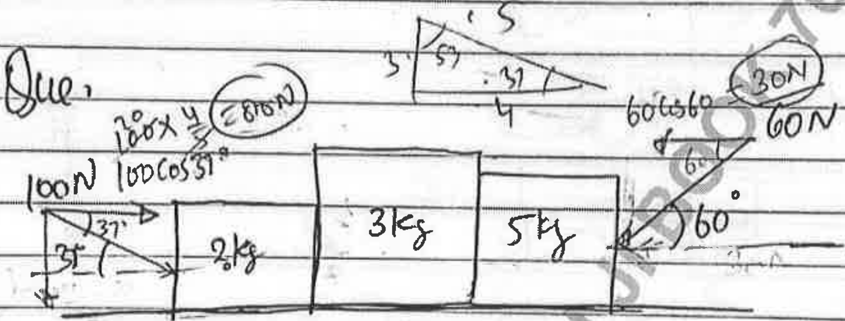
$F - f_1 = m_1 a$

$f_1 - f_2 = m_2 a$

$f_2 - 0 = m_2 a$

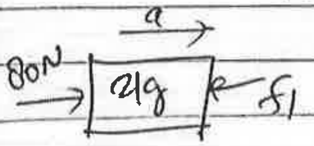
$f_2 = m_2 a$

Que.



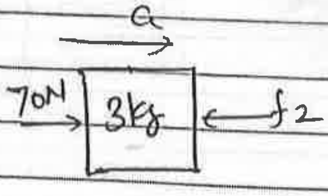
Find accl^n of block and Normal force b/w surface of 2kg, 3kg & 3kg-5kg

$$a = \frac{F_{ext}}{m} = \frac{80-30}{10} = 5\,m/s$$



$80 - f_1 = 2 \times 5$

$f_1 = 70N$



$70 - f_2 = 3 \times 5$

$f_2 = 55N$

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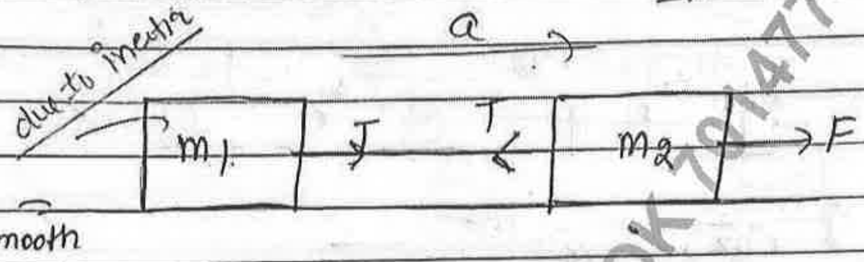
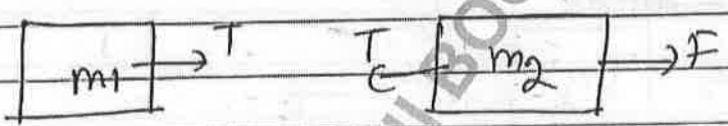
Block connected with String

- Tension arises due to Intermolecular Force (Electromagnetic)
- Tension directed away from the ends.
- String inextensible than accl^n of each point is same.
- String massless than Tension at each point is same.
  negligible mass
- String massive than Tension is d/F at d/F points.





$F - T = m_2 a$ —(I)

$T - 0 = m_1 a$ —(II)

$$\boxed{a = \frac{F_{ext}(Net)}{total\ mass}}$$

$F = m_2 a + T = m_2 a + m_1 a$

$F = (m_1 + m_2) a$

$$\boxed{a = \frac{F}{m_1 + m_2}}$$

$T = m_1 a$

$= \frac{m_1 F}{m_1 + m_2}$

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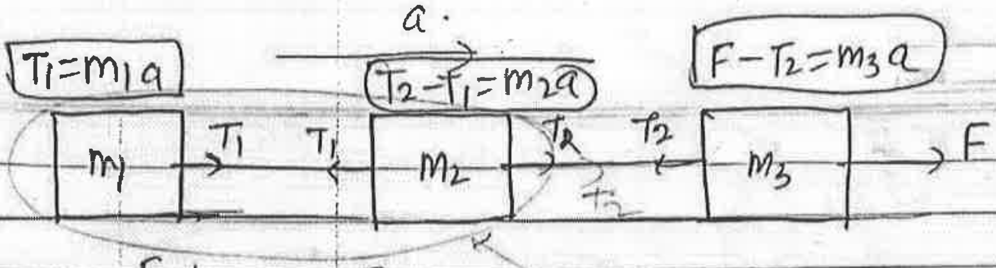
#

$\boxed{T_1 = m_1 a}$ $\boxed{T_2 - T_1 = m_2 a}$ $\boxed{F - T_2 = m_3 a}$

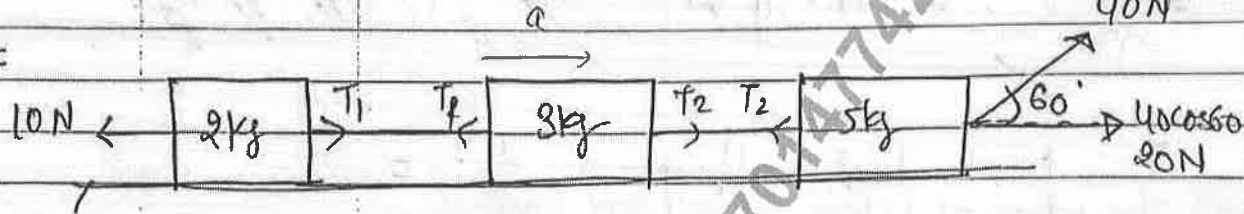


$$a = \frac{F_{ext}}{T.mass} = \frac{F}{m_1 + m_2 + m_2}$$

$$T_1 = \frac{m_1 F}{m_1 + m_2 + m_3} \qquad T_2 = \frac{(m_2 + m_1) F}{m_1 + m_2 + m_2}$$

"Tension is pulling" force.

#

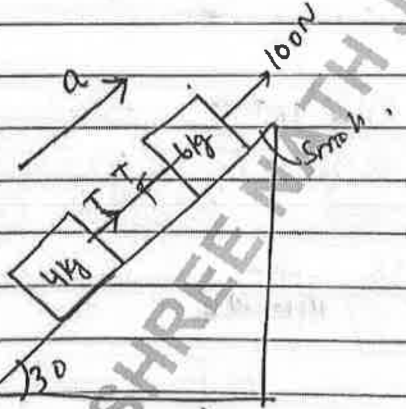


$$a = \frac{F_{net}}{T.mass} = \frac{20 - 10}{10} = \underline{1 m/s^2}$$

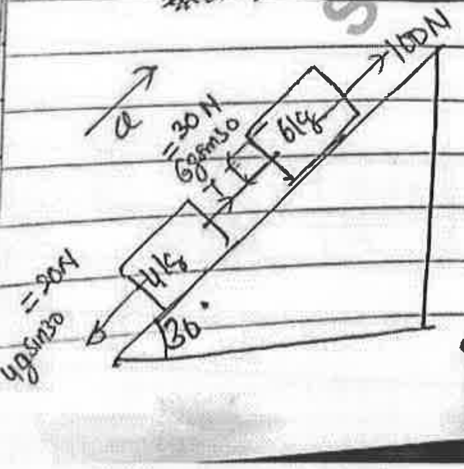
Find, $a = ?$ $1 m/s^2$
$T_1 = ?$ $12N$
$T_2 = ?$ $15N$

$T_1 - 10 = 2 \times 1$ $\qquad$ $T_2 - 12 = 3 \times 1$
$\underline{T_1 = 12N}$ $\qquad$ $\underline{T_2 = 15N}$

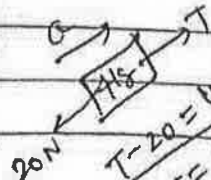
#



$a = 5 m/s^2$ $\quad T = 40N$



$$a = \frac{F_{net(ext)}}{T mass} = \frac{100 - 50}{10} = 5 m/s^2$$



$\boxed{T - 20 = 4 \times 5}$
$\boxed{T = 40N}$

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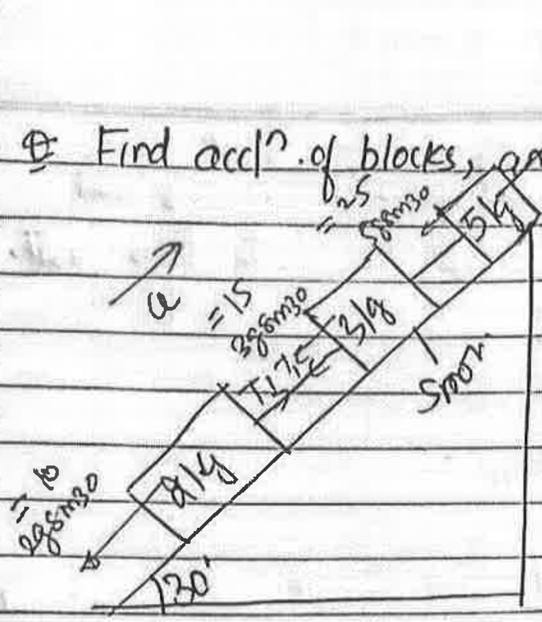
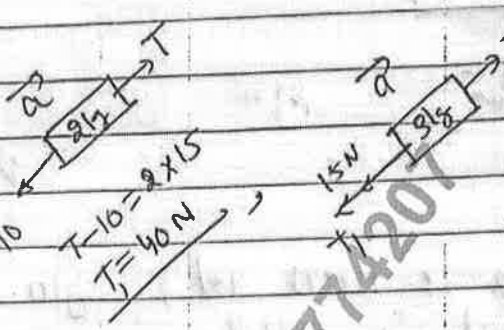
(Inclined plane पर weight का component भी लेना होगा)

Q. Find accl^n. of blocks, and Tension T_1 T_2. $a = 15\,m/s^2$ $T_1 = 40$ $T_2 = 100$



$$a = \frac{F_{net}}{T_{mass}} = \frac{200 - 50}{10} = \frac{150}{10} = 15\,m/s^2$$



$T - 10 = 2 \times 15$

$T_1 = 40\,N$

$T_2 - (15 + T_1) = 3 \times 15$

$T_2 - (15 + 40) = 45$

$T_2 = 55 + 45$

$T_2 = 100\,N$

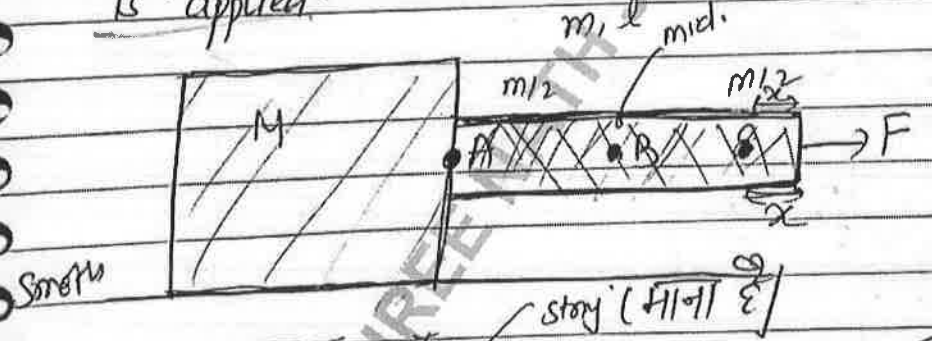
Q. The mass of string is 'm' and length is 'l' find ☆

(Break up and connect with massless string)

① Force by which string pulls blocks.

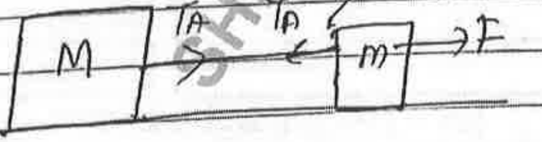
② Tension at mid-point of the string.

③ Tension at point 'x' distance away from the end where Force is applied.



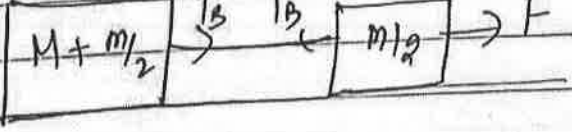
$$a = \frac{F}{M+m}$$

①



$$T_A = \frac{MF}{M+m}$$ ✓

②



$$T_B = \left[M + \frac{m}{2}\right] \times \frac{F}{M+m}$$ ✓

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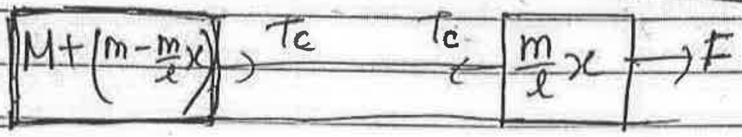
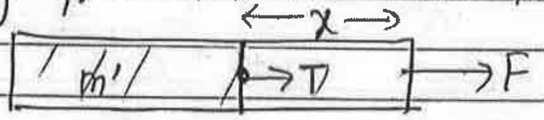
③ mass per unit length or Linear density $= \frac{m}{l}$ length से multiply करो और mass पाओगे।



$$T_c = \left[M+\left(m-\frac{m}{l}x\right)\right]\times\frac{F}{M+m}$$ ✓

Q A uniform Rop of mass M & length 'L' is placed on frictionless floor. Now it is pulled by force F at one end along the floor. Find Tension at point (x) distance away from the end where the force is applied.



$a = \frac{F}{M}$ Tension is pulling force.

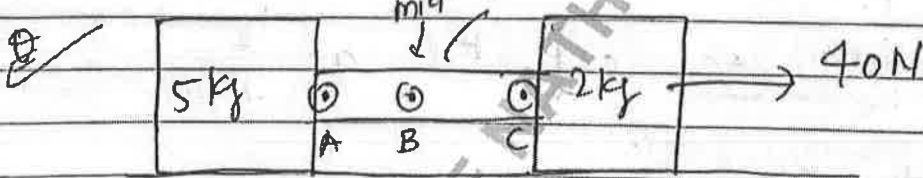
$L \to M$

$m' \to \frac{M}{L}[L-x]$

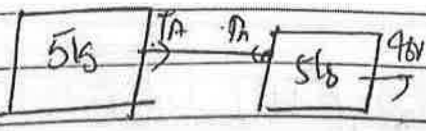
$$T = \frac{M}{L}[L-x]\times\frac{F}{M}$$

$$T = F\left[1-\frac{x}{L}\right]$$ ✓

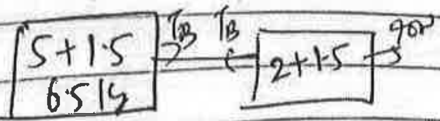
Q



$a = \frac{F_{ext}}{T.m} = \frac{40}{10} = 4 m/s^2$



$T_A = 5\times4 = 20N$



$T_B = 6.5\times4 = 26.0$
$= 26N$

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BB-3 4, 5, 7, 8

Ex-I ⇒ 72, 73-75, 76, 79, 89, 90, 92, 95, 96,

Ex-II ⇒ 19,   Ex-III ⇒ 14, 15, 16

Race ⇒ 12, 13

8 kg $\xrightarrow{T_c}$ 2 kg

$T_c = 8 \times g$

$= \cancel{80\,N}$ (32 N)

System of hanging block

direct

Case-I System is at rest or in uniform motion (up or down)

$\vec{v} = 0$, $\vec{v} = \text{const}$, $\vec{F}_{ext} = 0$ — equilibrium

$T_1 = (m_1 + m_2 + m_3)g$

m_1, $m_1 g$

$T_2 = (m_2 + m_3)g$

m_2, $m_2 g$

$T_3 = m_3 g$

m_3, $m_3 g$

rest: $T_3 = m_3 g$

T_3, m_3, $m_3 g$

T_2, m_2, $m_2 g$, T_3

$T_2 = m_2 g + T_3$

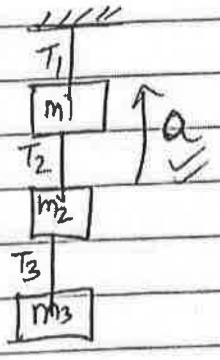
$= (m_2 + m_3)g$

T_1, m_1, $m_1 g$, T_2

$T_1 = m_1 g + T_2$

$T_1 = (m_1 + m_2 + m_3)g$

Case-II ↑ Ascending with uniform accⁿ



$T_1 = (m_1 + m_2 + m_3)\, g_{eff}$ $\boxed{g_{eff} = g + a}$

$T_1 = (m_1 + m_2 + m_3)(g + a)$

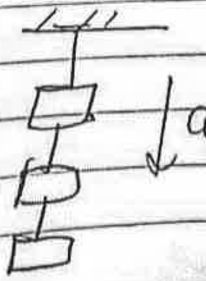
$T_2 = (m_2 + m_3)(g + a)$

$T_3 = m_3(g + a)$

~~down~~ upward Retardation

$g_{eff} = g - a$

Case-III



$T_1 = (m_1 + m_2 + m_3)\, g_{eff}$ $\boxed{g_{eff} = g - a}$

$T_2 = (m_2 + m_3)\, g - a$

$T_3 = m_3(g - a)$

downward Retardation

$\boxed{g_{eff} = g + a}$

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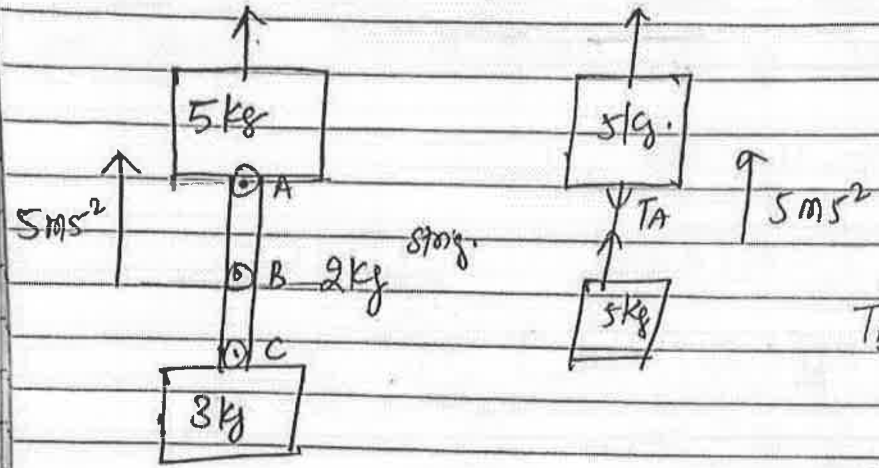
$T_A = 75,\ T_B = 60,\ T_C = 45$

Q Find Tension at pt A, B, C in following system of blocks.

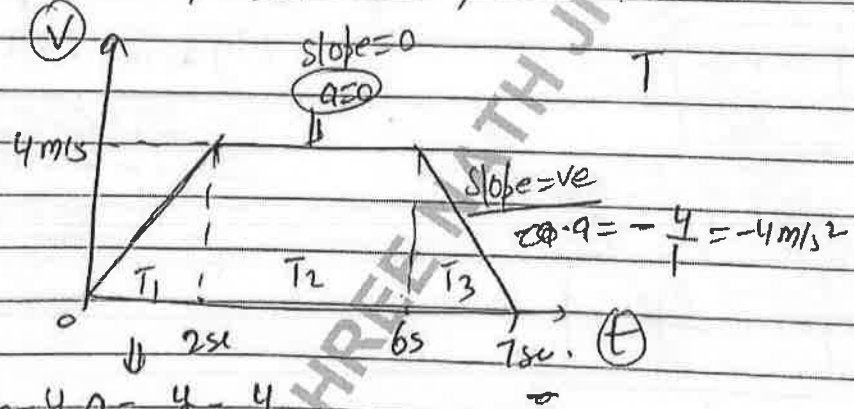


$T_A = 5(g+5)$
$= 5 \times 15 = 75\,N$

$T_B = 4(g+5)$
$= 4 \times 15 = 60\,N$

$T_C = 3(g+5) = 3 \times 15$
$= 45\,N.$

Q Lift starts Ascending it's velocity - time graph is given. Find Ratio of Tension in it's cable b/w 0 to 2 sec, 2 to 6 sec, 6 to 7 sec cir.



$\text{Slope} = \frac{y}{x} = a = \frac{4}{x} = \frac{4}{2}$
$= 2\,m/s^2$

$T_1 = M(g+a) = 10+2 = 12$

$T_2 = M(g+0) = 10 = 10$

$T_3 = M(g-a) = 10-4 = 6$

$12 : 10 : 6 \Rightarrow \boxed{6:5:3}$

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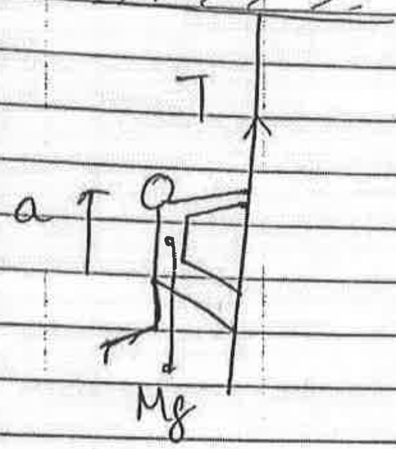
Q A mokey of mass 20kg is climbing up on a string which can ~~stan~~ with stands Max^m Tension of 250N. find max^m acel^n of the monkey without breaking string.



$T - Mg = Ma$

$\underline{T_{max} = M(g + a_{max})}$ $\quad \underline{a\uparrow \ T\uparrow}$

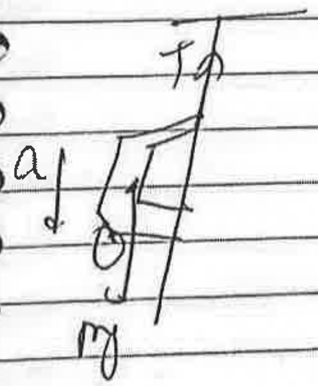
$250 = 20(10 + a_{max})$

$\frac{25}{2} - 10 = a_{max}$

$\frac{5}{2} = a_{max} = 2.5\ m/s^2$

Max^m Tension

Q Breaking strength of string is Half of the weight of Monkey. find the min^m acel^n by which the monkey move down without breaking the string.



$\boxed{T = \frac{Mg}{2}}$ (given)

$Mg - T = Ma$

$\underline{T_{max}} = M(g - a_{min})$

$\cancel{Mg - T = \frac{Mg}{2}}$

$\cancel{\boxed{T = \frac{Mg}{2}}}$

a

$a\downarrow \ T\uparrow$

$mg - T = ma$

$\underline{T} = m(g - a)$

$\frac{Mg}{2} = m(g - a)$

$\frac{g}{2} = g - a$

$a = g - \frac{g}{2} = \frac{g}{2}$

$\frac{10}{2} = 5\ m/s^2$

Ans.

$5\ m/s^2$

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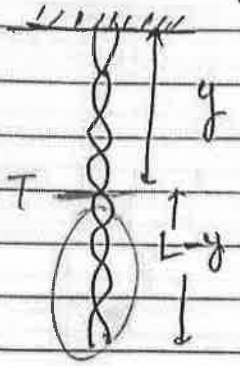
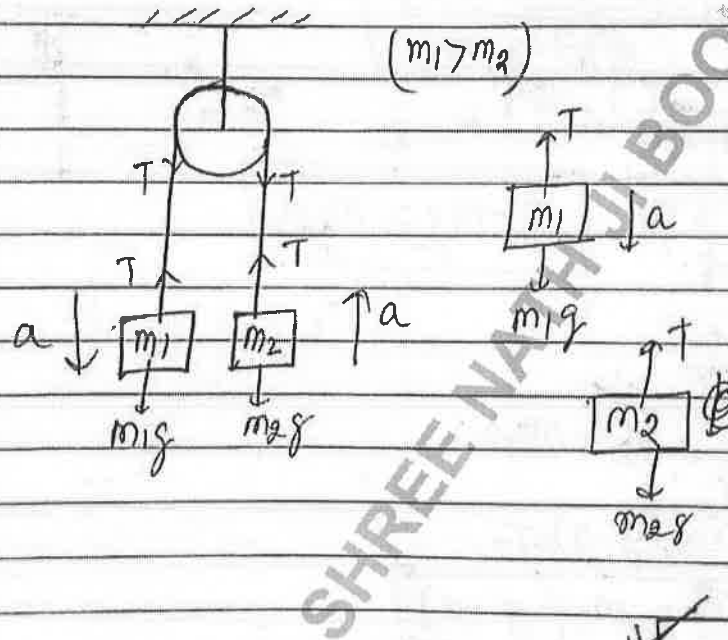
Q. A uniform chain of mass 'M' and length 'L' is suspended from one end. Find Tension at a point 'y' distance below the pt of suspension.



$$T = \frac{M}{L}(L-y)g$$

$$T = M\left(1-\frac{y}{L}\right)g$$

Pulley System ⇒ massless, frictionless ⇒ only change dirth of Force NOT magnitude.



$(m_1 > m_2)$

$m_1g - T = m_1a$ (I)

$T - m_2g = m_2a$ (II)

(I)+(II)

$m_1g - m_2g = (m_1+m_2)a$

$$\boxed{a = \frac{(m_1-m_2)g}{m_1+m_2}}$$ put

$$\boxed{T = \frac{2m_1m_2g}{m_1+m_2}}$$

$$T = \frac{2W_1W_2}{W_1+W_2}$$

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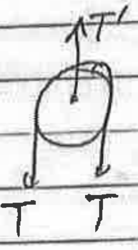
$$T = \frac{2 m_1 m_2 g}{(m_1+m_2)} \times \frac{g}{g}$$

$$T = \frac{2 m_1 g \; m_2 g}{m_1 g + m_2 g}$$

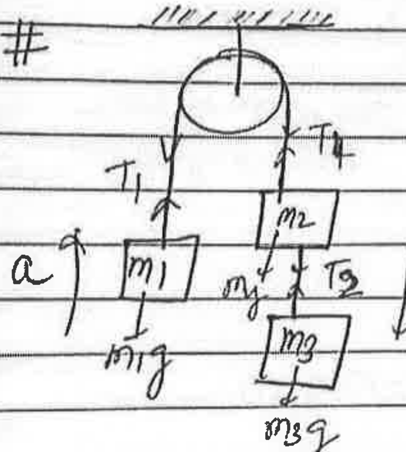
$$\boxed{T = \frac{2 w_1 w_2}{w_1 + w_2}}$$

Thrust on Pulley



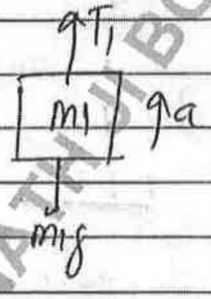
$$\boxed{T' = 2T}$$

#

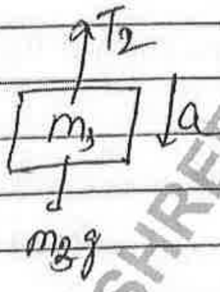


$(m_2+m_3) > m_1$

$$\boxed{a = \frac{(m_2+m_3-m_1)g}{m_1+m_2+m_3}}$$



$$\boxed{T_1 - mg = m_1 a}$$



$m_3 g - T_2 = m_3 a$

$m_3(g-a) = T_2$

$$\boxed{T_2 = m_3(g-a)}$$

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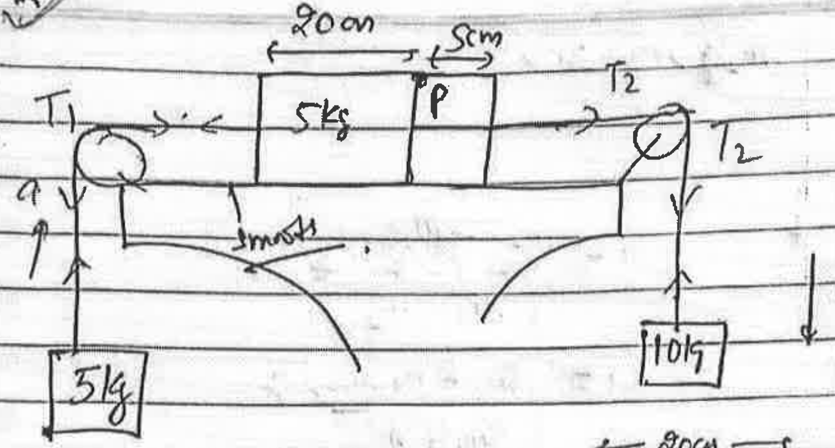
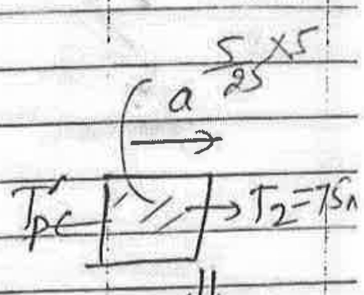
Find $T_p = ?$



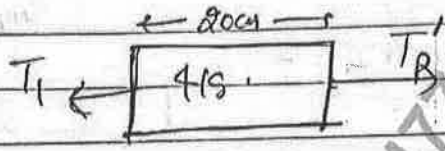
$a = \frac{5g}{20} = 2.5\ m/s^2$



$T_1 - 50 = 5 \times 2.5$

$T_1 = 50 + 12.5$

$T_1 = 62.5\ N$



$T_P' - T_1 = 4 \times 2.5$

$T_P' = 62.5 + 10.0$

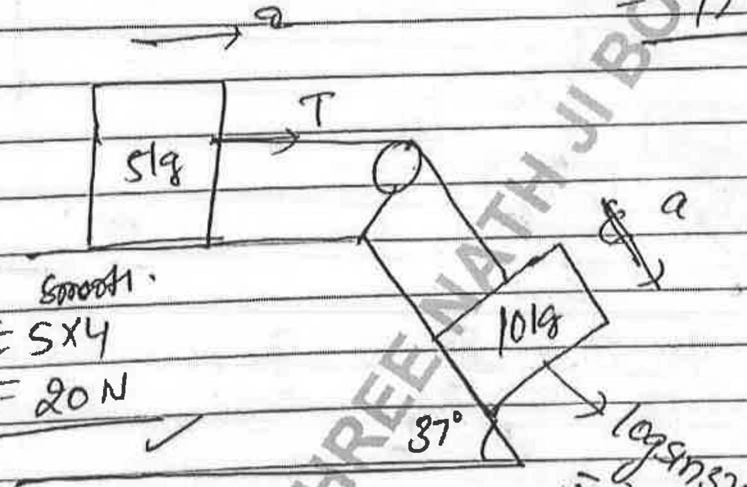
$= 72.5\ N$

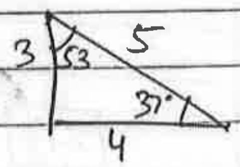
also

$T_2 - T_P' = m'a$

$75 - T_D' = 1 \times 2.5$

$T_P' = 72.5\ N$





$T = 5 \times 4$

$= 20\ N$

$10g \sin 37$

$= \cancel{100}^{2} \times \frac{3}{\cancel{5}}$

$= 60\ N$

$a = \frac{60}{15} = 4\ m/s^2$

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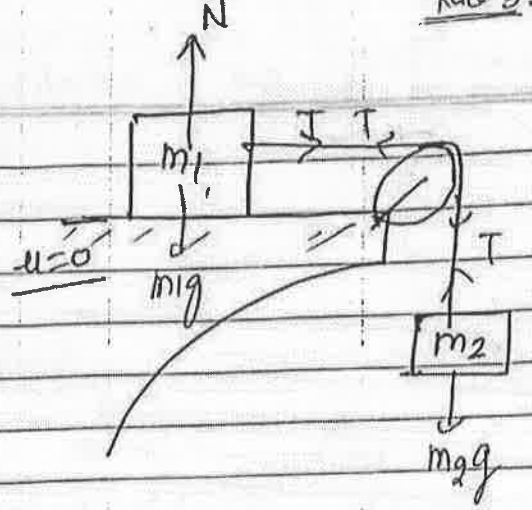
B.B-3 (complete), B.B-4 (1-4,) Ex-1 (39, 50, 71-76, 78-82, 88-92, 95, 96

Race-3 7, 8, ~~12~~ 13, 14, 15

Ex-II (4, 6, 8, 18, 19, 20,

Ex-III (5, 12, ~~14~~ 16.

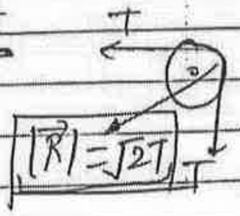


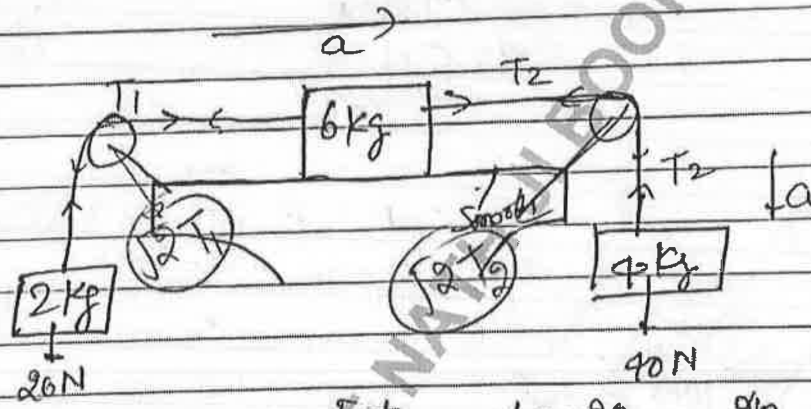
$$a = \frac{m_2 g}{m_1 + m_2} \qquad a = \frac{F_{net}}{T.mass}$$

$$T = m_1 (a)$$

$$T = m_1 \left(\frac{m_2 g}{m_1 + m_2} \right)$$

Thrust





$$a = \frac{F_{net}}{T.mass} = \frac{40 - 20}{12} = \frac{20}{12_6} = \frac{g}{6}$$

$$T_1 - 20 = 2 \times \frac{g}{6}$$

$$T_1 = \frac{g}{3} + 2g = \frac{7}{3} g$$

$$T_1 = \frac{70}{3} N$$

$$40 - T_2 = \cancel{4}^{2} \times \frac{g}{\cancel{6}_3}$$

$$4g - \frac{2g}{3} = T_2$$

$$\frac{10g}{3} = T_2$$

$$T_2 = \frac{100}{3} N$$

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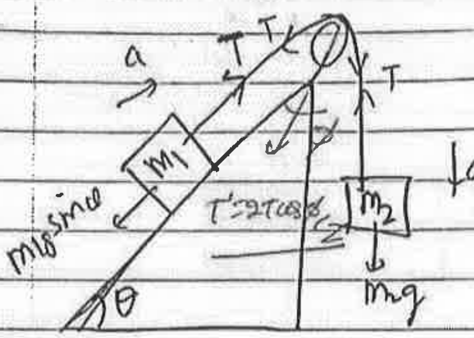
Find accl^n



$m_2 g > m_1 g\sin\theta$ $\quad |\vec{a}| = |\vec{b}| = a$

$|\vec{A}+\vec{B}| = 2a\cos\theta/2$

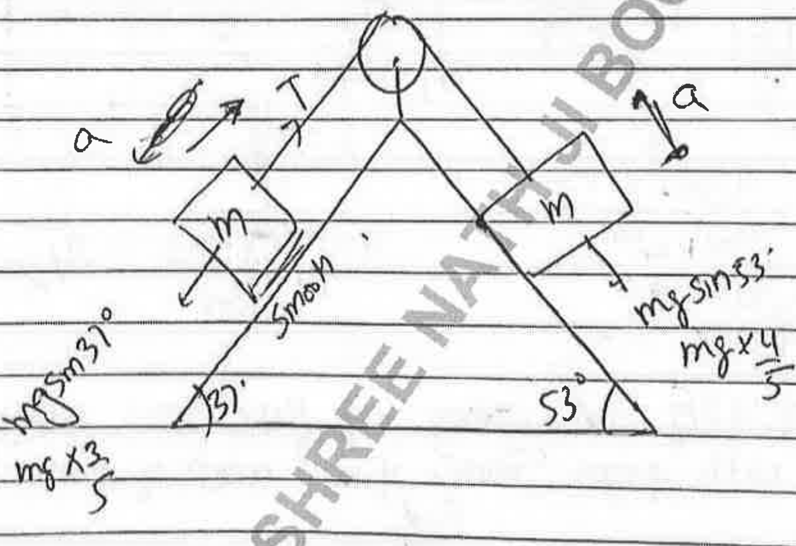
$$a = \frac{m_2 g - m_1 g\sin\theta}{m_1+m_2}$$

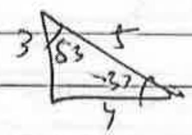
$$a = \frac{(m_2 - m_1\sin\theta)g}{m_1+m_2}$$

$$m_2 g - T = m_2\left(\frac{m_2 - m_1\sin\theta}{m_1+m_2}\right)g$$

$$T = m_2 g - m_2\left(\frac{m_2 - m_1\sin\theta}{m_1+m_2}\right)g$$

$T =$





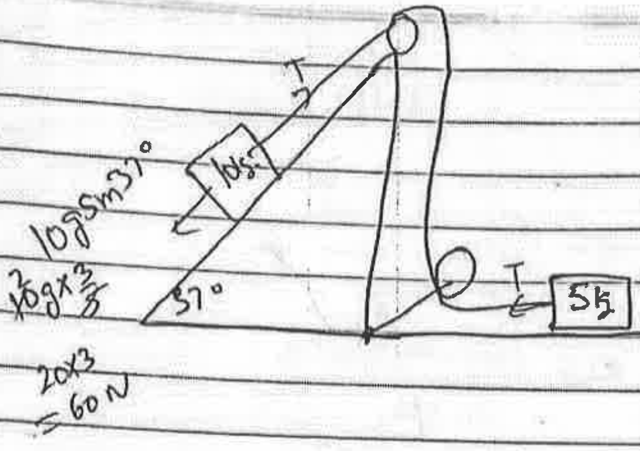
$$a = \frac{\frac{4mg}{5} - \frac{3mg}{5}}{2m} = \frac{mg}{5\times 2m} = 1\,m/s^2 = \frac{g}{10}$$

$T - \frac{3}{5}mg = m\times 1$

~~$T = \frac{3}{5}mg + m$~~

$$T - mg\frac{3}{5} = \frac{mg}{10}$$

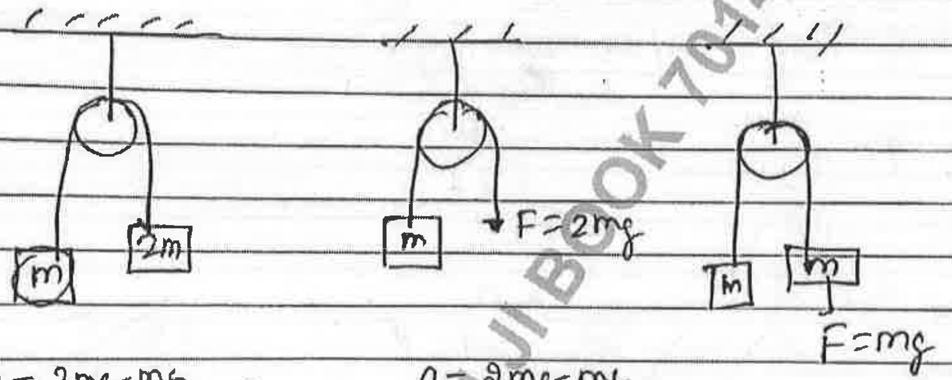
$$T = \frac{3}{5}mg + \frac{mg}{10} = \frac{7mg}{10}$$



$a = \frac{60}{15}$ $4 m/s^2$

$T = 5 \times 4 = 20$ N

Q. Find accln in Given diagram.



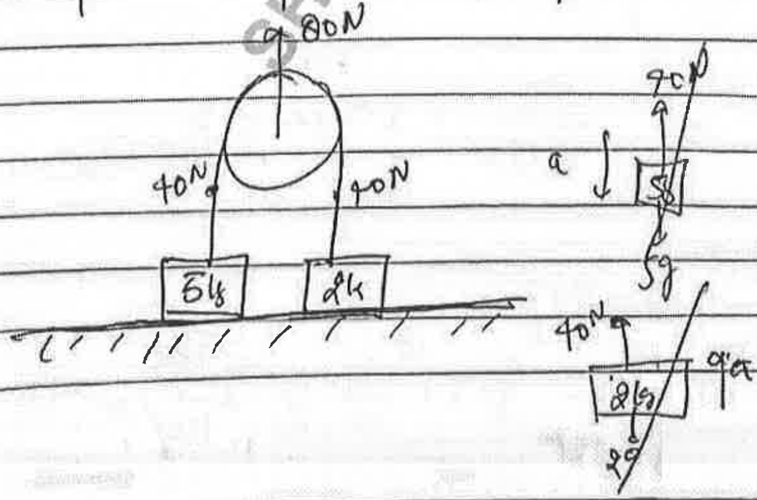
$a = \frac{2mg - mg}{3m} = g/3$

$a = \frac{2mg - mg}{m}$

$a = g$

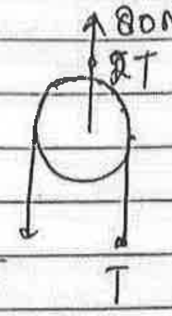
$a = \frac{2mg - mg}{2m} = g/2$

Q. Two blocks of mass 5kg & 2kg are resting on floor now pulley is pulled upward with force 80N. find acc^n of Both block.



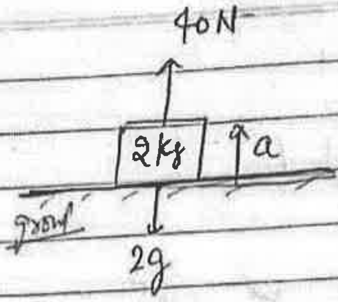
$a = \frac{55 - 40}{5} = \frac{10}{5} = 2 m/s^2$

$a_{2kg} = 10\ m/s^2$ $\quad a_{5kg} = 0$



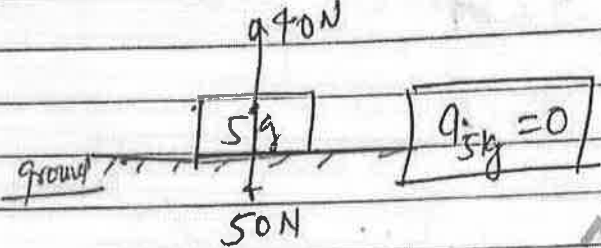
$2T = 80$

$\underline{T = 40N}$



$40 - 20 = 2a$

$\frac{20}{2} = a = \underline{10\ m/s^2}$

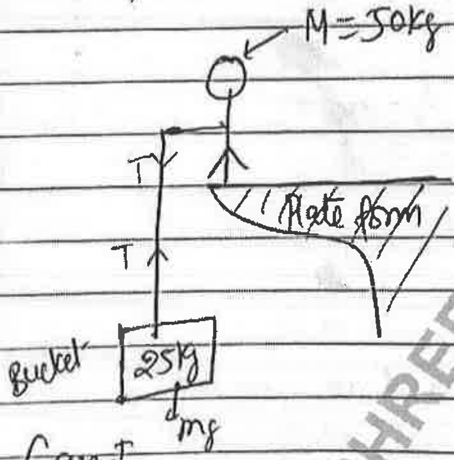


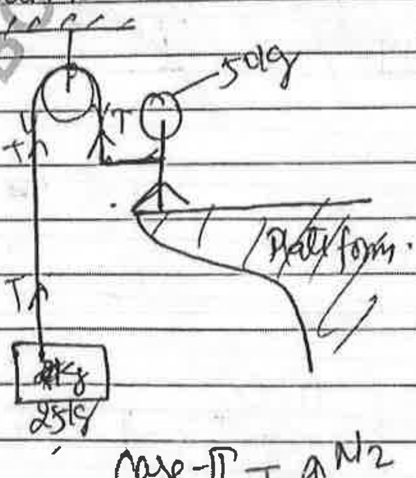
$\boxed{a_{5kg} = 0}$

NCERT

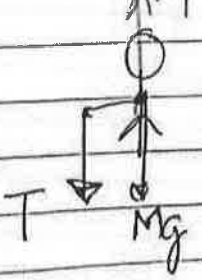
Find Force exerted by plateform on the man in Both case.

→ If the platform can with stand a force of 700 N, find which case is suitable.





Case-I



$N_1 = T + Mg$

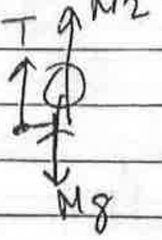
$= mg + Mg$

$= 75g$

$N_1 = 750N$

Not suitable because it is more than bearing strength

Case-II



$T + N_2 = Mg$

$N_2 = Mg - mg$

$N_2 = 250\ N$

Suitable.

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→ जी accl^n of block is d/F.

Movable Pulley (constrain motion)

↳ "Net work done by Tension is Zero" $\boxed{\vec{T}\cdot\vec{x} = \text{work} = 0}$

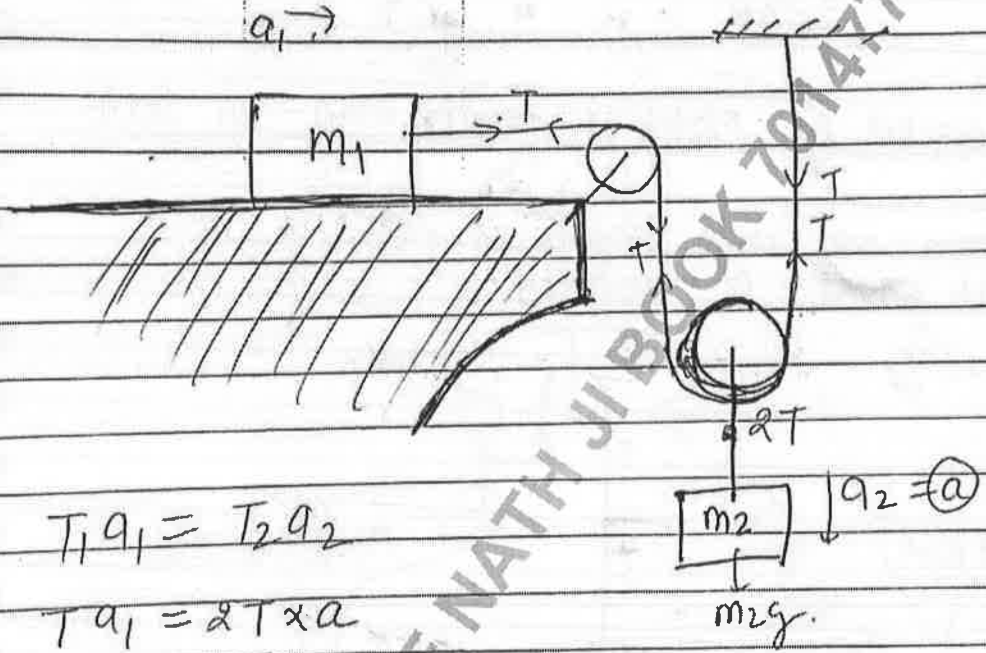
$$\left.\begin{aligned}\sum \vec{T}\cdot\vec{x} &= 0\\ \sum \vec{T}\cdot\vec{v} &= 0\\ \sum \vec{T}\cdot\vec{a} &= 0\end{aligned}\right]$$

Two blocks

$T_1x_1 = T_2x_2$

$T_1v_1 = T_2v_2$

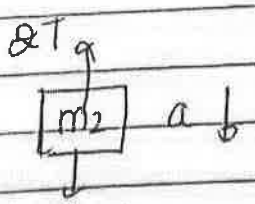
$\boxed{T_1a_1 = T_2a_2}$ *



$T_1a_1 = T_2a_2$

$Ta_1 = 2T \times a$

$\boxed{a_1 = 2a}$ (1)



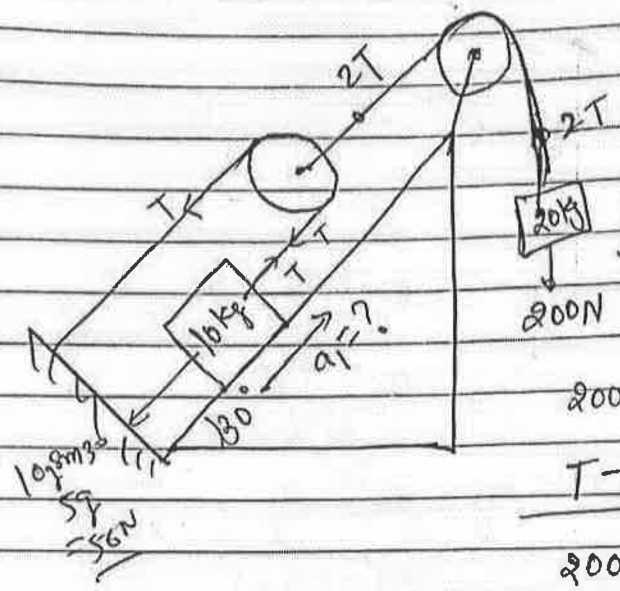
$m_2g - 2T = m_2a$ (1)

$T - 0 = m_1(2a)$ (11)

(1) + (11)

$$a = \frac{m_2g}{m_1+m_2}$$

Ex(I) → 71-76, 78-84, 86, 88-90, 95, 96
Ex(II) 18, 20, 19, 25*
Ex(III) Race-I & II



$T_1 a_1 = T_2 a_2$

$T a_1 = 2T a$

$\underline{a_1 = 2a}$

$200 - 2T = 20a$ (I)

$T - 50 = 10 \times (2a)$ (II)

$200 - 2T = T - 50$

$250 = 3T$

$T = \frac{250}{3}$ put in 1st.

$200 - 2\left(\frac{250}{3}\right) = 20a$

$\frac{600 - 500}{3} = 20a$

$\frac{5\cancel{100}}{3} = \cancel{20} a$

$a = 5/3 \rightarrow$ acc^n of 20kg

$a_1 = \frac{10}{3} \rightarrow$ acc^n of 10kg

$\underline{a_1 = (2a)}$

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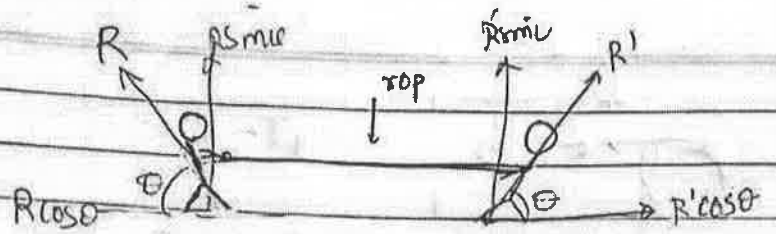
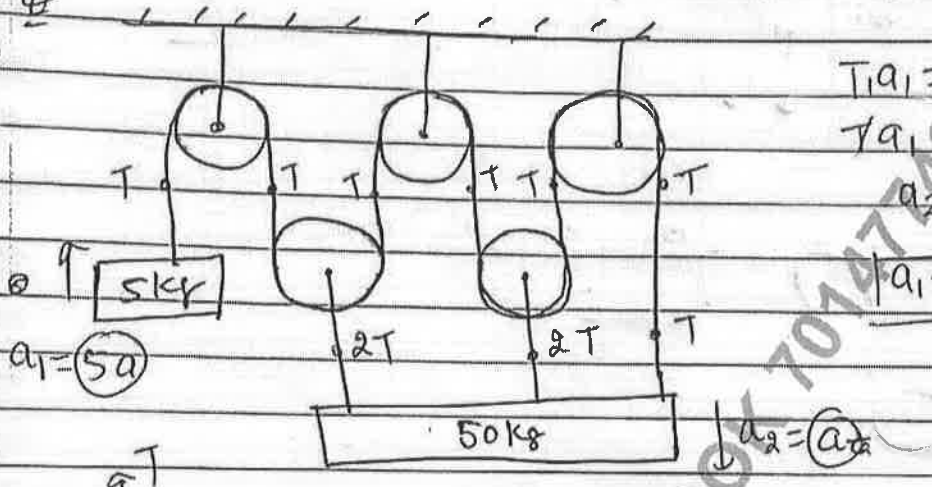
Q



Q

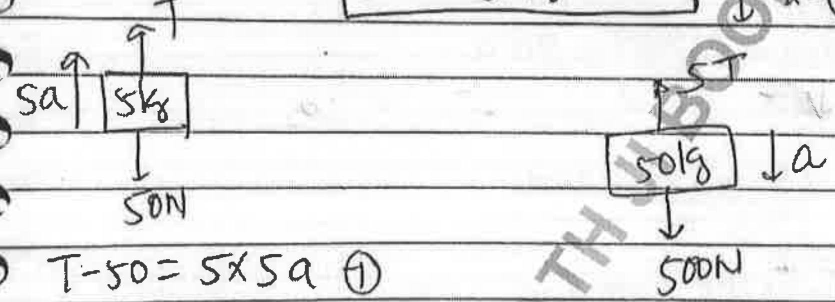


$T_1 a_1 = T_2 a_2$

$T a_1 = 5T \cdot a_2$

$a_2 = \frac{a_1}{5}$

$\boxed{a_1 = 5a_2}$

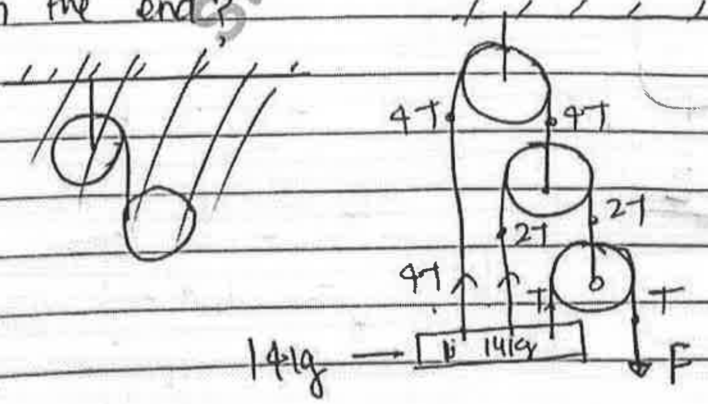


$500 - 5T = 50a$ — (2)

$T - 50 = 5 \times 5a$ (1)

solve eq (1) & (2) $a = \frac{10}{7}\ m/s^2$

Q To keep the block in eqb^m what force should be applied on the end?



$7T = mg$

$7T = 14g$

$T = \frac{14}{7}g$

$T = 2g$

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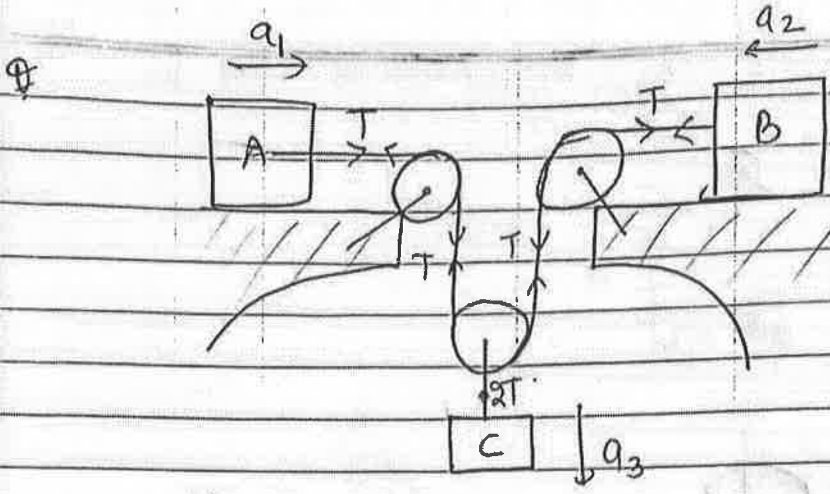
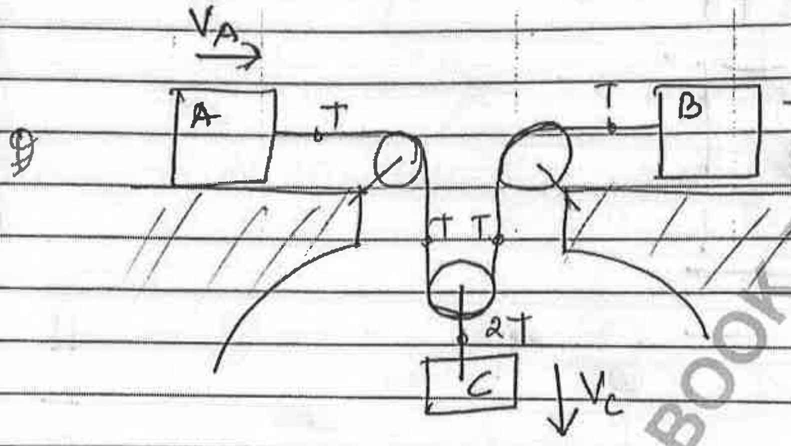


$$a_1T + a_2T = 2Ta_3$$

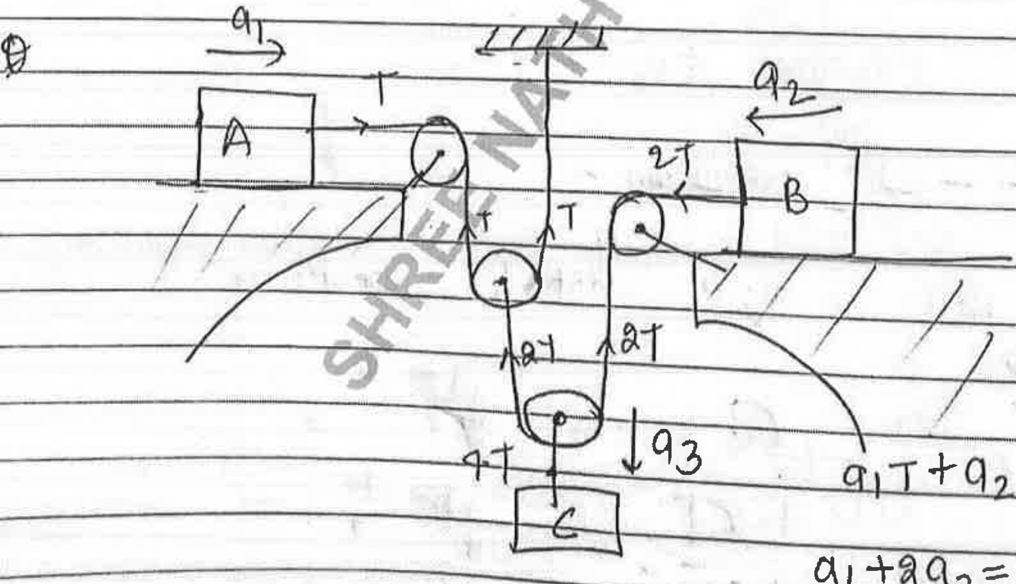
$$a_1 + a_2 = 2a_3$$

$$a_3 = \frac{a_1 + a_2}{2}$$



$$TV_A - TV_B = 2T\cdot V_C$$

$$V_C = \frac{V_A - V_B}{2}$$



$$a_1T + a_2\cdot 2T - 4Ta_3 = 0$$

$$a_1 + 2a_2 = 4a_3$$

$$a_3 = \frac{a_1 + 2a_2}{4}$$

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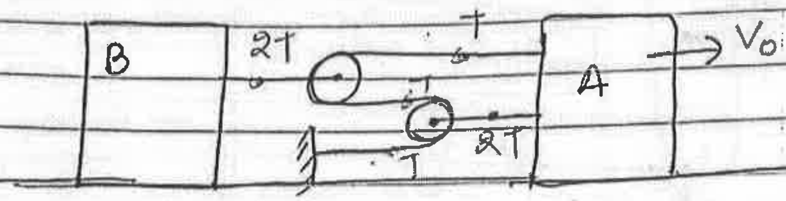
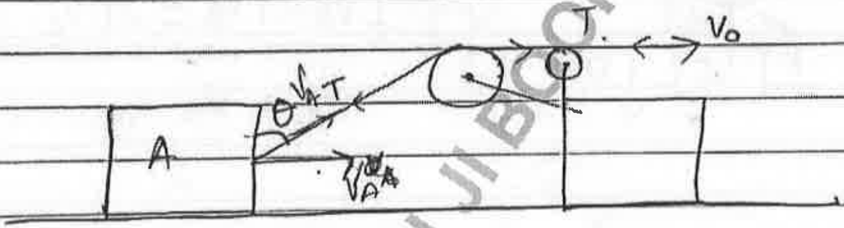
Q If speed of block A is V_0 then speed of B will be



$$2\cancel{T}\cdot V_B = 3\cancel{T}\cdot V_0$$

$$\boxed{V_B = \frac{3}{2}V_0}$$

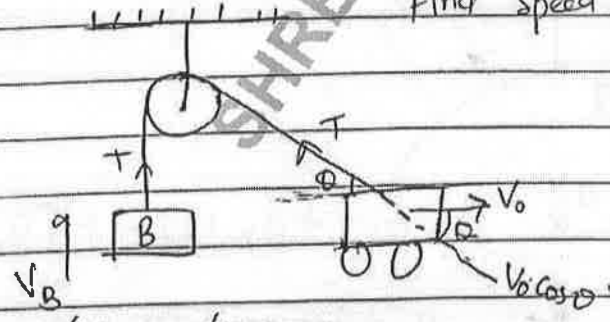
Q Find speed of block A if string is pulled with V_0 ?



$$\cancel{T}V_A \sin\theta = \cancel{T}V_0$$

$$V_A = \frac{V_0}{\sin\theta}$$

Q Find speed of Block 'B' in given situation,



$$\cancel{T}V_B = \cancel{T}V_0\cos\theta$$

$$\boxed{V_B = V_0\cos\theta}$$

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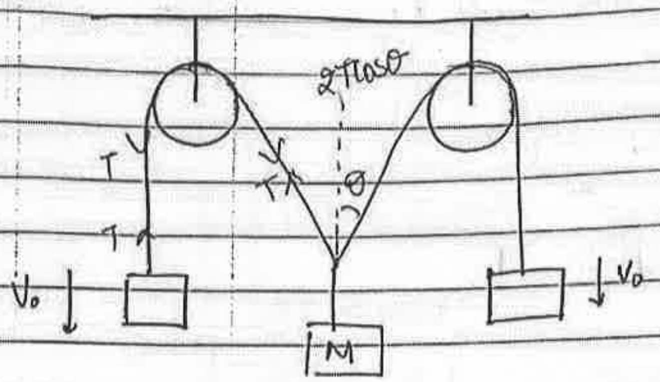
*

Q Find speed of 'M'

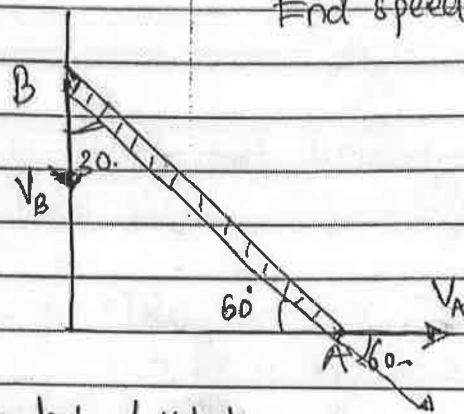


$-2TV_0 + 2T\cos\theta . V_m = 0$

$V_0 = V_m \cos\theta$

$V_M = \frac{V_0}{\cos\theta}$

Q Find speed of end 'B' in the given situation?



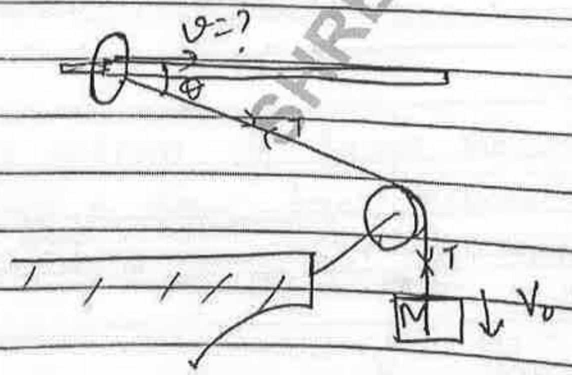
$V_B \cos 30 = V_A \cos 60$

$V_B \times \frac{\sqrt{3}}{2} = 10 \times \frac{1}{2}$

$V_B = \frac{10}{\sqrt{3}}$ m/s

Component of velocity are along the straight, are always equal.

Q The ring is sliding in the rod find speed of ring in given situation.



$v\cos\theta = V_0$

$v = \frac{V_0}{\cos\theta}$

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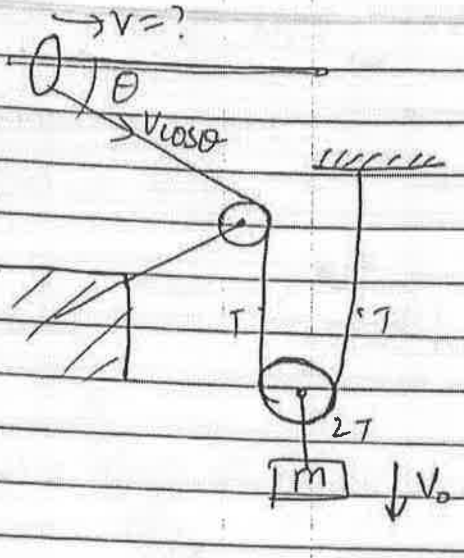
Q



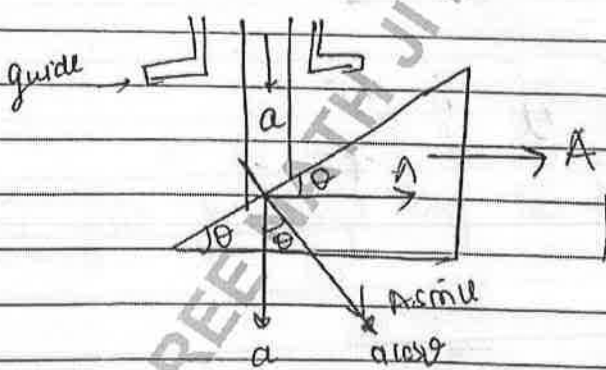
$\cancel{f}_0 v\cos\theta = V_0 (2\cancel{f})$

$v\cos\theta = 2V_0$

$$\boxed{V = \frac{2V_0}{\cos\theta}}$$

⟹ If two blocks are moving together in contact then values of velocity and accl^n ⊥ to contact surface will be same.

⟹ If rod is falling down with accl^n 'a' and accl^n of wedge is 'A' find relation b/w A and a?

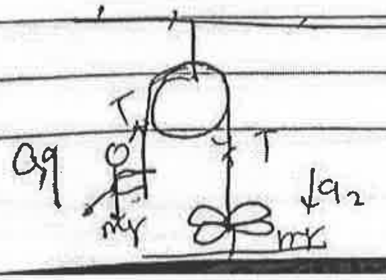


$$\boxed{A\sin\theta = a\cos\theta}$$

$a = A\tan\theta$

Ex-(9)

Q Mass of Monkey and bunch are equal. If monkey start climbing then separation b/w monkey and bunch is remains const, because both have same accl^n & same dir^n.



$T - mg = ma_1$ —(I)

$mg - T = ma_2$ —(II)

$ma_1 = ma_2$ ⟶ $a_1 = a_2$ ⟶ 0

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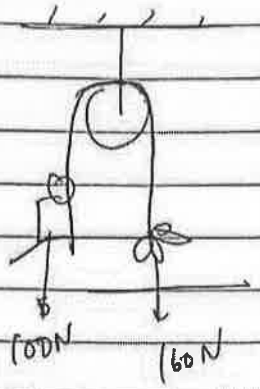
Q Mass of Banana bunch is 16 kg and that of monkey 12 kg. bunch is resting on floor. Find the max^m acc^n of Monkey it can climb without lifting the bunch?

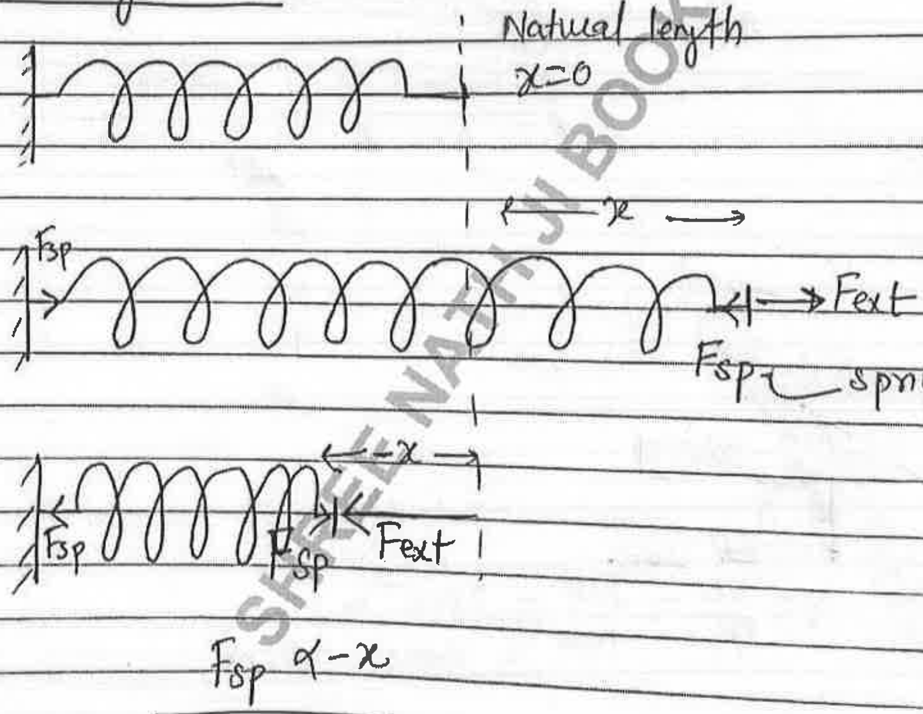


$$T_{max} - 120 = 12\, a_{max}$$

$$160 - 120 = 12\, a_{max}$$

$$a_{max} = \frac{10}{3}$$

Spring Force :



Fsp — spring force

$F_{sp} \propto -x$

$$\boxed{F_{sp} = -kx}$$

K = Spring const N/m

$K \propto \frac{1}{l}$ K↑ stiffness ज्यादा.

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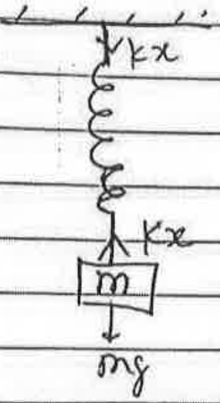
Steady state ⇒ max^m elongation 42

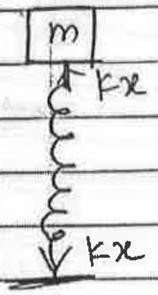
$kx \rightarrow$

In Eqb^m, $|\vec{F}_{ext}| = |\vec{F}_{sp}| = kx$

$$\vec{F}_{ext} = -\vec{F}_{sp}$$

Steady state



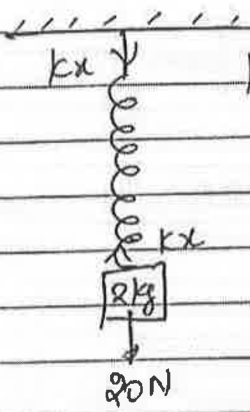


In Eqb^m

$$\boxed{kx = mg}$$

In Eqb^m

$$\boxed{kx = mg}$$



$k = 1000\ N/m$

what is extension.

$kx = 20$

$$x = \frac{20}{k} = \frac{200}{1000} = 2\times10^{-2}\ m$$

$$= \cancel{2\times10} = 2\,cm$$

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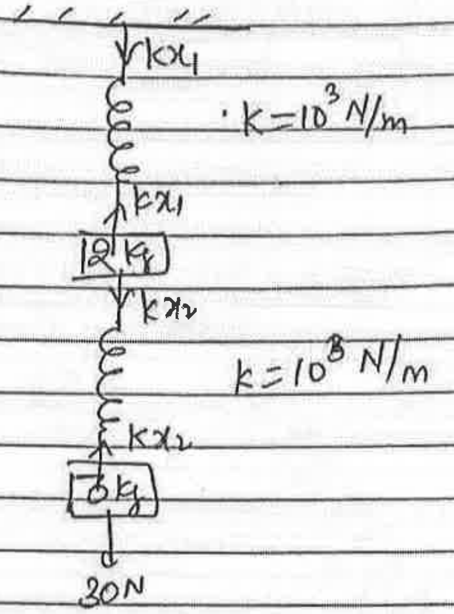
Steady state

extension = ?



$kx_1 = 50$ N

$x_1 = \frac{50}{10^3} = 5 \times 10^{-2}$ m

$= \underline{5 \text{ cm}}$

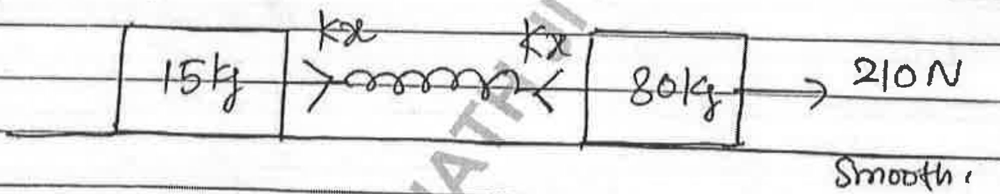
$kx_2 = 30$

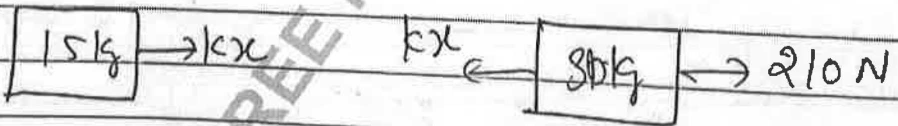
$x_2 = \frac{30}{1000} = 3 \times 10^{-2}$ m

$= \underline{3 \text{ cm}}$

unsteady state

Q. In the given fig. when accl^n of 15 kg is $\underline{6 \text{ m/s}^2}$. Find accl^n of 80 kg.





$kx - 0 = 15 \times 6$

$kx = 90$ N

$210 - kx = 30 \times a$

$210 - 90 = 30 \times a$

$\frac{120}{30} = a$

$\underline{a = 4 \text{ m/s}^2}$

diff accl^n ✓

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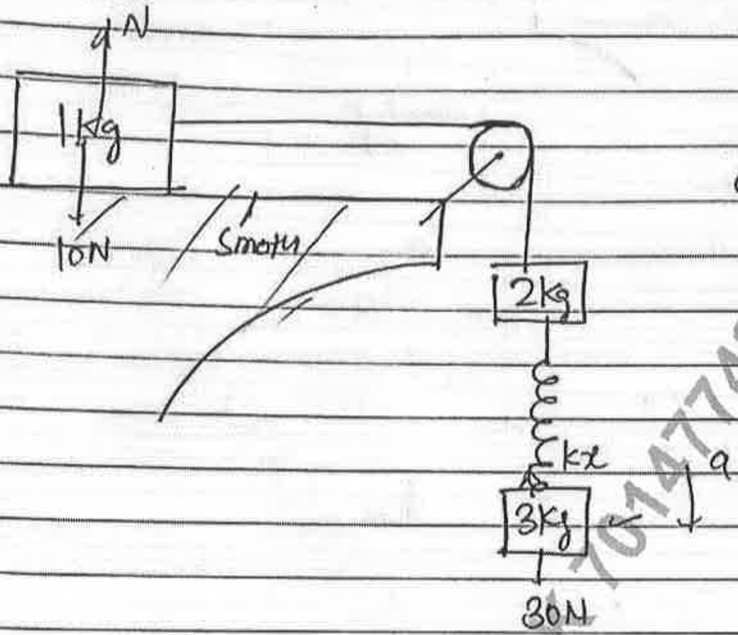
Q Find extension in spring in steady state (Max^m elongation).

$k = 50\ N/m$



$a = \frac{F_{ext}}{T.mass} = \frac{(2+3)g}{6}$

$= \frac{5g}{6} = \frac{50}{6}$

$30 - kx = \cancel{3} \times \frac{\cancel{50}\ 25}{\cancel{6}\ 2}$

$kx = 5$

$x = \frac{5}{50} = \frac{1}{10}\ m$

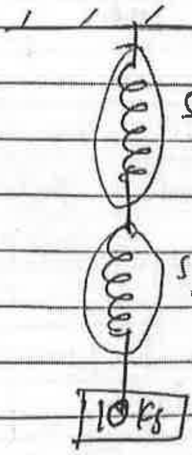
$x = \frac{1}{10} \times 100 = \underline{10\ cm}$

Spring Balance → measure 'Tension'

Light spring ⇒

$kx = mg = 100\ N$

Reading = $\underline{10\ kg\text{-}wt}$



$S_1 = 10\ kg\text{-}wt$

$S_2 = 10\ kg\text{-}wt$

(∵ light spring)

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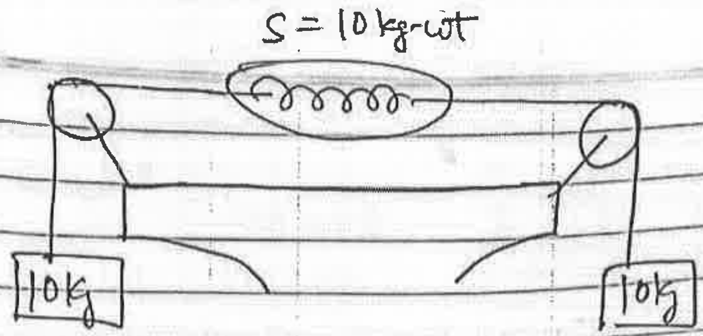
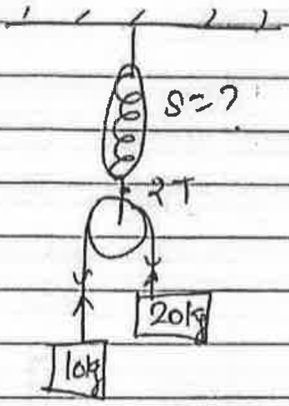




$$T = \left(\frac{2m_1m_2}{m_1+m_2}\right)g$$

$$= \left(\frac{2\times10\times20}{10+20}\right)10$$

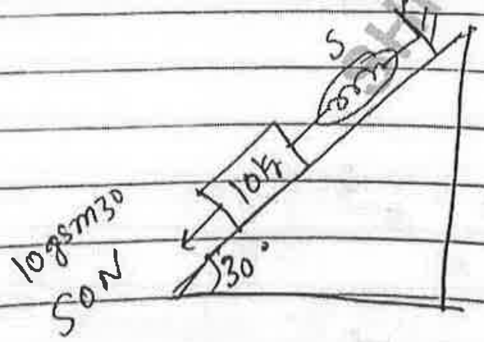
$$T = \frac{400}{3}\ N$$

Spring Balance $T' = 2T$

$$T' = \frac{800}{3}\ N$$

$$\text{Reading} = \frac{80}{3}\ \text{kg-wt}$$

$$= \underline{26.6\ \text{kg-wt}}$$



~~kx = 50N~~

$$\text{Reding} = \frac{50}{10}\ \text{kgwt}$$

$$= \underline{5\ \text{kg-wt}}$$

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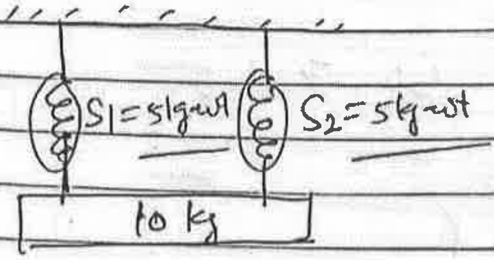
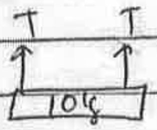
$\left[Kx \equiv T \right]$

Parallel



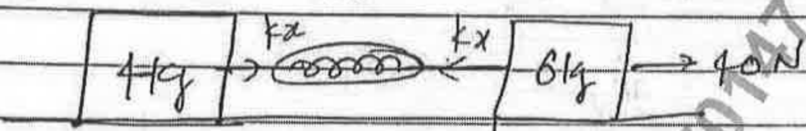


$2T = 100$

$T = 50N$

Readig ⇒ 5 kg-wt

Q Find reading of dynamometer in steady state.
dyna. ⟶ horizontal



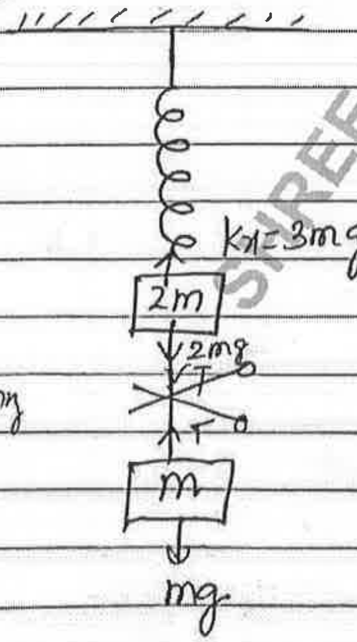
$a = \frac{40}{10} = 4 m/s^2$ $\qquad kx = 4 \times 4 = 16 N$

Readig = 1.6 kg-wt

Cutting of Spring → [cut ⇒ T = 0, kx = 0]

Neet

When the string b/w m & 2m is cut Find acc^n of the blocks at exactly at this instant of time?



✓cut → T = 0 or kx = 0

upwards $\quad a = \frac{3mg - 2mg}{2m}$

$a = g/2$ upwards

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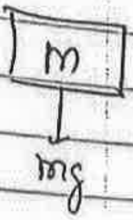
B.B ⇒ 334 Cov
Ex-I (85,
Ex-II (25, 27, 30, 22*,
Ex-III (10)

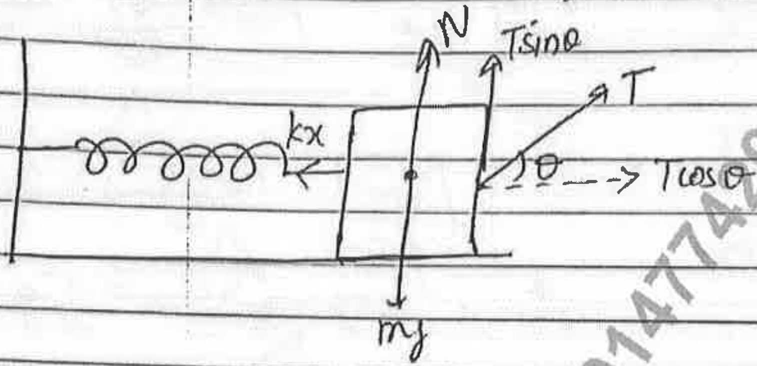
Page =



$a = g$ (downwards)

$\left\{ \text{Cut} \Rightarrow \underline{T=0} \text{ or } \underline{kx=0} \right\}$

B.B.4
last Que.



$N = mg - T\sin\theta$

No contact $N = 0$

$mg = T\sin\theta$

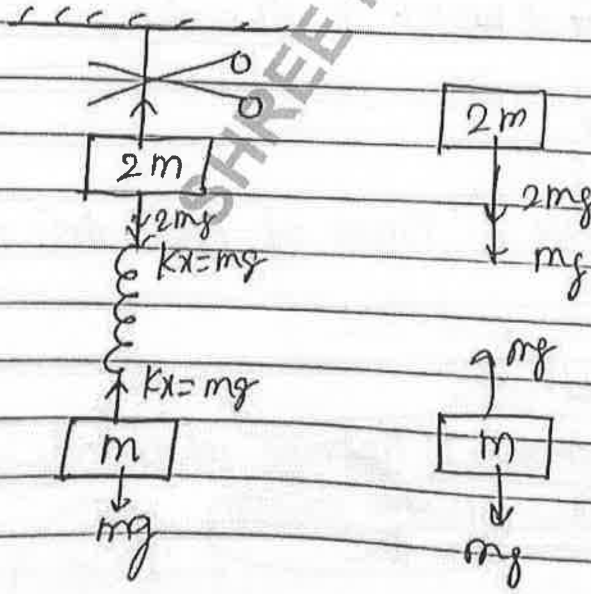
$T = \dfrac{mg}{\sin\theta}$ ✓

$T\cos\theta = kx$

$x = \dfrac{T\cos\theta}{k} = \dfrac{mg}{\sin\theta} \times \dfrac{\cos\theta}{k}$

$= \dfrac{mg}{k}\cot\theta$ ✓

Q At the time of cutting (Just after) Find acc/n of the blocks.



$3mg - 0 = 2m\,a_{(2m)}$

$\dfrac{3g}{2} = a_{(2m)}$, downwards.

$a_m = \dfrac{mg - mg}{m} = 0$

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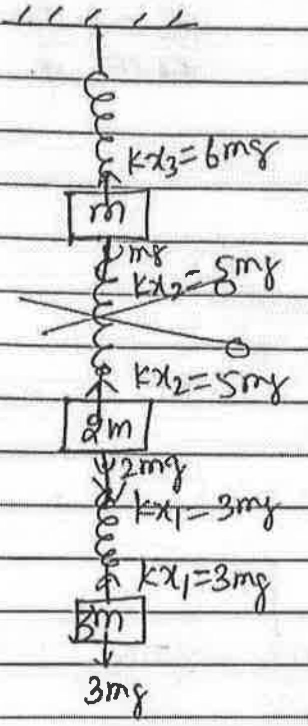
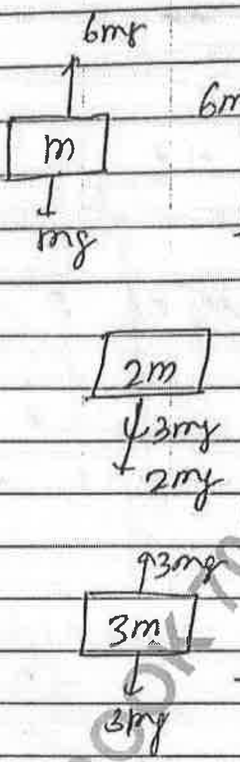
Q. If the given spring is cut f/o acc^n of each Block?





$6mg - mg = m a_m$

$\boxed{a_m = 5g}$ upwards

$5mg + 0 = 2m\, a_{(2m)}$

$a_{2m} = \frac{5}{2} g = 2.5g$

downwards.

$a_{3m} = 0$

Frame of Reference

It is a system respect to which motion of a particle is Studied. (Coordinate system + watch + observer)

Types of Frame of refrence

(I) Inertial frame of reference : Frame at rest or in uniform motion. $\vec{V} = const$

Laws of motion are valid.

(II) Non-inertial frame of reference : A frame which is in accelerated motion or Rotational motion.

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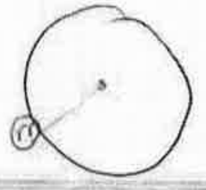
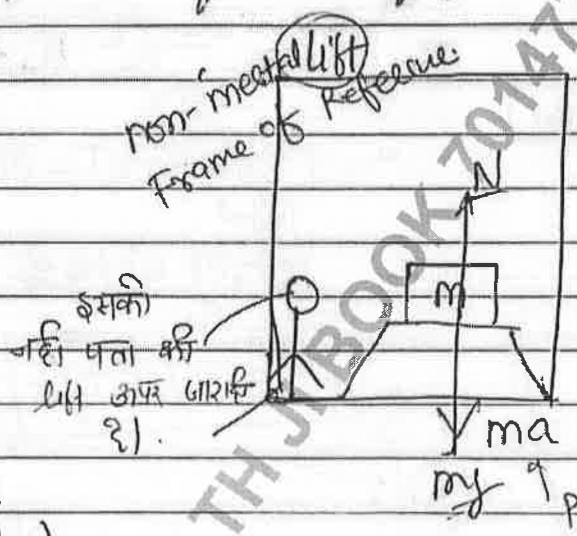


Pseudoforce: It is a False force or fictitious force it is considered when we have to apply laws of Motion in Non-inertial frame. Means it is correction.
→ observer को "Pseudoforce" ऐसा मालूम है।

→ Does not constitute Action-Rxn pair

→ dir^n ⇒ opp. to acc^n of Frame.
magnitude ⇒ product of mass of particle & acc^n of Frame.

*

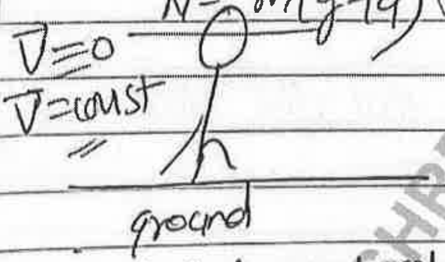


$\uparrow a$

Pseudo $\downarrow$

$N_2 = mg + ma$
$= m(g+a)$

$N - mg = ma$
$N = m(g+a)$ ✓



inertial frame of reference
NLM ✓

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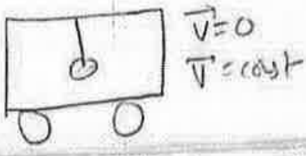
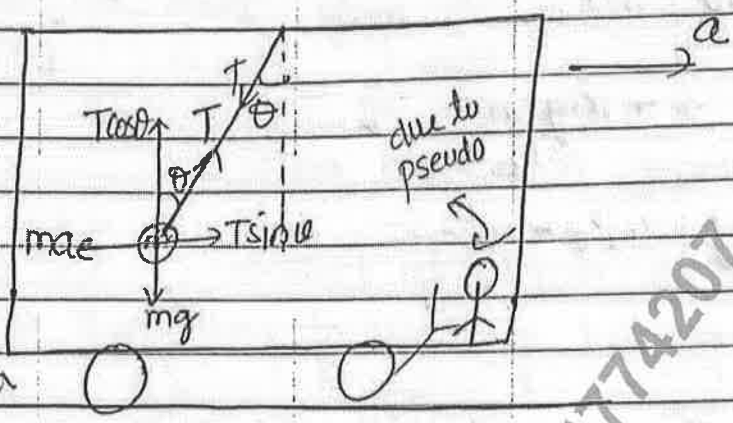


*What is the angle substended by string from vertical & Tension.? in Eqb^m?



ground frame

$T\sin\theta - 0 = ma$

$T\cos\theta = mg$

due to Inertia

Car frame (non-inertial)

$T\sin\theta = ma$

$T\cos\theta = mg$

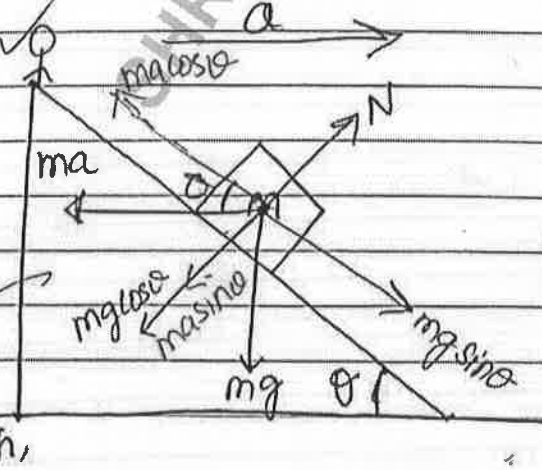
$$\boxed{\begin{aligned} \tan\theta &= \frac{a}{g} \\ \theta &= \tan^{-1}(a/g) \end{aligned}}$$

$T^2(\sin^2\theta + \cos^2\theta) = (ma)^2 + (mg)^2$

$T = m\sqrt{a^2+g^2}$

$$\boxed{T = m\sqrt{a^2+g^2}}$$

Q. What should be the acel^n of wedge so that the block will remains rest relative to wedge.



$ma\cos\theta = mg\sin\theta$

$\boxed{a = g\tan\theta}$

Neet

Normal Force on block

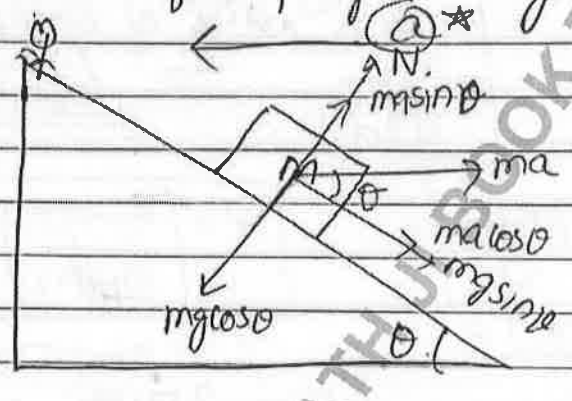
$N = mg\cos\theta + ma\sin\theta$

$$\boxed{N = \frac{mg}{\cos\theta} = mg\sec\theta}$$

$N = mg\cos\theta + mg\frac{\sin\theta}{\cos\theta}\sin\theta$

$N = mg\left[\frac{\cos^2\theta + \sin^2\theta}{\cos\theta}\right]$

* in Previous Question what should be accl^n of wedge so that blocks fall freely vertically downwards. ($N=0$)



$N = mg\cos\theta - ma\sin\theta$

block leaves the surface $N=0$

$0 = mg\cos\theta - ma\sin\theta$

$\cancel{m}g\cos\theta = \cancel{m}a\sin\theta$

$$\boxed{a = g\cot\theta}$$

min

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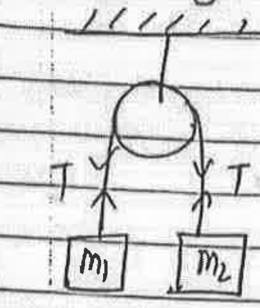
BB-5/2 Ex-I (19, 61, 62, 64, 65, 66, 70)
Ex-II (28)
Ex-III (6, 12)

Stationary or uniform motion of lift

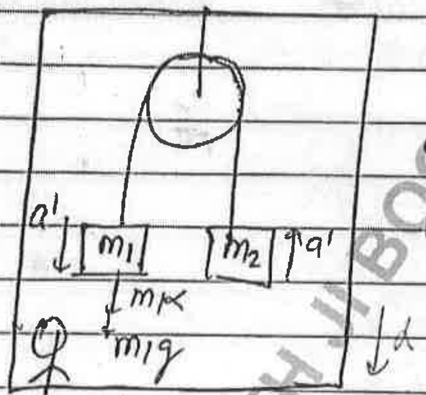


$m_1 > m_2$

$$a = \frac{(m_1 - m_2)g}{m_1 + m_2}$$

$$T = \frac{2m_1m_2}{m_1+m_2} g$$

Lift accelerating upwards with 'α'



Accl^n of Blocks relative to lift *

$$\boxed{a' = \left(\frac{m_1 - m_2}{m_1+m_2}\right)(g+\alpha)}$$ $(g-\alpha)$

$$\boxed{T = \frac{2m_1m_2}{m_1+m_2}(g+\alpha)}$$ $(g-\alpha)$

generally NOT asked
Rarer Tough

From ground $\Rightarrow$ accl^n of Both block is d/F

$$\vec{a}_{bL} = \vec{a}_b - \vec{a}_L$$

$$\vec{a}_b = \vec{a}_{bL} + \vec{a}_L$$

T $\Rightarrow$ same
only accl^n d/F आएगा।

$m_1 \Rightarrow$

$$\vec{a}_b = a'(-\hat{j}) + \alpha\hat{j}$$

$$\vec{a}_b = (\alpha - a')\hat{j}$$

$m_2 \Rightarrow$

$$\vec{a}_b = a'\hat{j} + \alpha\hat{j}$$

$$= (a' + \alpha)\hat{j}$$

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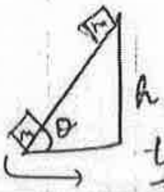
$V = \sqrt{2gh}$ ✓ $t = \sqrt{\frac{2h}{g}}$



$v = \sqrt{2gh}$ — $t_{in} = \frac{1}{\sin\theta}\sqrt{\frac{2h}{g}}$

⇒ Speed will be same $\sqrt{2gh}$ in Both case ~~b/w~~ but time is diff

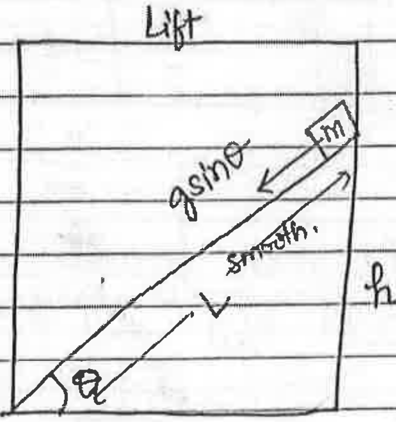
Q. A Small block of mass 'm' is placed on inclined plane of inclination 'θ' which is resting on the floor of lift. Find time taken by block to slid down when (Height of inclined place relative to floor h)

(I) Lift is at rest

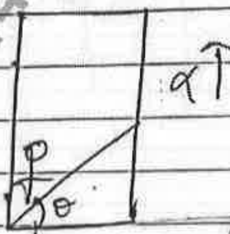
(II) Ascending with uniform acc^n 'α'

(III) dscending " " " 'α' (α<g)

(i)



②

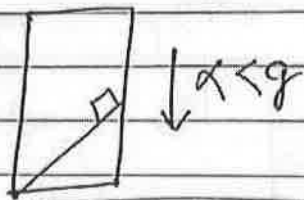


$$\boxed{t_2 = \frac{1}{\sin\theta}\sqrt{\frac{2h}{g+\alpha}}}$$

$S = ut + \frac{1}{2}at^2$

$L = 0 + \frac{1}{2} g\sin\theta\, t^2$

③



$\sin\theta = \frac{h}{L}$

$$\boxed{t_3 = \frac{1}{\sin\theta}\sqrt{\frac{2h}{g-\alpha}}}$$

$\frac{h}{\sin\theta} = 0 + \frac{1}{2} g\sin\theta\, t^2$

$$\boxed{t = \frac{1}{\sin\theta}\sqrt{\frac{2h}{g}}}$$

$t_{inclined} > t_{free\text{-}fall}$.

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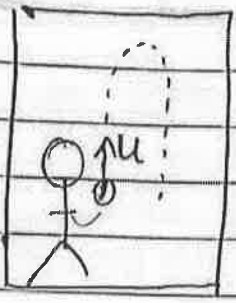
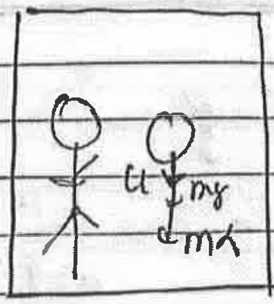


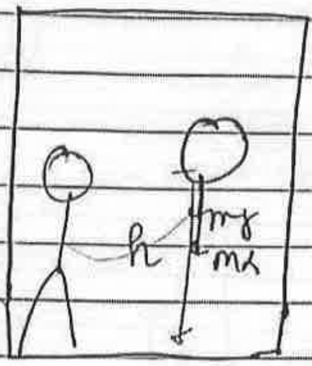
$\vec{v} = 0$ of lift

$T = \frac{2u}{g}$



$\uparrow \alpha$

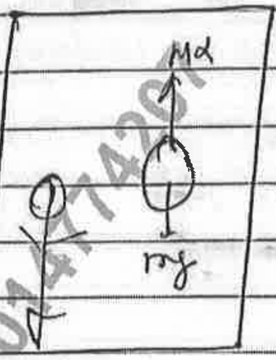
$T = \frac{2u}{g+\alpha}$



dropped.

$\uparrow \alpha$

$t = \sqrt{\frac{2h}{g+\alpha}}$

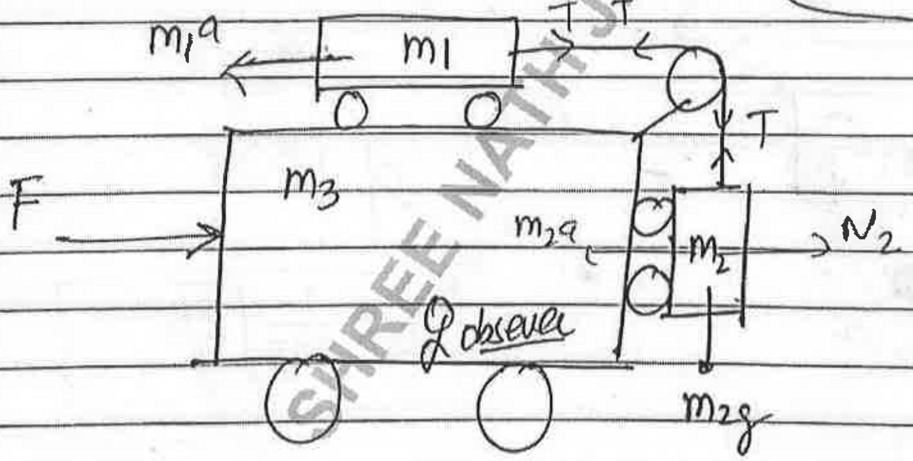


$\downarrow \alpha$

$t = \sqrt{\frac{2h}{g-\alpha}}$

Q. Find applied value of Force in the given figure so that There is NO Relative Motion b/w M_1 & M_3

Eqb^m में है।



$m_1 a = T$ — (i)

$m_2 g = T$ — (ii)

$m_1 a = m_2 g$

$a = \left(\frac{m_2}{m_1}\right) g$

$$F = (m_1 + m_2 + m_3)\, a$$

$$F = (m_1 + m_2 + M_3) \frac{m_2}{m_1} g$$

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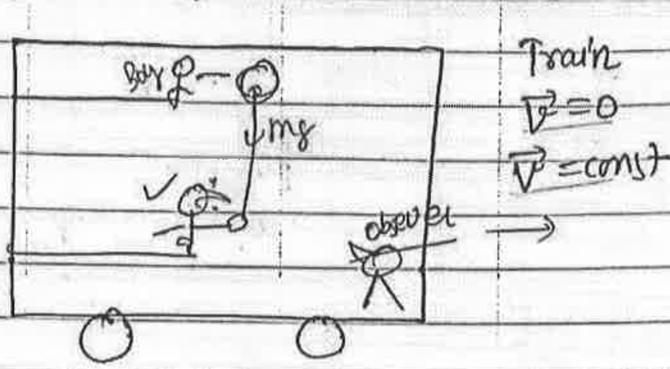
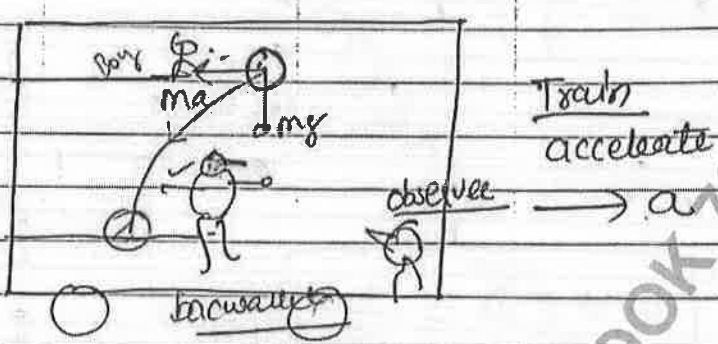
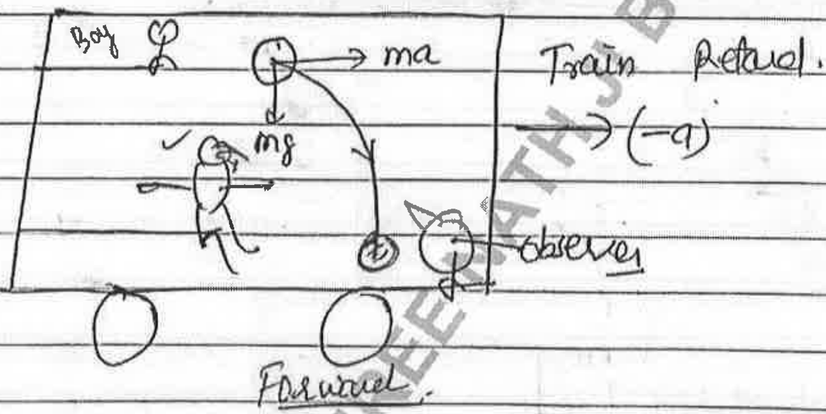
NCERT.

Face-in, direction of Motion.







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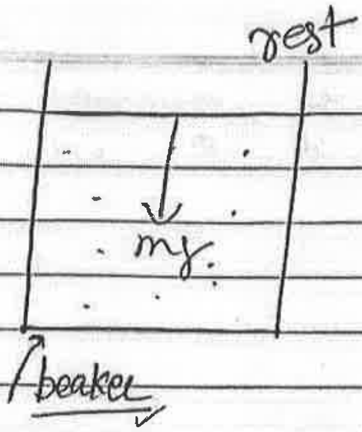
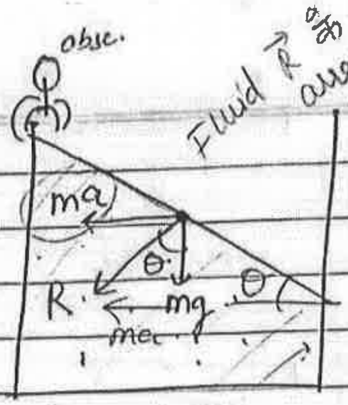
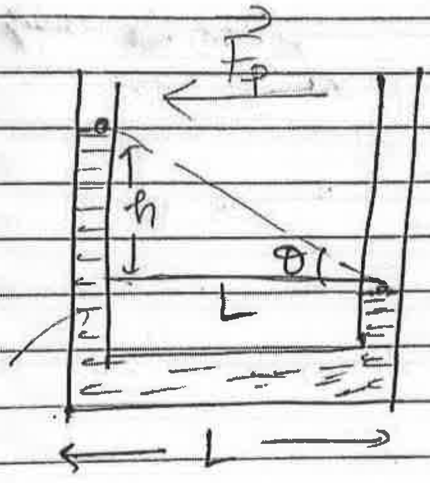
Fluid $\vec{R}$ को $\perp$ always ही चाहिए।





$$\tan\theta = \frac{ma}{mg}$$

$$\boxed{\tan\theta = \frac{a}{g}}$$

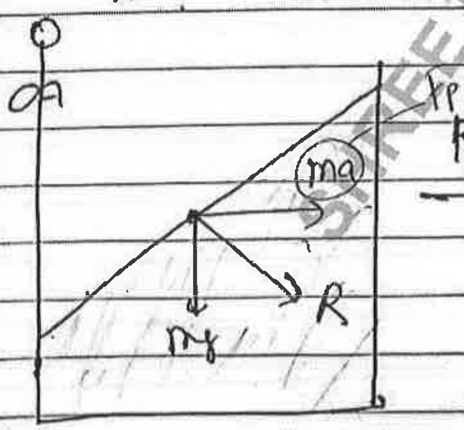


$\vec{a}$

$$\boxed{\tan\theta = \frac{h}{L} = \frac{a}{g}}$$

Level difference

$h = ?$



Retardation

$\longrightarrow$ $(-a)$

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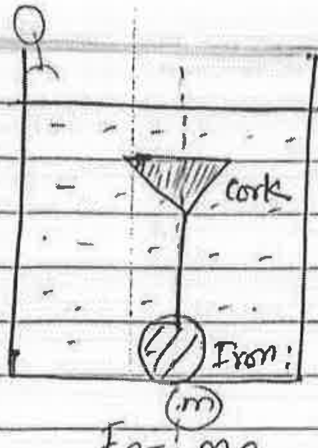
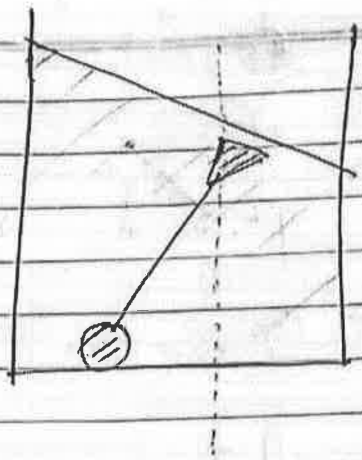
*





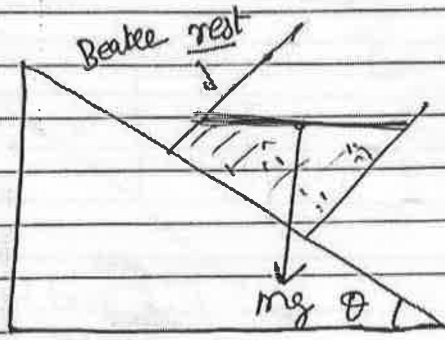
जिसकी density कम होगी, वो आगे shift

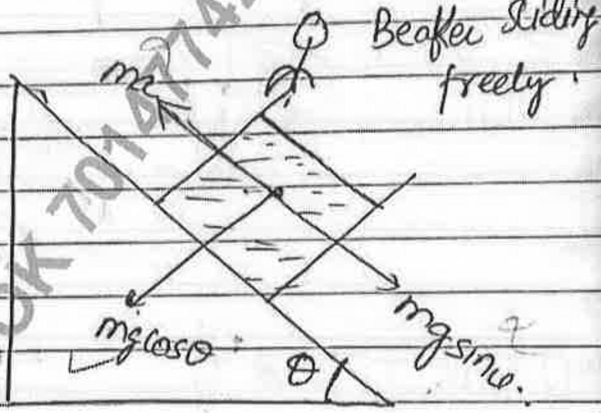
accl $\vec{a}$

$F_p = ma$

$m \uparrow \quad F_p \uparrow$

*



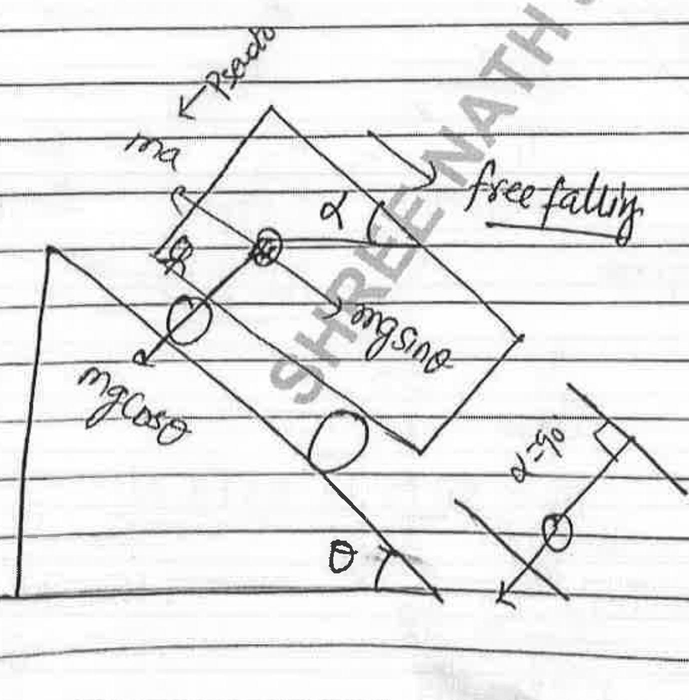


Beaker sliding freely,

$a = g\sin\theta$

$\underline{ma = mg\sin\theta}$

*
?



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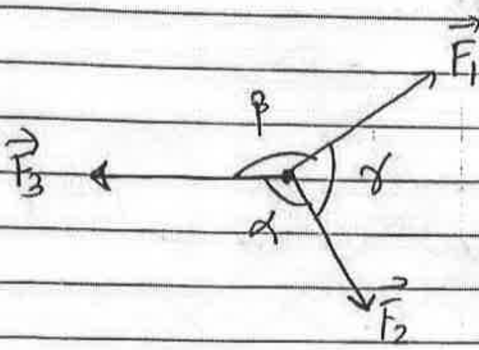
Translatory Eqb^m $F_{ext}(net) = 0$

$\vec{v} = 0$ $\vec{v} = const$

Lami's Theorm

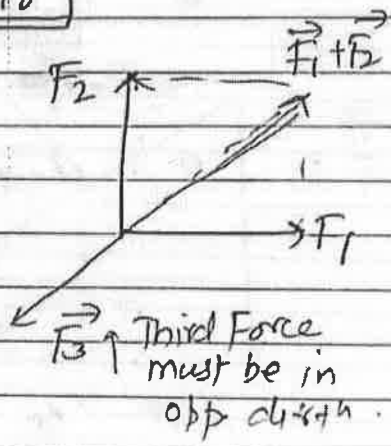
Three Concurrent Coplanar in Equilibrium



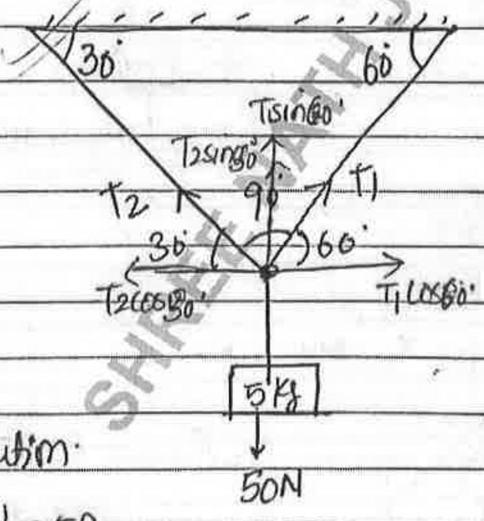
$$\boxed{\frac{F_1}{\sin\alpha} = \frac{F_2}{\sin\beta} = \frac{F_3}{\sin\gamma}}$$

$$\vec{F_1} + \vec{F_2} + \vec{F_3} = 0$$

$$\vec{F_1} + \vec{F_2} = -\vec{F_3}$$



Q. Find T_1 & T_2



M-I

$$\frac{T_1}{\sin(90+30)} = \frac{T_2}{\sin(90+60)} = \frac{50}{\sin 90°}$$

$$\frac{T_1}{\cos 30} = \frac{50}{1}$$

$$T_1 = 50\frac{\sqrt{3}}{2} = 25\sqrt{3}\ N$$

$$\frac{T_2}{\cos 60°} = 50$$

$$T_2 = 25N$$

method ②

Vector Resolution

$$T_1 \times \frac{\sqrt{3}}{2} + T_2 \times \frac{1}{2} = 50$$

$$T_1\sqrt{3} + T_2 = 100$$

$$\cancel{T_2 \times \frac{1}{2}} = T_1 \times \frac{1}{2}$$

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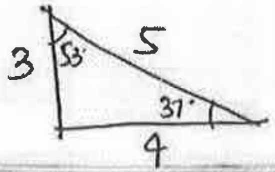

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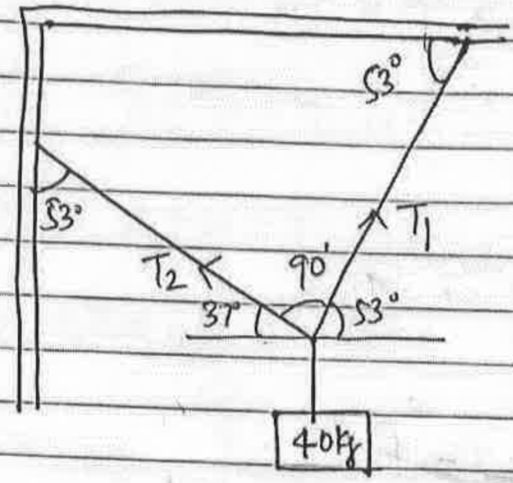


$T_1 = 320 \quad T_2 = 240$

$$\frac{T_1}{\sin(90+37)} = \frac{T_2}{\sin(90+53)} = \frac{400}{\sin(90)}$$

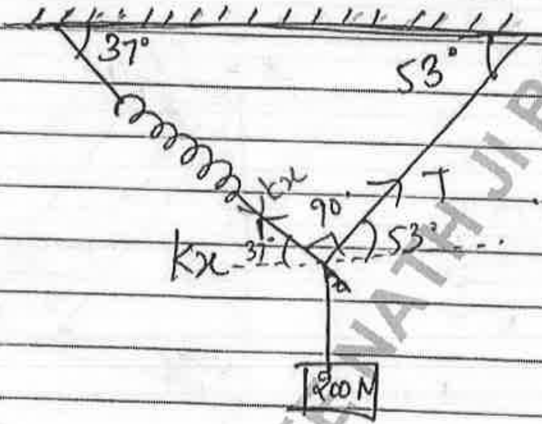
$$\frac{T_1}{\cos 37} = \frac{400}{1}$$

$$T_1 = \cancel{400}^{80} \times \frac{4}{\cancel{5}} = 320\,N$$

$$\frac{T_2}{\cos 53} = \frac{400}{1}$$

$$T_2 = \cancel{400}^{80} \times \frac{3}{\cancel{5}} = 240\,N$$

* If Extension in the spring is 4cm. f/o spring const.



$$\frac{kx}{\sin(90+53)} = \frac{T}{\sin(90+37)} = \frac{200}{\sin 90}$$

$$\frac{kx}{\cos 53} = \frac{200}{1}$$

$$kx = 200 \cos 53$$

$$= \cancel{200}^{40} \times \frac{3}{\cancel{5}}$$

$$kx = 120$$

$$k = \frac{120}{4/100} = 3000\,N/m$$

also,

T

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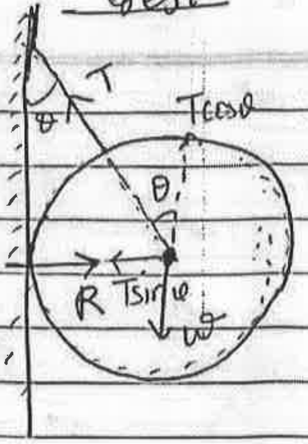
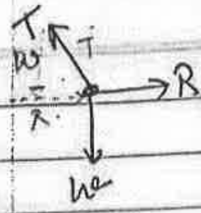
Tension Normal Rxn Weight

* System is at rest, Then correct relation b/w T & R & W





(a) $\vec{T} + \vec{W} + \vec{R} = 0$ ✓ Concurrent

(b) $T^2 = W^2 + R^2$ ✓

(c) $T = W + R$ ✗

(d) $R = W \tan\theta$ ✓

$T\sin\theta = R$

$T\cos\theta = W$

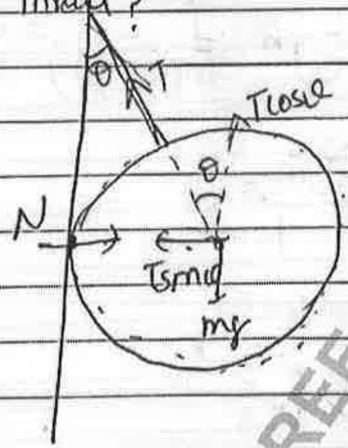
$T^2 = R^2 + W^2$

$\tan\theta = \frac{R}{W}$

$R = W\tan\theta$

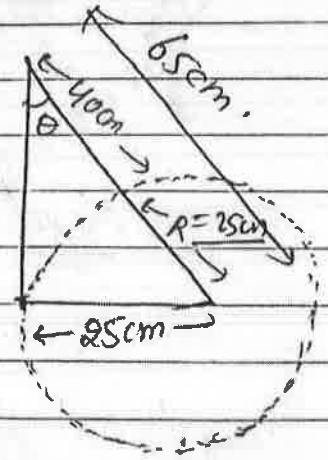
$\vec{T} = -(\vec{W} + \vec{R})$

* Mass of sphere 1kg, length of string 40 cm and radius of sphere 25cm. Find Tension in thread?

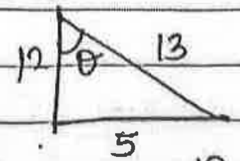


$T\cos\theta = mg$

$T = \frac{1 \times 10}{\cos\theta}$



$\sin\theta = \frac{25}{65} = \frac{5}{13}$



$\cos\theta = \frac{12}{13}$

$T = \frac{10}{\cos\theta} = \frac{10 \times 13}{12}$

$= \frac{130}{12} = \frac{65}{12}$

$= 10.8 N$ ✓

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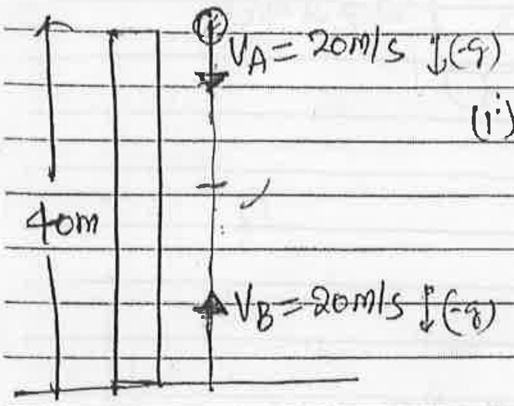
Date ______ Page ______

Q. Two particle A & B are simultaneously projected with same speed 20 m/s. Particle A vertically downwards from Top of a tower of height 40 m & 'B' vertically upwards from the foot of the Tower in same line. When & where they will collide.
(I) (II)

Sol^n (I)



(I) $\vec{a}_{rel} = 0$

↓

$\vec{S}_{rel} = \vec{u}_{rel} t + 0$

$40 = (20+20) t$

$\boxed{t = 1 sec}$

↑ (+)
↓ (−)

↓ g → alway downwards

$a_{rel} = \vec{a}_A - \vec{a}_B$

$= -g - (-g)$

$\vec{a}_{rel} = 0$

⇓

If Both particle motion under gravity Then

$\boxed{\vec{a}_{rel} = 0}$

(II) $S_A = u_A t + \frac{1}{2} g t^2$

$= 20 \times 1 + \frac{1}{2} \times 10 \times 1$

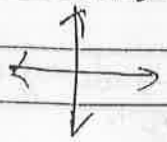
Ans
$= 25 m$ → from Top
or
15 m → height above ground.

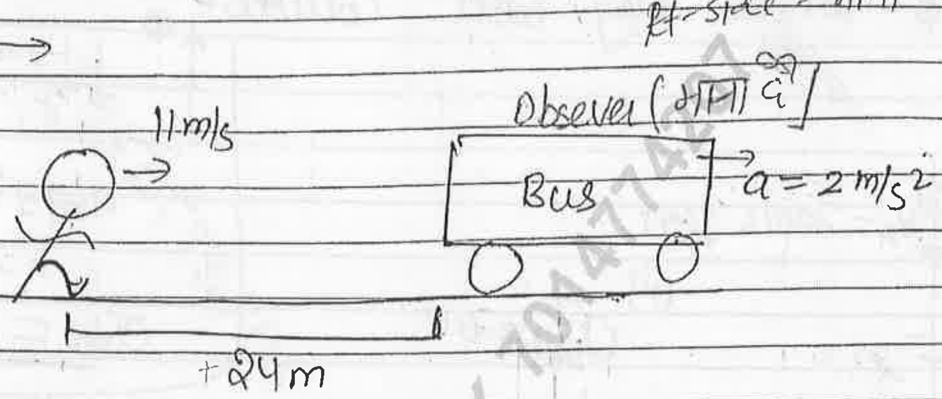
$\vec{V}_{rel} = \overrightarrow{\text{object}} - \overrightarrow{\text{Observer}}$

$a_{rel} \Rightarrow$ acc^n.

Q. A man is standing 24 m distance behind a bus on a straight road. Both starts ~~no~~ their motion simulatenously Bus with acc^n $2 m/s^2$ and the man with speed ~~Dm~~ 11 m/s. How long it will take for the man to over take Bus.

Soln





$$\vec{S}_{rel} = \vec{U}_{rel}\, t + \frac{1}{2} a_{rel} t^2$$

Obje - Obse

$$+24 = [\vec{V}_m - \vec{V}_B] t + \frac{1}{2} [\vec{a}_m - \vec{a}_B] t^2$$

$$24 = [11 - 0] t + \frac{1}{2} [0 - 2] t^2$$

$$24 = 11t - t^2$$

$$t^2 - 11t + 24 = 0$$

$$t = \frac{11 \pm \sqrt{121 - 96}}{2 \times 1} = \frac{11 \pm 5}{2}$$

$$t = \frac{11 - 5}{2} = 3 \text{ sec}$$

Man over take Bus

$$t = \frac{11 + 5}{2} = 8 \text{ sec}$$

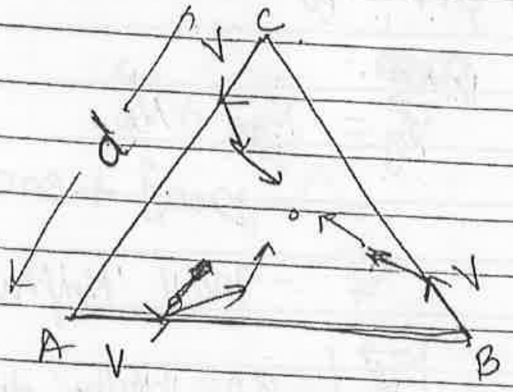
again Bus overtake the man after 8 second.

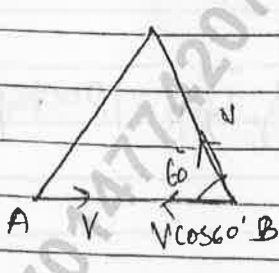
Date ______ Page ______

Q. 3 persons are standing on the vertex of an equilateral Δ of side 'd'. All the 3 starts moving simultaneously with same speed 'v' such that each person always faces the preceding, when they will meet. $t = ?$

meet at centroid

Sol^n



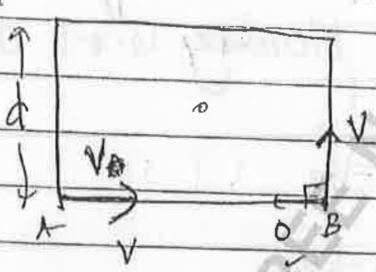


$$\boxed{V_{rel} = \frac{d_{rel}}{t}}$$

$$v + \frac{v}{2} = \frac{d}{t}$$

$$\boxed{t = \frac{2d}{3v}}$$

(*) square

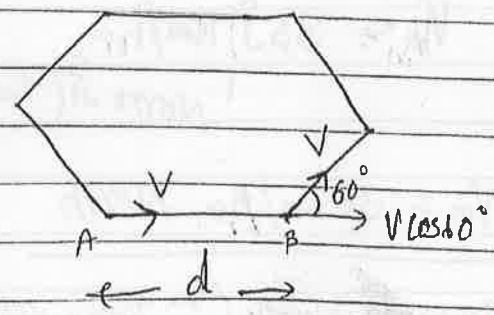


$$\boxed{V_{rel} = \frac{d_{rel}}{t}}$$

$$v + 0 = \frac{d}{t}$$

$$t = \frac{d}{v}$$

(*) hexagon



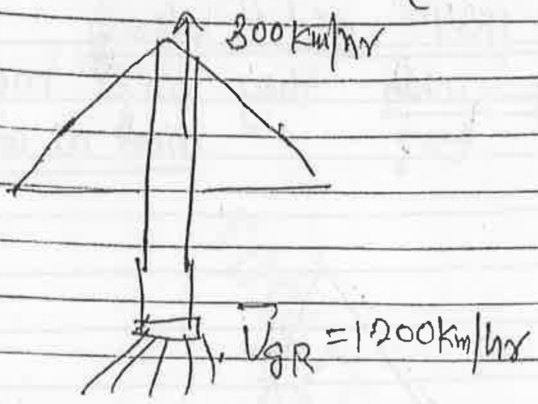
$$\boxed{V_{rel} = \frac{d_{rel}}{t}}$$

$$v - \frac{v}{2} = \frac{d}{t}$$

$$\boxed{t = \frac{2d}{v}}$$

NCERT

Q. A Jet Plane (or Rocket) is moving Vertically upward with speed 300 km/hr. It is ejecting gases with speed 1200 km/hr w.r.t itself. Then find actual speed of gas. (w.r.t Ground).



R → Rocket

$$\vec{V}_{gR} = \vec{V}_g - \vec{V}_R$$

$$\vec{V}_g = \vec{V}_{gR} + \vec{V}_R$$

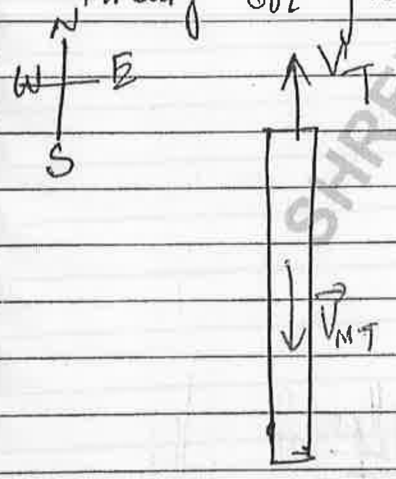
$$= -1200\hat{j} + 300\hat{j}$$

$$= -900\hat{j} \text{ km/hr}$$

$$|\vec{V}_g| = 900 \text{ km/hr downwards.}$$

NCERT

Q. A Train moves in North dirn with a speed of 54 km/hr. A monkey is running on the roof of the Train against it's motion with velocity 18 km/hr w.r.t Train. The velocity of Monkey w.r.t a Man standing on ground.



$$\vec{V}_{MT} = \vec{V}_M - \vec{V}_T$$

$$\vec{V}_M = \vec{V}_{MT} + \vec{V}_T$$

$$= -18\hat{j} + 54\hat{j}$$

$$\vec{V}_M = 36\hat{j} \text{ km/hr}$$

North में (Train Velocity ज्यादा है)

$V_m = 36$ km/hr North

$\vec{V}_m = 36$ km/hr towards North ($\because$ Train velocity dominates over $\vec{V}_{MT}$)

* Muzzle velocity ⇒ Vel. of bullet w.r.t to Barrel or Police.

⇓ or

$\vec{V}_{bullet/Police}$

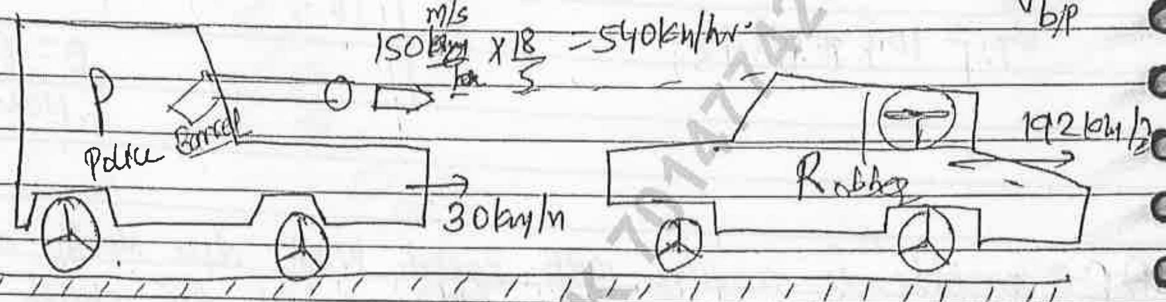
CNF NCERT exercise

Q. A police Jeep moving with speed 30 km/hr is chasing Robber's Car which is moving with speed 192 km/hr. Now police fire bullet towards Robber with muzzle velocity 150 m/s towards Robbers. Find the speed by which the bullet hits Robbers car to create hard damage?

Speed of b w.r.t. Barrel $\vec{V}_{b/P}$

↳ $V_{bR} = ?$

Soln



$$\vec{V}_{bR} = \vec{V}_b - \vec{V}_R$$

$$= 570\hat{i} - 192\hat{i}$$

$$= 378\hat{i} \text{ km/hr.}$$

$$\vec{V}_{bp} = \vec{V}_b - \vec{V}_p$$

$$\vec{V}_b = \vec{V}_{bp} + \vec{V}_p$$

$$= 540\hat{i} + 30\hat{i}$$

$$= 570\hat{i} \text{ km/hr}$$

Put

$$|\vec{V}_{bR}| = 378 \text{ km/hr}$$

$$= 378 \times \frac{5}{18} = 105 \text{ m/s}$$

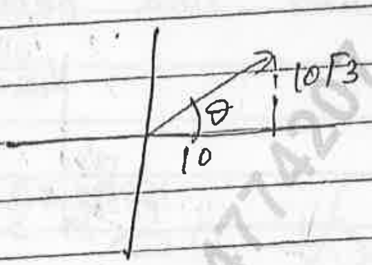
Q. A man is moving in his car with speed of 10 km/h due east. A train appears to him moving with speed $10\sqrt{3}$ km/h due north. find actual velocity of the train?

$\vec{V_C} = 10\hat{i}$

$\vec{V_{TC}} = 10\sqrt{3}\hat{j}$

$\vec{V_T} = 10\hat{i} + 10\sqrt{3}\hat{j}$

$|\vec{V_T}| = \sqrt{100+300} = 20 m/s$



$\tan\theta = \frac{10\sqrt{3}}{10}$

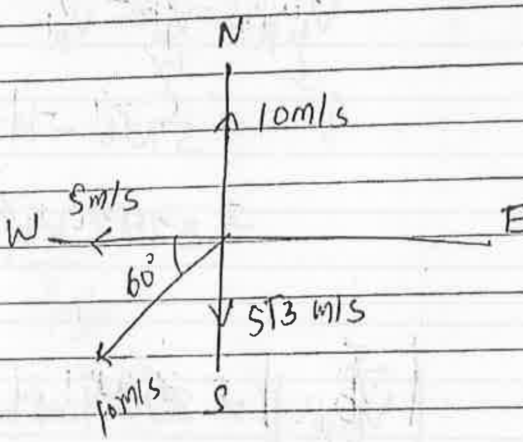
$\theta = 60^\circ$
North of east

Q A particle A is moving with speed 10m/s due North another particle B is moving with same speed 60° south of west. Find velocity of A wrt B?

soln $\vec{V_{AB}} = \vec{V_A} - V_B$

$= 10\hat{j} - (-5\hat{i} - 5\sqrt{3}\hat{j})$

$= 5\hat{i} + 5(2+\sqrt{3})\hat{j}$



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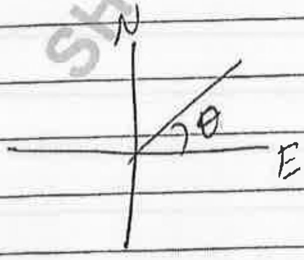


$\tan\theta = \frac{5(2+\sqrt{3})}{5} = 2+\sqrt{3}$

$\theta = \tan^{-1}(2+\sqrt{3})$ North of East.

Q. Car A is moving with speed 10 m/s due south and car B is moving with speed $10\sqrt{3}$ m/s due west.

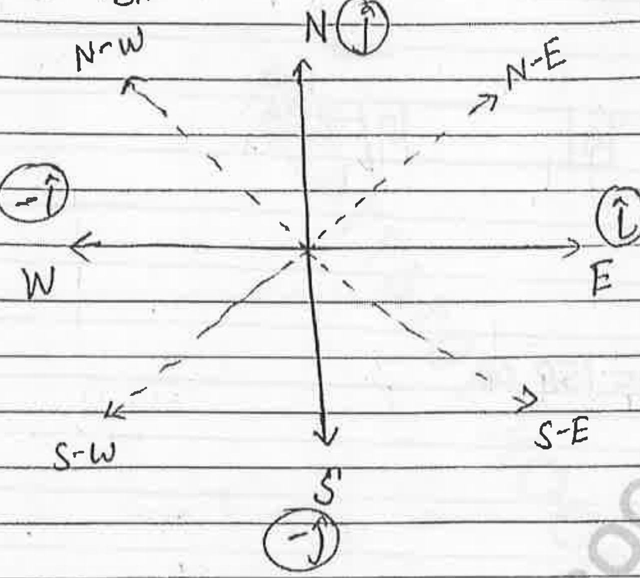
Find ① $\vec{V}_{AB}$

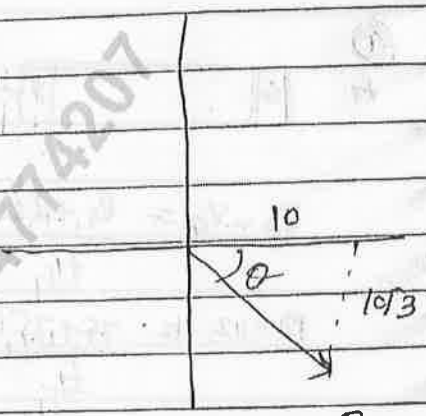
② $\vec{V}_{BA}$

$\vec{x} = 10\hat{i} - 10\sqrt{3}\hat{j}$

$|\vec{x}| = \sqrt{(10)^2 - (10\sqrt{3})^2}$

Notes:





$\tan\theta = \frac{10\sqrt{3}}{10}$

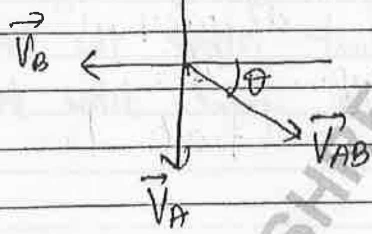
θ = 60° south of west

or

30° west of south.

① $\vec{V}_{AB} = -10\hat{j} - (-10\sqrt{3}\hat{i})$

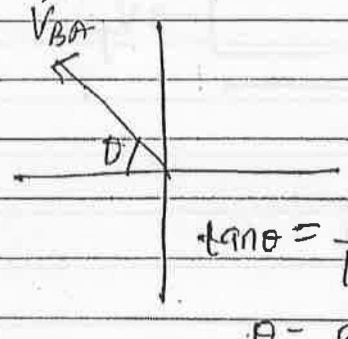
$= 10\sqrt{3}\hat{i} - 10\hat{j}$



θ = 60° south of east

② $\vec{V}_{BA} = -10\sqrt{3}\hat{i} - (-10\hat{j})$

$= -10\sqrt{3}\hat{i} + 10\hat{j}$



$\tan\theta = \frac{10}{10\sqrt{3}}$

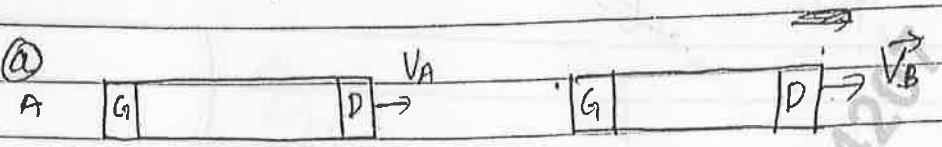
θ = 30° north of west

Q. Two trains of equal length 75m are moving with speed 13 m/s and 12 m/s on parallel track find time taken to cross each other.

(a) In same dirth

(b) In Opp. dirth.

(a)



$$V_A - V_B = \frac{l_1 + l_2}{t_1}$$

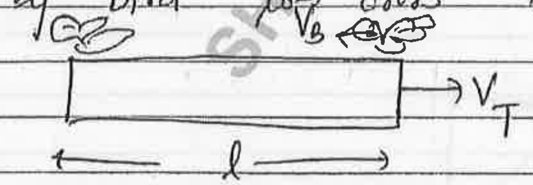
$$13 - 12 = \frac{75+75}{t_1} \qquad \underline{t_1 = 150 \text{ sec}}$$

(b) Opposite

$$V_A + V_B = \frac{75+75}{t_2}$$

$$t_2 = \frac{150}{25} = \underline{6 \text{ sec}}$$

Q A Train of length 500m is moving with speed 15 m/s along east. a bird is flying just above the train with speed 10m/s opposite to train find time taken by bird to cross the train?



$$V_{rel} = \frac{l}{t}$$

$$V_T + V_B = \frac{l}{t}$$

$$t = \frac{500}{15+10} = \frac{500}{25} = 20 \text{ Sec.}$$

eg- $\boxed{A} \rightarrow 20\,m/s$ $\boxed{B} \rightarrow 10\,m/s$

$$\vec{V}_{BA} = \vec{V}_B - \vec{V}_A = 10\hat{i} - 20\hat{i} = \underline{-10\hat{i}}$$

$$|\vec{V}_{BA}| = 10\,m/s$$

$$\vec{V}_{AB} = \vec{V}_A - \vec{V}_B = 20\hat{i} - 10\hat{i} = \underline{10\hat{i}}$$

② If particle are moving in opp. dirtn

$\boxed{A} \rightarrow \vec{V}_A$ $\vec{V}_B \leftarrow \boxed{B}$

$$\vec{V}_{BA} = \vec{V}_B - \vec{V}_A$$

$$= -V_B\hat{i} - V_A\hat{i}$$

$$= (V_B - V_A)(-\hat{i})$$

$$\boxed{\vec{V}_{BA} = |\vec{V}_{AB}| = V_B + V_A}$$

Accl^n

① $\vec{a}_{rel} = 0$

$V_{rel} = \frac{d_{rel}}{t}$

② $\vec{a}_{rel} \neq 0$

$\vec{a}_{AB} = \vec{a}_A - \vec{a}_B$

$\vec{a}_A = 0$

$\vec{a}_{AB} = -\vec{a}_B$

③ $\vec{a}_{rel} = \text{const}$

$\vec{V}_{rel} = \vec{u}_{rel} + \vec{a}_{rel} \cdot t$

$\vec{S}_{rel} = \vec{u}_{rel} t + \frac{1}{2} a_{rel} t^2$

$\vec{V}_{rel}^2 - \vec{u}_{rel}^2 = 2\vec{a}_{rel}\vec{S}_{rel}$

Q. A police jeep moving with speed 108 km/h chasing Robbers car moving with 54 km/h They are 300 m ahead How long it will take for police to caught robbers?

$\boxed{P} \rightarrow V_P = 108 \times \frac{5}{18} = 30\,m/s$ $\boxed{R} \rightarrow V_R = 54 \times \frac{5}{18} = 15\,m/s$

300 m

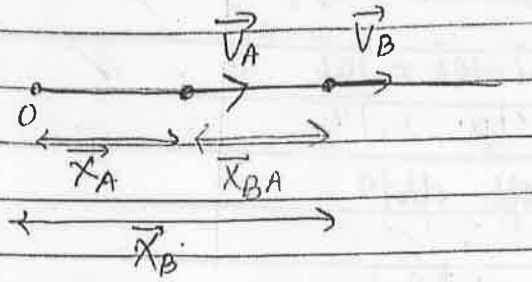
$$V_{rel} = \frac{d_{rel}}{t}$$

$$30 - 15 = \frac{300}{t}$$

$$t = \frac{300}{15} = 20\ \text{sec}$$

Relative Motion

Relative velocity



$$\boxed{\vec{V}_{BA} = \vec{V}_B - \vec{V}_A}$$

$$\boxed{\vec{V}_{AB} = \vec{V}_A - \vec{V}_B}$$

Ground

$$\boxed{\vec{V}_{rel} = \overrightarrow{object} - \overrightarrow{observer}}$$

Ground.

$$\vec{x}_{BA} = \vec{x}_B - \vec{x}_A$$

$$\frac{d\vec{x}_{BA}}{dt} = \frac{d\vec{x}_B}{dt} - \frac{d\vec{x}_A}{dt}$$

$$\boxed{\vec{V}_{BA} = \vec{V}_B - \vec{V}_A}$$

$$\vec{a}_{AB} = \vec{a}_A - \vec{a}_B$$

↓

acc^n of $\vec{A}$ w.r.t $\vec{B}$

① If particle are moving in same direction along st. line.

$\boxed{A} \rightarrow V_A$ $\boxed{B} \rightarrow V_B$

$\vec{V}_{AB} = \vec{V}_A - \vec{V}_B$	$\vec{V}_{BA} = \vec{V}_B - \vec{V}_A$
$= V_A\hat{i} - V_B\hat{i}$	$= V_B\hat{i} - V_A\hat{i}$
$= (V_A - V_B)\hat{i}$	$= (V_B - V_A)\hat{i}$

$$\boxed{|\vec{V}_{AB}| = V_A - V_B = |\vec{V}_A| - |\vec{V}_B|}$$

Numerical solve,

$$\boxed{|\vec{V}_{AB}| = |\vec{V}_{BA}| = V_A \sim V_B}$$

Same dir^n

+ve difference

* inertial mass → ratio of Force to accⁿ $F = ma$

$m_i = \frac{F}{a}$

Gravitational mass → ratio of Force to accⁿ due gravity

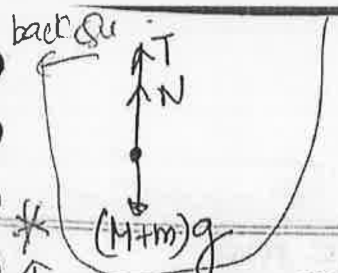
$F = mg$

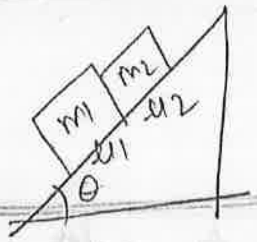
$m_g = \frac{F}{g}$

Both are equal,

$$\boxed{\frac{m_i}{m_g} = 1}$$

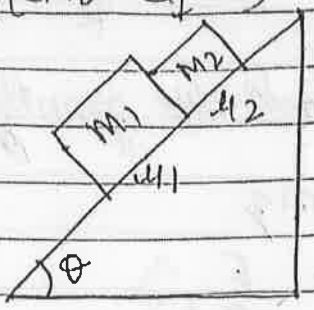
*





$\mu_1 < \mu_2$
$a_1 > a_2$ will separate

$a = g(\sin\theta - \mu_1 \cos\theta)$



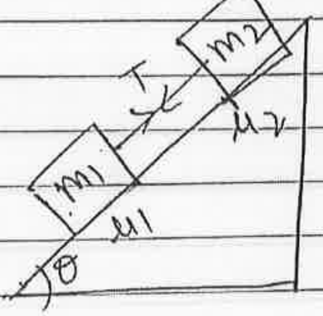
$$\mu_c = \frac{\mu_1 m_1 + \mu_2 m_2}{m_1 + m_2}$$

$a = g(\sin\theta - \mu_c \cos\theta)$

acc^n of (Both block) a_c

if, $\mu_1 > \mu_2$
$a_1 < a_2$ (pull)

*



$\mu_1 < \mu_2$ & acc^n will same as above,
$a_1 > a_2$, then Tension is produced.

⊛ $f_{roll} << f_K < f_L$

Rolling friction: at the point of contact of body & surface point the surfaces are slightly deformed the

Fact जो FP X

$f_k = \mu_k N$
$= 0.4 \times 100$
$= 40N$

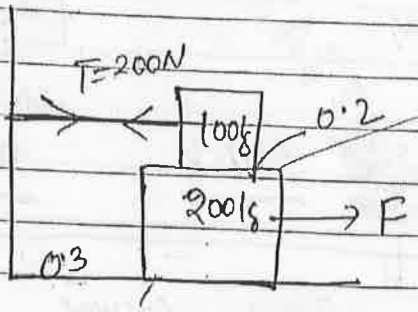
100N ← 10kg → $f_k = 40N$ ⇒ $100 - 40 = ma$

$\frac{60}{10} = a \quad a = 6\,m/s^2$

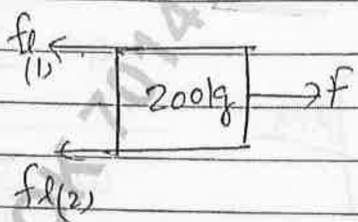
f_k - 40N ← 40kg →

$40 = 40a$
$a = 1\,m/s^2$

Q. Find min^m value of 'F' so that the 100kg begins to slide-



$f_L = 0.2 \times 1000$
$= 200N$



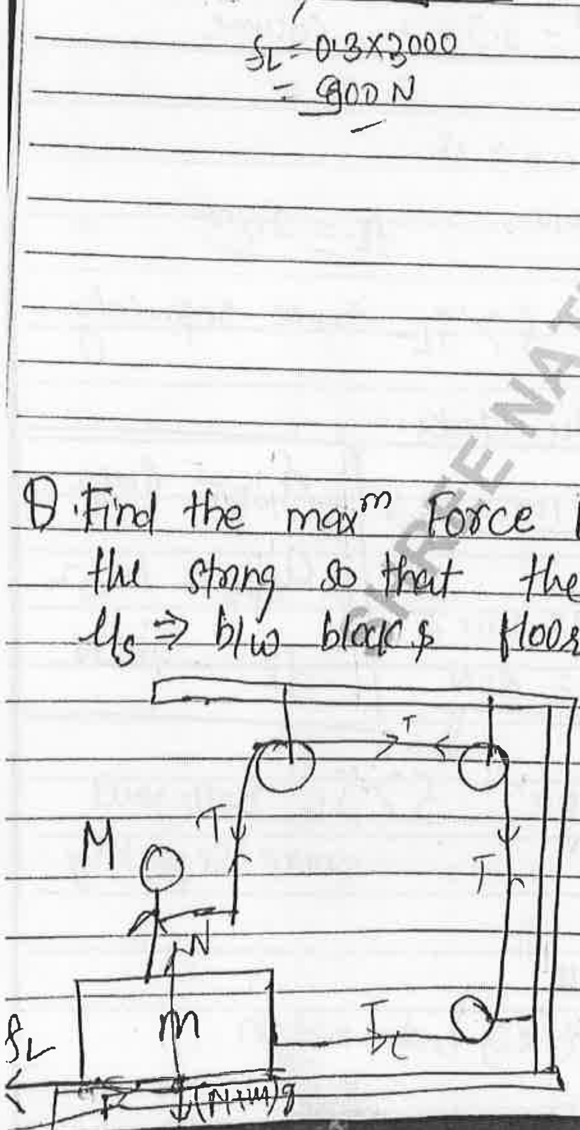
$f_L = 0.3 \times 3000$
$= 900\,N$

$F = f_{l_1} + f_{l_2}$
$= 200 + 900$
$= 1100\,N$

$\boxed{F \geq f_{l_1} + f_{l_2}}$ For Relative Motion.

$F_{min} = f_{l_1} + f_{l_2}$

Q. Find the max^m Force by which the man can pull the string so that the block does not slide.
μ_s ⇒ b/w block & floor. Mass of Man = M



$\frac{max^m\ T}{so\ no\ sliding}$

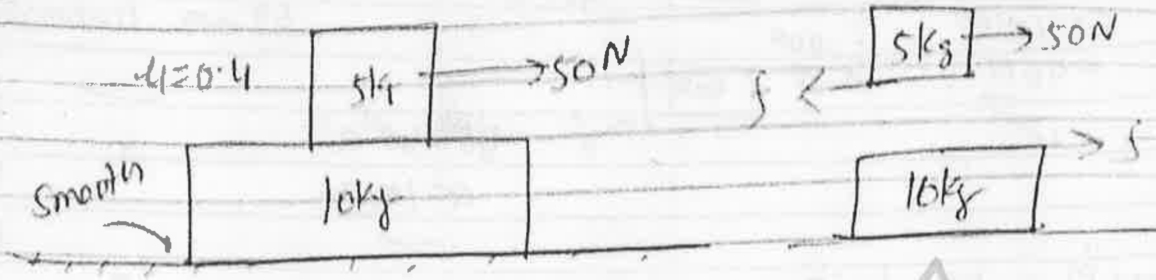
$T = f_L$

$T = \mu_s N$

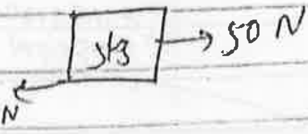
$N = (M+m)g - T$

$\boxed{T = \frac{\mu_s (M+m) g}{1+\mu_s}}$

Q. Find acc^n of the Blocks and friction b/w the blocks.



$f_L = \mu mg$
$= 0.4 \times 50$
$= \underline{20N}$

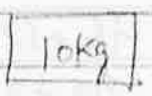


$20 = 10 \times a$
$a = 2\ m/s^2$

$a = \frac{50-20}{5} = \cancel{\frac{30}{5}}\ \underline{6m/s^2}$

$a_{5kg} = 6m/s^2$
$a_{10kg} = 2\ m/s^2$
$f = 20N$
Ans

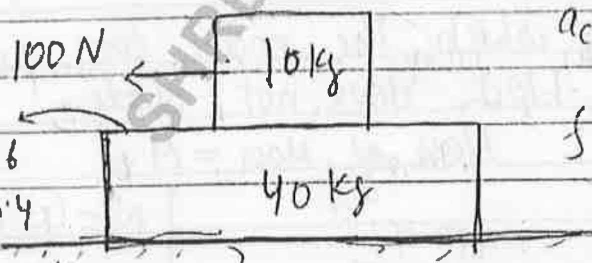
$a_c = \frac{50}{15} = \frac{10}{3} = 3.3\ m/s^2$ assume.



$f - 0 = 10 \times 3.3$
$\underline{f = 33N}$, $\underline{f_L = 20N}$

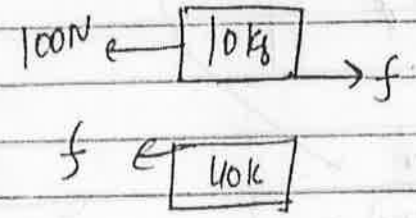
$f > f_L$ move sepately.

BA 8 (1)
Q. Find acc^n of blocks & friction b/w the blocks.



$a_c = \frac{100}{50} = 2m/s^2$
$f - 0 = 40 \times 2$
$f = 80N$
⇓

$a_{10kg} = 6m/s^2$
$a_{40kg} = 1m/s^2$
$f_k = 40\ N$



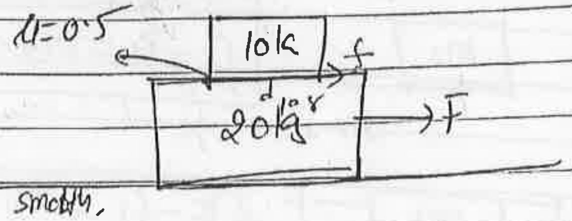
$f_L = 0.6 \times 100$
$= \underline{60\ N}$

$f > f_L$ Both will move sepeately

⇓
f_k comes under action ;;

Date ____ Page ____

Q. Find the max^m value of applied force 'F' so that Both blocks move together.



$a_c = \frac{F}{30}$

10kg → f

$f - 0 = 10 \times \frac{F}{30}$

$f_L = 0.5 \times 100 = 50\ N$

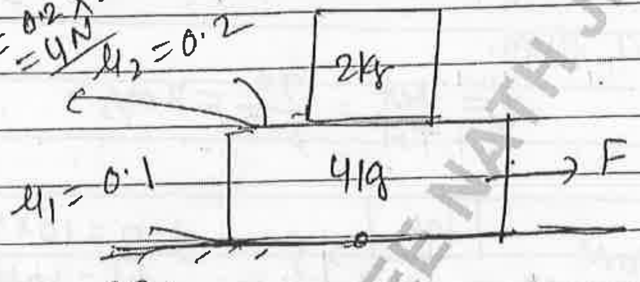
$f = 50N$

$\frac{F}{3} = 50$

$\boxed{F = 150\ N}$

Q. Find max^M value of (F) so that Both blocks move together.

$(f_L)_2 = 0.2 \times 20 = 4N$



move together ⇒ $f = f_L$

$a = \frac{F - f}{6}$

2kg → f

$(f_L)_1 = 0.1 \times 6g = 6N$

$f - 0 = 2\left(\frac{F-6}{6}\right)$

$4 = \frac{\cancel{2}(F-6)}{\cancel{6}\,3}$

$12 = F - 6$

$F = 18\ N$

$f = m(a_c)$ / Force eqn.

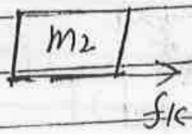
(4) (a) $f \leq f_L$

Both moving together

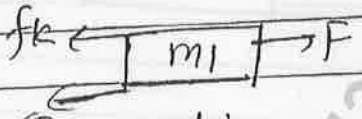
a_c, f

(b) $f > f_L$ → Relative motion b/w Both Blocks.

Both move seprately

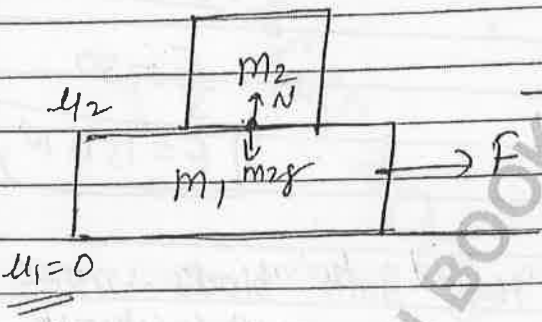


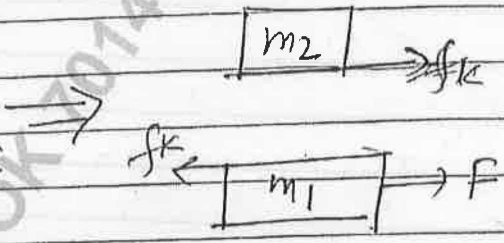
$\boxed{f_k - 0 = m_2 a_2}$



$\boxed{F - f_k = m_1 a_1}$

(f) if ground is also Rough.

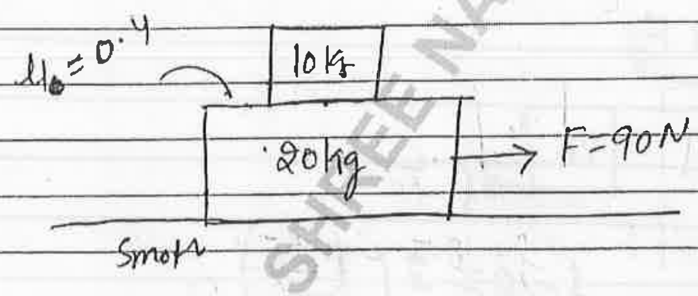




$(f_L = \mu m_2 g)$

Q Find accln of blocks & friction b/w them.

$a_c = \frac{F_{ext}}{T.M} = \frac{90}{30} = 3\ m/s^2$



10k → f

$f - 0 = 10 \times 3$

$\boxed{f = 30N}$

f_L ← 20kg → $F = 90N$

$a = 3\ m/s^2$ of both Blocks

Static friction $f_s = 30\ N$

$f_L = \mu m_2 g$

$= 0.4 \times 100$

$= 40N$

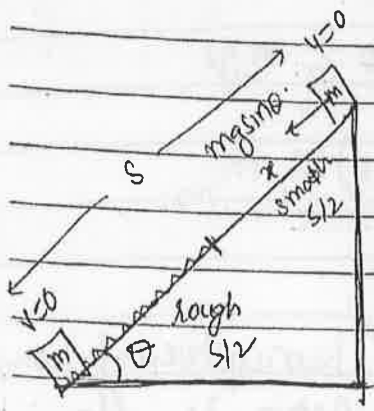
$f < f_L$

No Relative motion

Both will move together.

Neet

Q. block is drooped, and finally comes at rest, find $\mu_k = ?$



$$W_{all\ force} = k_f - k_i$$

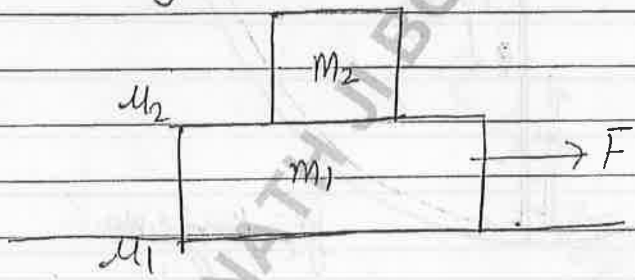
$$W_g + W_f = 0 - 0$$

$$(mg\sin\theta)\cancel{S} - \mu_k mg\cos\theta\left(\frac{\cancel{S}}{2}\right) = 0$$

$$\sin\theta = \mu_k \frac{\cos\theta}{2}$$

$$\boxed{\mu_k = 2\tan\theta}$$

Two blocks System

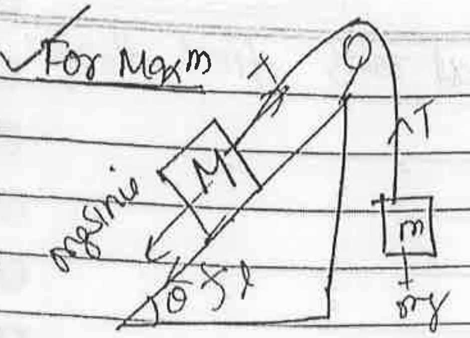


① Check whether lower block is moving or not

② Assume both moving together and find Common accl^n (even they are not moving together)

③ F.B.D of the block on which force is not acting. Find friction on it and find f_L

For $M \to m$

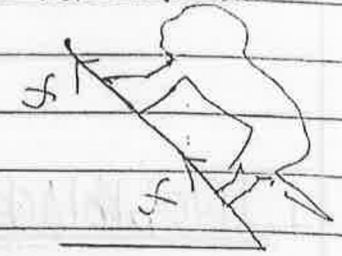


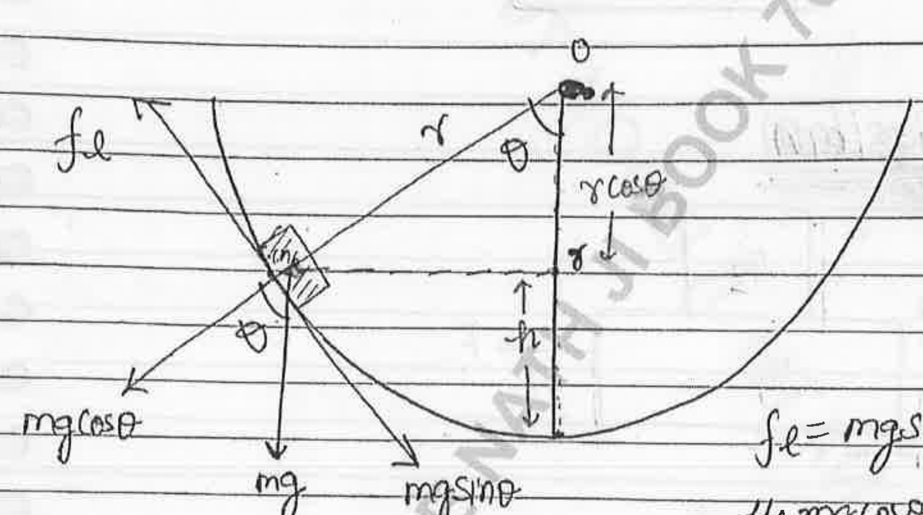
$$Mg\sin\theta + f_L = T$$

$$Mg\sin\theta + \mu Mg\cos\theta = m_{max}g$$

$$\underline{M(\sin\theta + \mu\cos\theta) = m_{max}}$$

Q. An Insect starts crawling up in an hemispherical bowl of radius 'r' If coefficient of static friction is μ_s find ~~find the~~ Maximum height up to which it crawl.

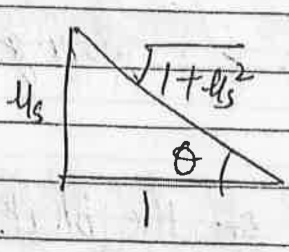




$$f_l = mg\sin\theta$$

$$\mu_s mg\cos\theta = mg\sin\theta$$

$$\boxed{\tan\theta = \mu_s}$$



$$h = r - r\cos\theta$$

$$= r(1-\cos\theta)$$

$$h = r\left(1 - \frac{1}{\sqrt{1+\mu_s^2}}\right)$$

Neet

Q. A block is placed on horizontal plank now one end of the plank is gradually raised when angle of inclination is 30° the block starts sliding and covers a distance 4m in 4 second. Find μ_s & μ_k?

30° angle of repose,

$\tan\theta = \mu_s$

$\tan 30 = \mu_s$

$\boxed{\cancel{\mu_s = \sqrt{3}}}$ $\mu_s = \frac{1}{\sqrt{3}}$

$\mu_s = \frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{1.732}{3}$ 0.577

$\mu_s = 0.577$

$s = ut + \frac{1}{2}at^2$

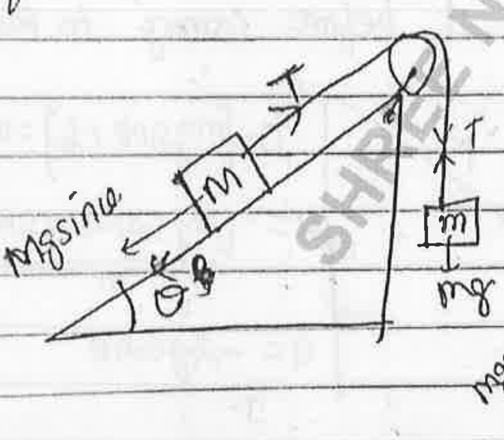
$4 = 0 + \frac{1}{2}g(\sin 30 - \mu_k \cos 30)4^2$

$\sqrt{3}\mu_k = 1 - 0.1$

$\mu_k = \frac{0.9}{\sqrt{3}} = 0.3\sqrt{3}$

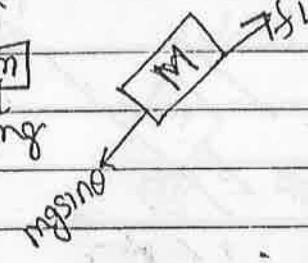
$= 0.5196$

Q. In the given Fig. What should be the min^m & max^m value of 'm' so that M does not slide (μ).



For min^m

M



$T = mg$

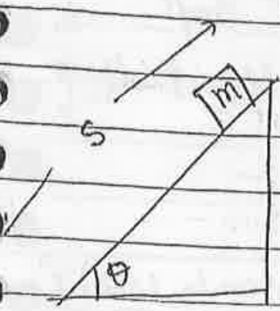
$Mg\sin\theta = T + f_L$

$Mg\sin\theta - f_L = T$

$Mg\sin\theta - \mu Mg\cos\theta = mg$

$m_{min} = M(\sin\theta - \mu\cos\theta)$

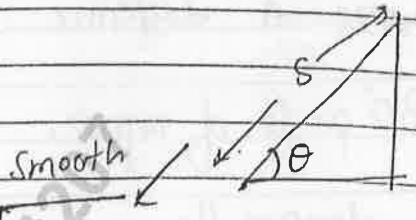
Q. Time taken by a block to slide down on a rough inclined plane of inclination 'θ' is 'n' times that of time taken on a smooth identical inclined plane from same height. f/o coefficient of friction.



$s = ut + \frac{1}{2}at^2$

$s = 0 + \frac{1}{2} g(\sin\theta - \mu_k \cos\theta) t^2$

rough



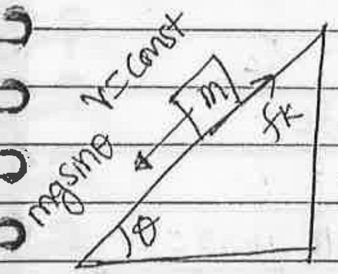
$$t = \sqrt{\frac{2s}{g(\sin\theta - \mu_k\cos\theta)}} = n\sqrt{\frac{2s}{g\sin\theta}}$$

$$\sin\theta = n^2\sin\theta - n^2\mu_k\cos\theta$$

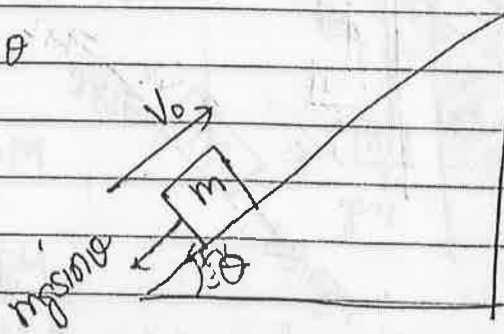
$$n^2\mu_k\cos\theta = \sin\theta(n^2-1)$$

$$\boxed{\mu_k = \tan\theta\left(\frac{n^2-1}{n^2}\right)} \qquad \mu_k = \tan\theta\left(1-\frac{1}{n^2}\right)$$

Q. A block is sliding down on in inclined plane with const velocity if the same block is projected up along the same inclined plane with speed V_0. Find distance travelled by it before coming to rest? (θ) → angle of inclination?



$f_k = mg\sin\theta$



$0 - [mg\sin\theta + f_k] = ma$

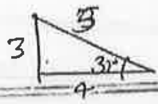
$a = -\frac{(mg\sin\theta + mg\sin\theta)}{m}$

$a = -2g\sin\theta$

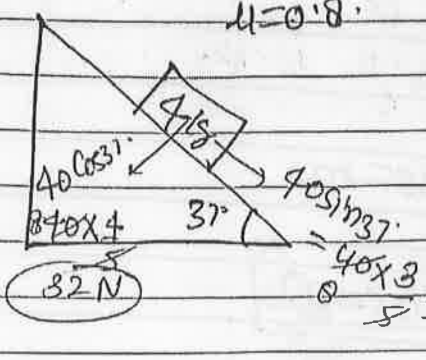
$$a = \frac{v^2-u^2}{2s}$$

$$\frac{0 - V_0^2}{2s} = -2g\sin\theta$$

$$s = \frac{V_0^2}{4g\sin\theta}$$



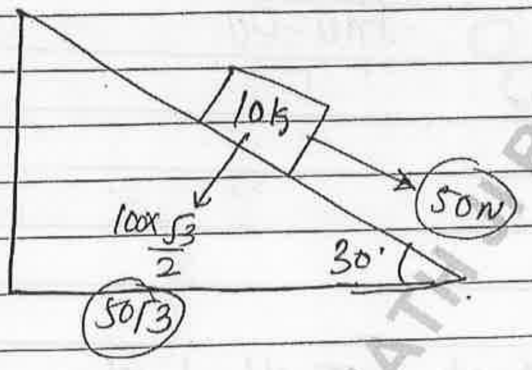
Q Find accl^n of block and friction force acting on it.



$f_L = \mu N$
$= 0.8 \times 32$
$f_L = 25.6$

$F_{ext} < f_L$
No Relative Motion
$f_s = F_{ext} = 24N$

Q Find accl^n & f acting on block.



$f_L = \mu_s N$
$= 0.4 \times 50\sqrt{3}$
$= 20\sqrt{3} = 20 \times 1.7$
sliding down $= 34N$

$F_{ext} = 10g\sin 30$
$= 50N$

$F_{ext} > f_L$ there will be Relative motion.

$f_k = \mu_k N = 0.3 \times 50\sqrt{3}$
$= 15 \times 1.7$
$= 25.5N$

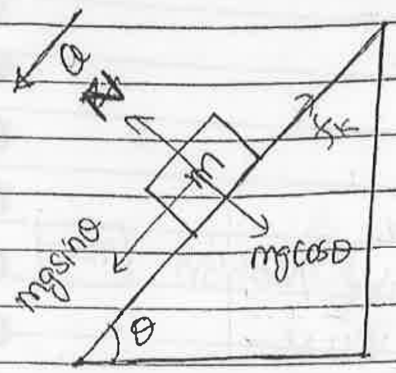
$a = ?$

$50 - 25.5 = 10a$

$\frac{24.5}{10} = a = 2.45\ m/s^2$

Inclined Plane

Case (I) accl^n of the block while Sliding down.



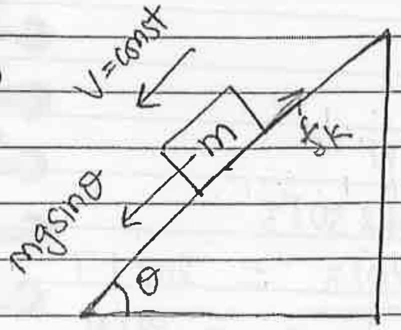
$mg\sin\theta - f_k = ma$

$mg\sin\theta - \mu_k mg\cos\theta = ma$

$$\boxed{a = g(\sin\theta - \mu_k\cos\theta)}$$

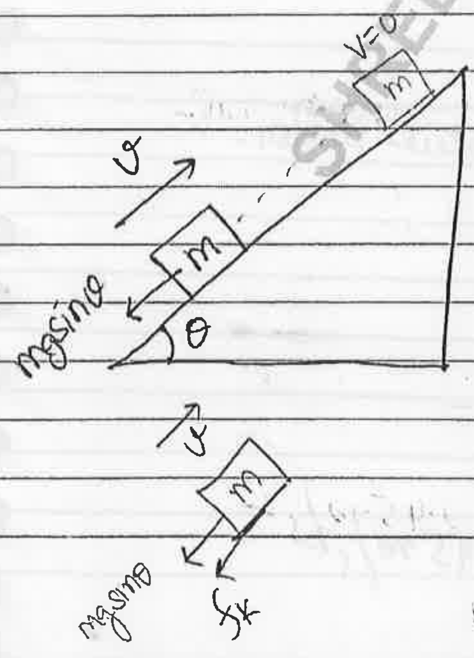
Case (II)

Block sliding down with const velocity. ⇒ $F_{net} = 0$



$f_k = mg\sin\theta$

Case-(III) Block Projected up along Inclined then.



$0 - [mg\sin\theta + f_k] = ma$

$$\boxed{a = -g[\sin\theta + \mu_k\cos\theta]}$$

"Retardation"

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Time taken by Block to fall

$ma = 80N$ ← [40kg] → $f_k = 60$

$80 - 60 = 40a$

$a = 0.5\ m/s^2$ relative to truck.

$$S = ut + \frac{1}{2}at^2$$

$$S = 0 + \frac{1}{2} \times 0.5\, t^2$$

$$t = \sqrt{20}\ s$$

Now for truck

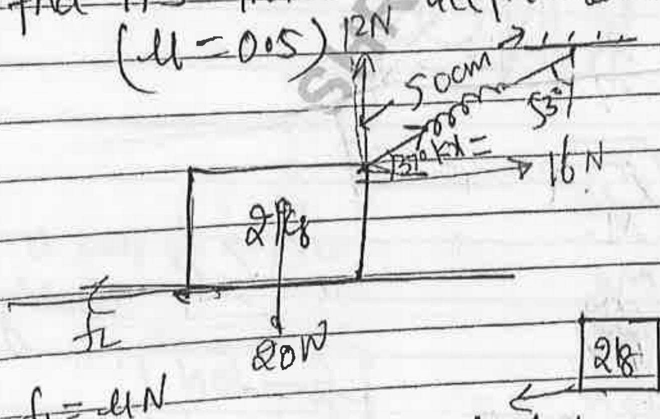
$$t = \sqrt{20}\ s$$

$$S = ut + \frac{1}{2}at^2$$

$$S = 0 + \frac{1}{2} \times 2 \times 20 = 20m$$

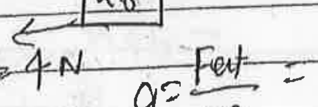
Q. The natural length of spring is 40cm and spring const 200 N/m. If the block in the fig is released from the given position find it's initial acc^n when

$(\mu = 0.5)$



extension = 50 − 40 = 10 cm

$$F = kx = \frac{200 \times 10}{100} = 20N$$

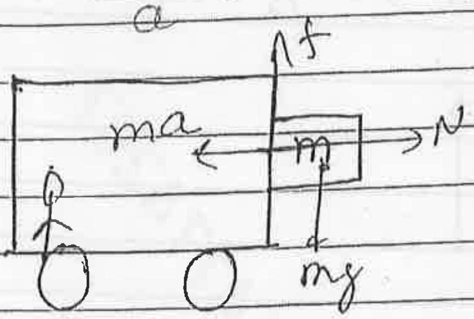


$f_L = \mu N$
$= 0.5 \times 8$
$= 4N$

$$a = \frac{F_{net}}{m} = \frac{12}{2} = 6\ m/s^2$$

Note:
If Force is acting horizontally and NO friction b/w block & wall, then it can't be in Eqbm.

Q. What should be the min^m accl^n of cart in given fig so that the block does not fall, (lls)



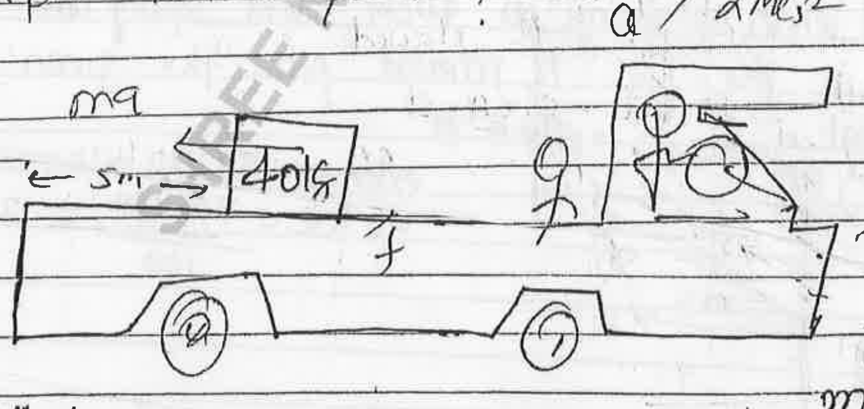
$f_L \geq mg$

$\mu N \geq mg$

$\mu ma \geq mg$

Q. A block of mass 40 kg is placed on a open Truck at a distance 5m from open end. coefficient of friction 0.15. The Truck starts its motion with accl^n $2 m/s^2$. When the block falls f/o the distance travelled by Truck from initial point. ?

$a = 2 m/s^2$

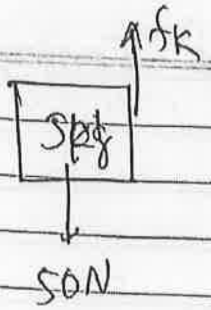


$f_l = \mu_s N$
$= 0.15 \times 400$
$= 60N$

$F_{spd} = ma$
$= 40 \times 2$
$= 80N$

ma > fl there is sliding

$\boxed{f_K = 60N}$



$$50 - 30 = 5a$$

$$\boxed{a = 4\ m/s^2 \text{ downwards}}$$

(2)

$$f_L = \mu_s N = 0.4 \times 500$$
$$= 200\ N$$

$$f_L > mg$$

No Relative motion

$$\boxed{a = 0}$$

$\cancel{f_s = \mu_s N}$ $\cancel{= 0.4 \times}$

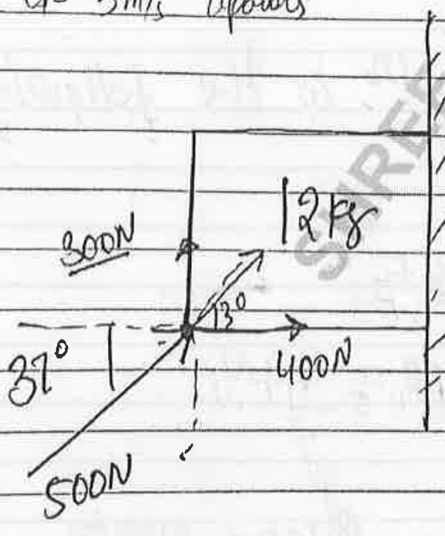
f_s = weight के बराबर होगा।

$$\boxed{f_s = 50N}$$

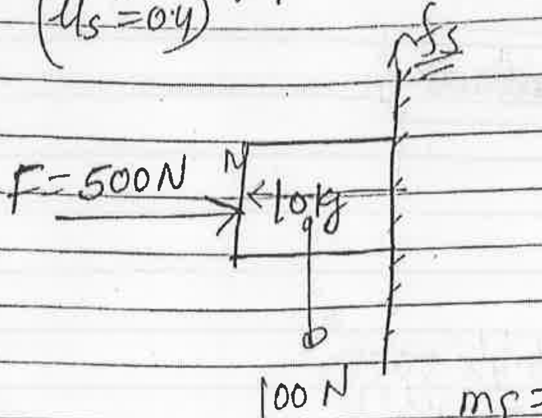
there will be relative motion

$f_k = 120N$

(3) $a = 5m/s^2$ upwards



Q In given fig f/o value of friction & Contact force.
$(\mu_s = 0.4)$



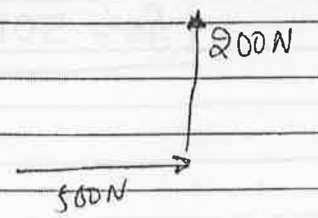
$f_L = \mu N$
$= \frac{0.4 \times 500}{10}$
$f_L = 200N$.

$mg = 100N$

$mg < f_L$ No relative Motion

at this situation;
$\boxed{f_s = 100N}$ = weight के बराबर.

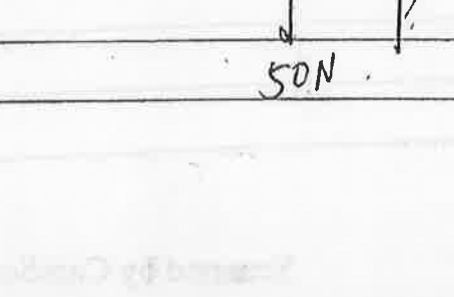
(ii)



$f_c = \sqrt{N^2 + f_s^2}$
$= \sqrt{(500)^2 + (100)^2}$
$= 100\sqrt{26}\ N$

Q. Find value of friction and acc^n in the following cases.
$(\mu_s = 0.4)$, $(\mu_k = 0.3)$ in ③ cases

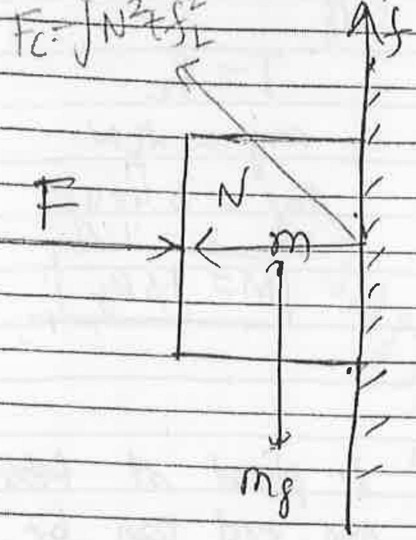
(i)



$f_l = \mu_s N = \mu_s F$
$= 0.4 \times 100 = 40N$

$mg > f_l$
there will be Relative Motion
$f_k = \mu_k N = 0.3 \times 100$
$= 30N$.

Find Min^m value 'F' to hold the block against wall.
coefficient of static friction μ_s.



No sliding against wall

$mg \leq f_L$

$mg \leq \mu_s N$

$mg \leq \mu_s F$

$\mu_s F_{min} = mg$

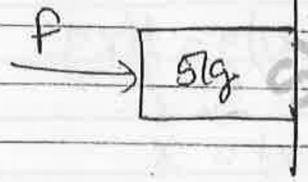
$$\boxed{F_{min} = \frac{mg}{\mu_s}}$$

$F_C = \sqrt{N^2 + f_L^2}$

$= \sqrt{F^2 + \mu_s^2 F^2}$

$F_C = F\sqrt{1+\mu_s^2}$

Find Min^m value of Force to hold the Block against wall ($\mu_s = 0.4$)



$f_L \geq mg$

$\mu F_{min} = mg$

$0.4\, F_{min} = 5 \times 10$

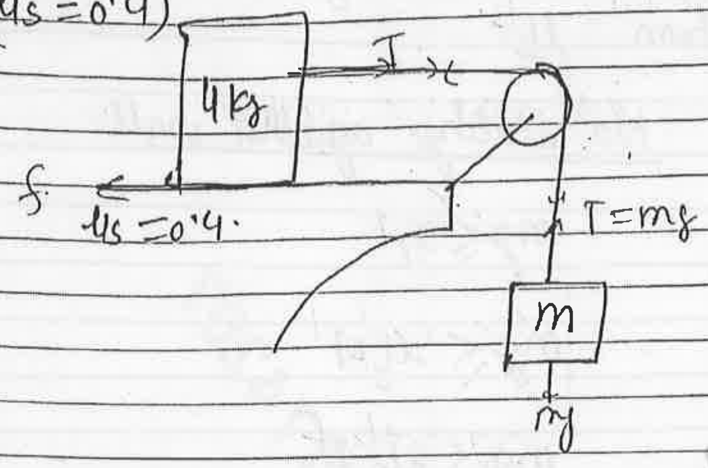
$F_{min} = \frac{5 \times 100}{04} = \underline{125\, N}$

ASP sir ⇒ 9414242313

Rau ⇒ (1,2,3,7,8,9)
Fnd.

B.B-6 (1,2,4,5,7-10)
Ex-1 (99,100,101,102,107,113,116,119,)
Ex-2 (9,12,
Ex-13 (13,15)

Q.11 Find max^m value of m so that block does not slide.
($\mu_s = 0.4$)



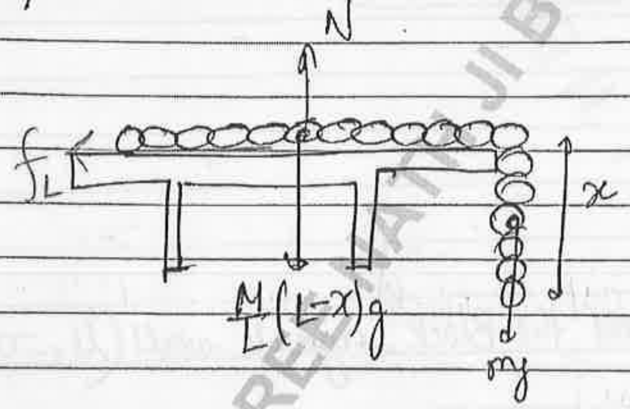
$T = f_L$

$mg = \mu_s N$

$m\cancel{g} = 0.4 \times 4\cancel{g}$

$\boxed{m = 1.6\,kg}$

Q.12 A uniform chain of length 'L' & 'M' is placed at table Top find the max^m length by which its one end can be suspended without slipping. Coefficient of static friction is μ_s.?



For No slipping

$f_L \geq mg$

$f_L = mg$

$\mu_s N = mg$

$\mu_s\left(\frac{M}{L}(L-x)\right)g = \frac{M}{L}xg$

$\mu_s(L-x) = x$

$\mu_s L - \mu_s x = x$

$\mu_s L = (\mu_s + 1)x$

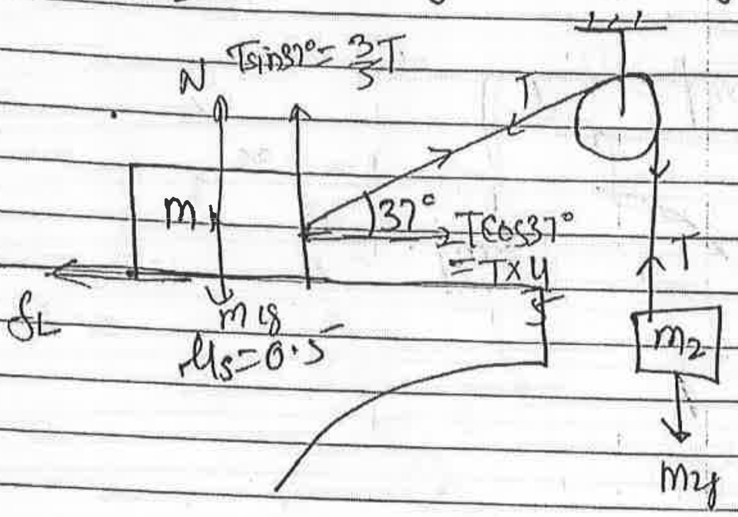
$$\boxed{x = \frac{\mu_s L}{\mu_s + 1}}$$

Date ______ Page ______

(10)

Q. Find $\frac{m_1}{m_2}$ if the system is just going to start motion.

$\boxed{\frac{m_1}{m_2} = \frac{11}{5}}$



$T = m_2 g$

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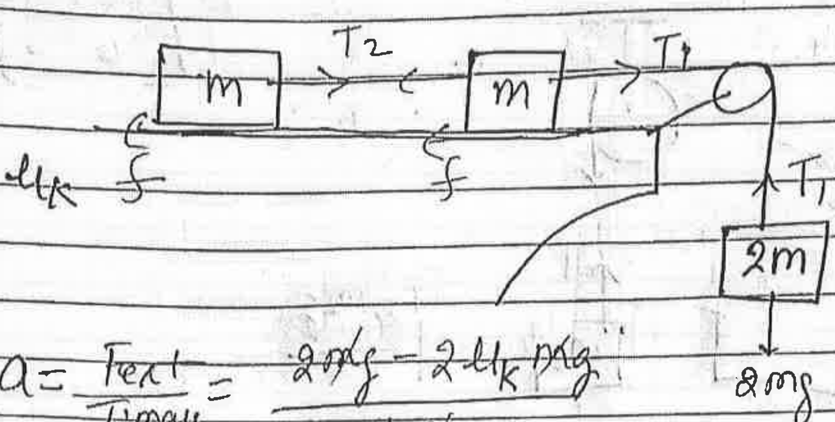
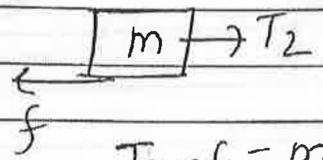
Q-9 Find accl^n of the system & Tension in the connecting strings



$a = \frac{F_{ext}}{T\ mass} = \frac{2mg - 2\mu_k mg}{4m}$

$$\boxed{a = \frac{g}{2}(1-\mu_k)}$$



$T_2 - f = ma$

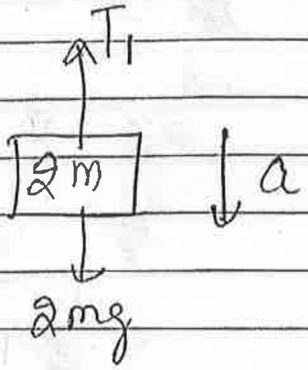
$T_2 = f + ma$

$= \mu_k mg + m\left[\frac{g}{2}(1-\mu_k)\right]$

$= \mu_k mg + \frac{mg}{2} - \frac{\mu_k mg}{2}$

$= \frac{\mu_k mg}{2} + \frac{mg}{2}$

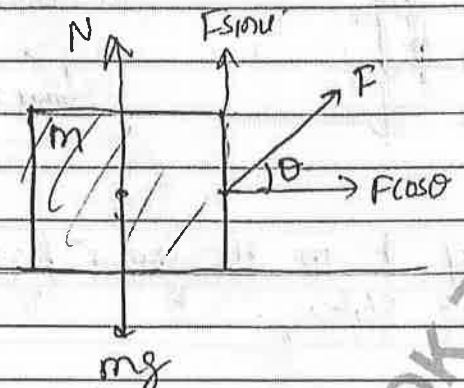
$$\boxed{T_2 = \frac{mg}{2}\left[\mu_k + 1\right]}$$



$2mg - T_1 = 2m\left(\frac{g}{2}(1-\mu_k)\right)$

inclined pulling is easier than inclined Pushing because in pulling "Normal" is less.

Q In the given fig Find the $\min^m$ possible value of F to slide the block? ($\theta \rightarrow$ can be suitably adjusted)



$N = mg - F\sin\theta$

$F\cos\theta = \mu_s(mg - F\sin\theta)$

$$\downarrow F = \frac{\mu_s mg}{(\cos\theta + \mu_s \sin\theta)} \uparrow \text{maxima}$$

$$F = \frac{\mu_s mg}{\cos\theta(1 + \mu_s \tan\theta)}$$

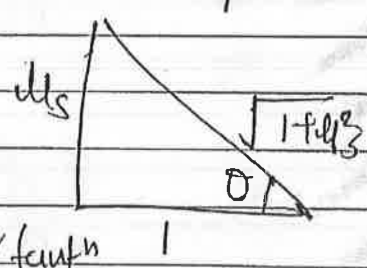
$$F = \frac{\mu_s mg}{\frac{1}{\sqrt{1+\mu_s^2}}(1+\mu_s^2)}$$

$$\boxed{F = \frac{\mu_s mg}{\sqrt{1+\mu_s^2}}}$$

① $\frac{d}{d\theta}(\cos\theta + \mu_s \sin\theta)$

② $-\sin\theta + \mu_s\cos\theta = 0$

$\tan\theta = \frac{\mu_s}{1}$



$\max^m$ value of funct^n

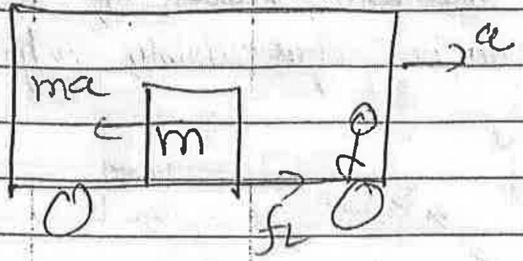
$= \sqrt{1^2 + \mu_s^2}$

$= \sqrt{1+\mu_s^2}$

Date ______ Page ______

Q(5) A box is placed on the floor of a train then f/o the max^m accl^n of Train so that the box is not slide relative to train.

$(\mu_s = 0.15)$



$$m a_{max} = f_L$$

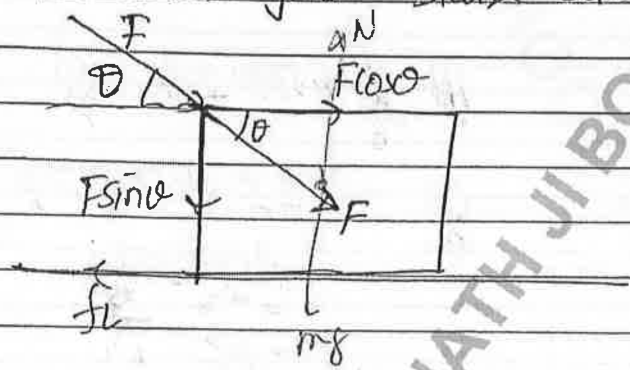
$$\cancel{m} a_{max} = \mu_s \cancel{m} g$$

$$a_{max} = 0.15 \times 10$$

$$= 1.5\ m/s^2$$

Case-I

Q-6 Find min^m value of F in the given fig. so that the block just starts sliding.



$$N = mg + F\sin\theta$$

to just start sliding

$$F\cos\theta = f_L$$

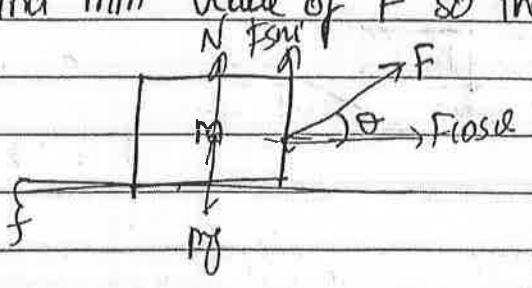
$$F\cos\theta = \mu_s (mg + F\sin\theta)$$

$$F\cos\theta - \mu_s F\sin\theta = \mu_s mg$$

$$\boxed{F = \frac{\mu_s mg}{(\cos\theta - \mu_s \sin\theta)}}$$

Case-II

Q7 Find min^m value of F so that block just starts sliding.



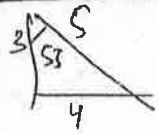
$$N + F\sin\theta = mg$$

$$N = mg - F\sin\theta$$

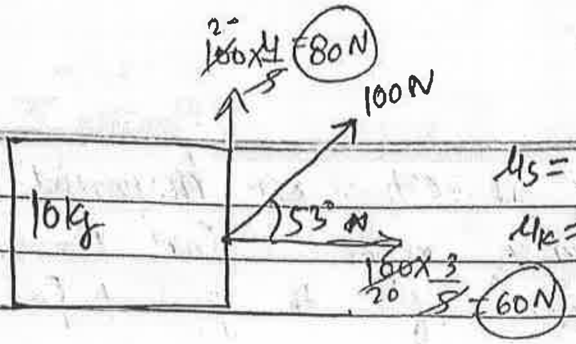
$$F\cos\theta = f_L$$

$$F\cos\theta = \mu_s (mg - F\sin\theta)$$

2:55 pm



Q3



$\mu_s = 0.6$

$\mu_k = 0.4$

find accⁿ & Friction Force

$f_L = \mu_s N$
$= 0.6\times 20$
$f_L = 12\,N$

$N + 80 = 100$
$N = 20$

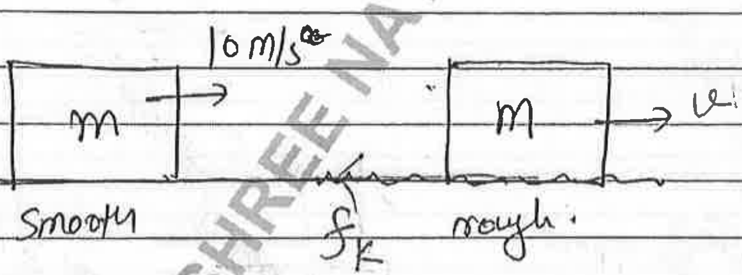
$\boxed{\begin{aligned} f_k &= \mu_k N \\ &= 0.4\times 20 \\ f_k &= 8N \end{aligned}}$

accⁿ.

Force eqⁿ $\Rightarrow$ $a = \dfrac{60-8}{10}$

$= \dfrac{52}{10} = 5.2\ m/s^2$

Q4 A block is sliding on smooth horizontal surface with velocity 10 m/s now it starts moving on rough surface & after travelling a distance 40m on rough surface it comes to rest. f/o value of coefficient of friction.



$a = \dfrac{0 - f_k}{m}$

$a = -\dfrac{\mu_k \cancel{m} g}{\cancel{m}}$

$\boxed{a = -\mu_k g}$

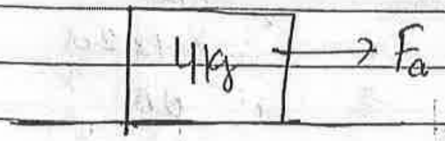
$a = \dfrac{v^2 - u^2}{2S}$

$-\mu_k g = \dfrac{0-(10)^2}{2\times 40}$

$\mu_k = \dfrac{1}{8} = 0.125$

Q① A block of mass 4kg is resting on horizontal floor μ_s & μ_k are $\frac{2}{3}$ and $\frac{1}{3}$ respec. Find the value of friction force & accln if the Applied Force have the Values

(i) 0 N (ii) 20 N (iii) 60 N



$f_L = \mu_s N$

$f_L = \frac{2}{3} \times 40 = \frac{80}{3} = \underline{26.6 N}$

(i) $F_{ext} = 0$

No tendency of relative motion

$f = 0$

② $F_{ext} = 20 N$

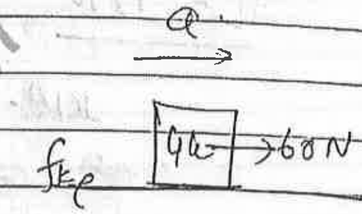
$F_{ext} < f_L$

No relative motion

$\boxed{a = 0}$

$f_s = F_{ext} = 20 N$

③



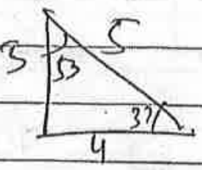
$60 - \mu_k N = 4a$

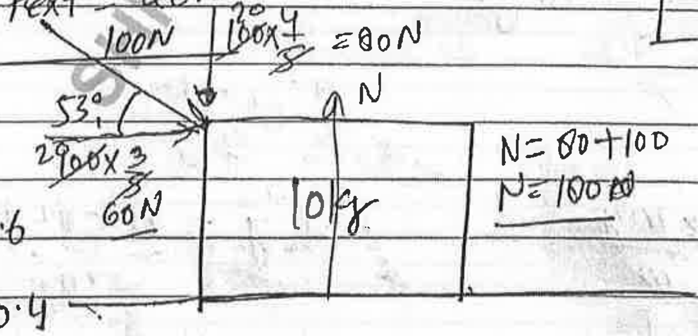
$15\cancel{60} - \frac{1}{3} \times \cancel{4}10 = \cancel{4} a$

$\frac{45 - 10}{3} = a$

$\frac{35}{3} = a \ m/s^2$

Q.② find acc^n & frictn of the block on





$\mu_s = 0.6$

$\mu_k = 0.4$

$N = 80 + 100$

$N = 180 N$

$f_L = \mu_s N$

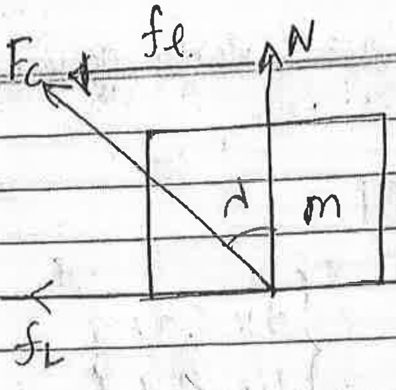
$f_L = 0.6 \times 180$

$f_L = 108$

$a = 0$, $f = F_{ext}$

No Relative Motion. $f_s = 60 N$

$\lambda \rightarrow$ angle of friction



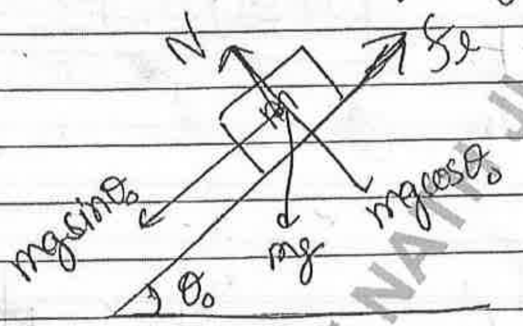
* The resultant of limiting friction & Normal make from Normal.

$$\tan\lambda = \frac{f_\ell}{N} \qquad \boxed{\tan\lambda = \mu_s}$$

$$\tan\lambda = \frac{\mu_s \cancel{N}}{\cancel{N}} \qquad \lambda = \tan^{-1}(\mu_s)$$

Angle Of Repose (θ_0)

$\hookrightarrow$ Block just going slide



$$mg\sin\theta_0 = f_\ell$$

$$mg\sin\theta_0 = \mu_s N$$

$$\cancel{mg}\sin\theta_0 = \mu_s \cancel{mg}\cos\theta_0$$

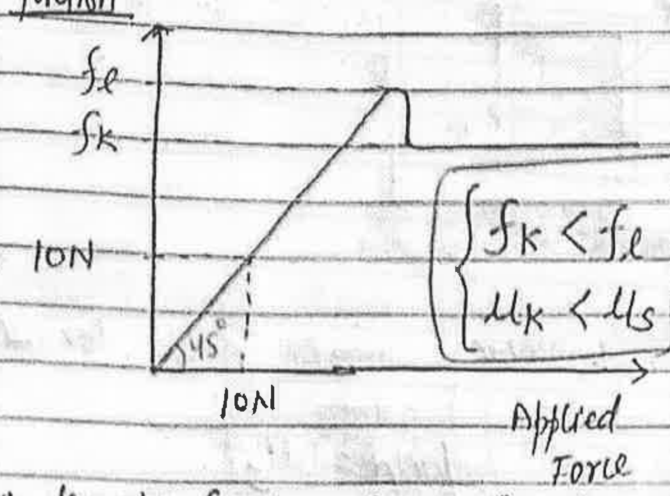
$$\boxed{\tan\theta_0 = \mu_s}$$

$$\boxed{\theta_0 = \lambda}$$

Coefficient of static & kinetic friction generally have value b/w 0 & 1

But in some exceptional case there value may exceeds one.

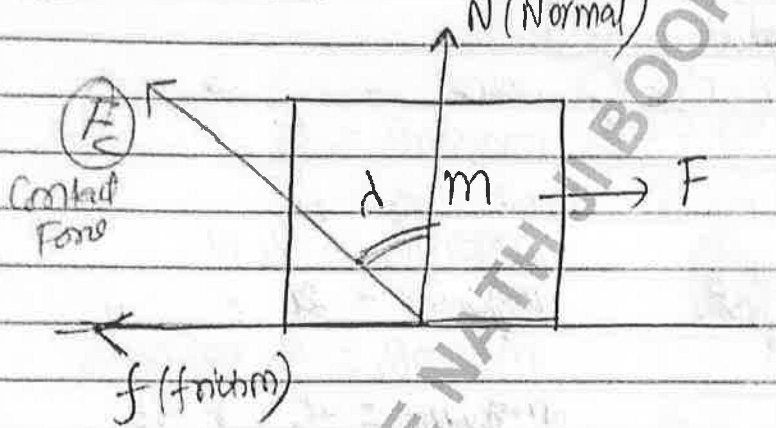
* graph b/w friction & Applied force.



$$\boxed{\begin{cases} f_k < f_l \\ \mu_k < \mu_s \end{cases}}^{*}$$

* Kinetic friction force is less than limiting friction but it may be more than static friction.

Contact Force



$$\boxed{F_c = \sqrt{N^2 + f^2}}$$

(1) $f = 0$

$(f_c)_{min} = N$

(2) $f = f_l$

(3) $(F_c)_{max} = \sqrt{N^2 + f_l^2}$

$= \sqrt{N^2 + \mu_s^2 N^2}$

$(f_c)_{max} = N\sqrt{1 + \mu_s^2}$

$$\boxed{N \leq f_c \leq N\sqrt{1+\mu_s^2}}$$

$\lambda \rightarrow$ angle of friction

Coefficient of kinetic friction'

$$\boxed{\mu_K = \frac{f_K}{N}}$$ unitless, dimensionless

depend Nature of surface

Question solving Method

① $F_{ext} > f_K$ speed up

$$F_{ext} - f_K = ma$$

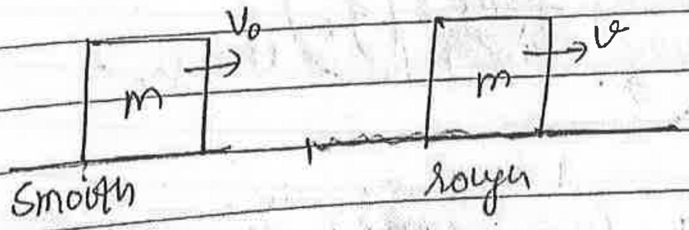
$$a = \frac{F_{ext} - f_K}{m}$$

② $F_{ext} = f_K$

$$\boxed{\begin{array}{l} a = 0 \\ \vec{v} = \text{const} \end{array}}$$

③ $F_{ext} < f_K$ speeding down $a = \frac{F_{ext} - f_K}{m}$

④ $F_{ext} = 0$ speed down $\boxed{a = 0 - \frac{f_K}{m}}$ $a = \mu_K g$



$f_K = \mu_K N$

$f_K = \mu_K Mg$

 Date ______ Page ______

② Limiting friction Force (f_ℓ) Max^m value of static friction force.

$$F_\ell \propto N \text{(Normal)}$$

$$\boxed{f_\ell = \mu_s N}$$

* limiting friction is independent of apparent area of contact.

coefficient of static friction

⊛ $$\boxed{\mu_s = \frac{f_L}{N}}$$ → unitless, dimensionless

↳ depends on Nature of surface.

① $F_{ext} = 0$ $f = 0$

② $F_{ext} < f_L$ No relative motion $\boxed{F_{ext} = f_s}$

③ $F_{ext} = f_L$ relative motion just going to start [Impending motion going to start]

④ $F_{ext} > f_L$ relative motion starts

↳ Static friction disappears
Kinetic or sliding friction appear. *

③ Kinetic or sliding friction force: (f_k)
Force of friction acting during the Relative Motion.

$$f_K \propto N$$

$$\boxed{f_K = \mu_K N}$$

f_K → independent of apparent area of contact (actual)

nearly independent of velocity.

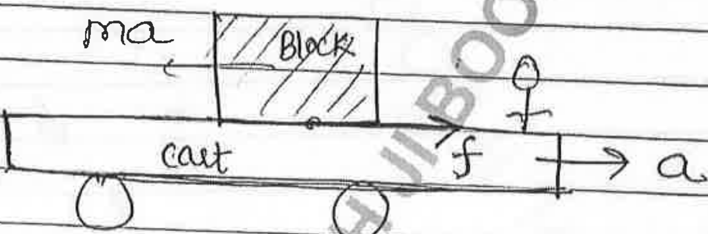
⊛ Friction ⊛

Component of contact force which is ∥ to the contact surface is c/a friction. & dirⁿ opp. to Relative Motion.

* Friction does NOT opposses Motion, it support Motion.

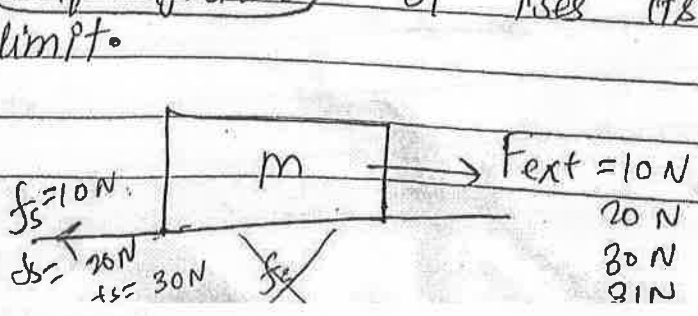
* Friction apposses Relative Motion

Reason: (I) Ancient view: Interlocking b/w the surfaces.
(II) Modern view: Due to interatomic or intermolecular (I.M.F) force of attraction b/w the atoms of surfaces. (Nature → Electromagnetic)



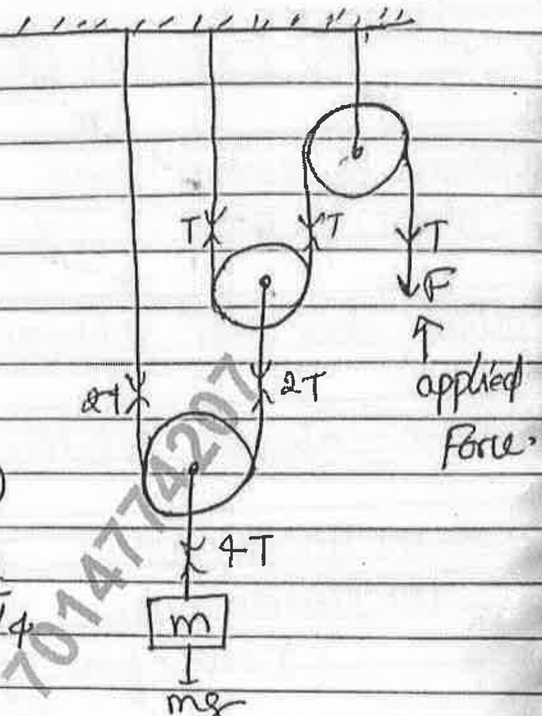
Types of friction:
① Static friction.
② Limiting ,,
③ Kinetic ,,

① Static friction Force (f_s) the force of friction acts before the Relative motion starts.
It is self adjustable. It ↑ses its value up to a certain limit.



*

M·A → ज्यादा
मतलब machine
अच्छी·



$$4T = mg$$

$$T = \frac{mg}{4}$$

$$F = \frac{mg}{4} \text{ (Effort)}$$

$$M \cdot A = \frac{\text{Load}}{\text{Effort}} = \frac{mg}{mg/4}$$

$$M \cdot A = 4$$

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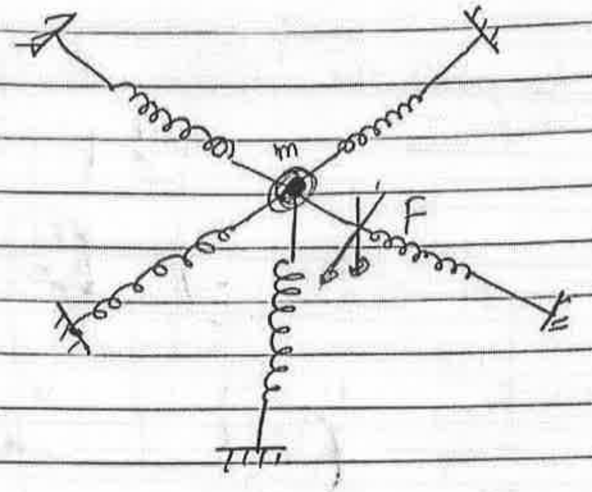
*The particle is in Eqbm, One force ~~F~~ is 'F' if this ~~two~~ particle spring is 'Cut' F/o accln of particle at this moment.

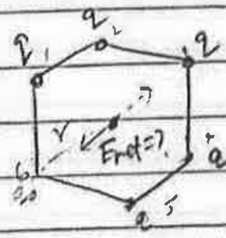


$$\vec{F_1}+\vec{F_2}+\vec{F_3}+\vec{F_4}+\vec{F}=0$$

$$\vec{F_1}+\vec{F_2}+\vec{F_3}+\vec{F_4}=-\vec{F}$$

$$\vec{a}=\frac{\vec{F}_{net}}{m}=\frac{\vec{F_1}+\vec{F_2}+\vec{F_3}+\vec{F_4}}{m}$$

$$\vec{a}=\frac{-\vec{F}}{m}$$

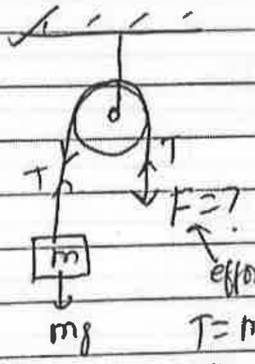


$F = kq_1 q_2$

$\vec{F_1}+\vec{F_2}+\vec{F_3}+\vec{F_4}+\vec{F_5}+\vec{F_6}=0$

$\vec{F_1}+\vec{F_2}+\vec{F_3}+\vec{F_4}+\vec{F_5}=-\vec{F_6}$

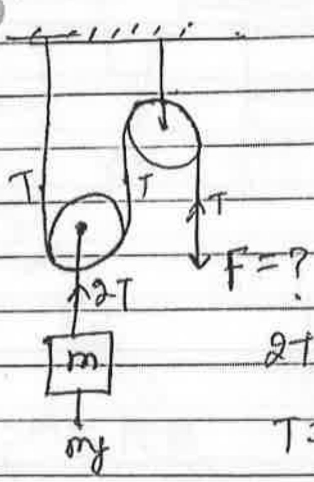
Machanical Advantage $= \dfrac{\text{Load}}{\text{Effort}}$



$M.A = \dfrac{mg}{mg} = 1$

$T = mg$

$F = mg$ (effort)



$2T = mg$

$T = \dfrac{mg}{2}$

$F = \dfrac{mg}{2}$ (Effort)

~~M.A~~

$M.A = \dfrac{mg}{mg/2}$

$= 2$

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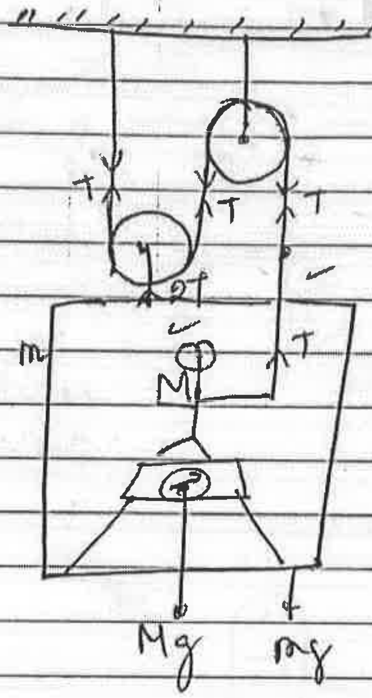
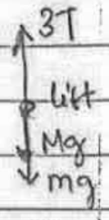
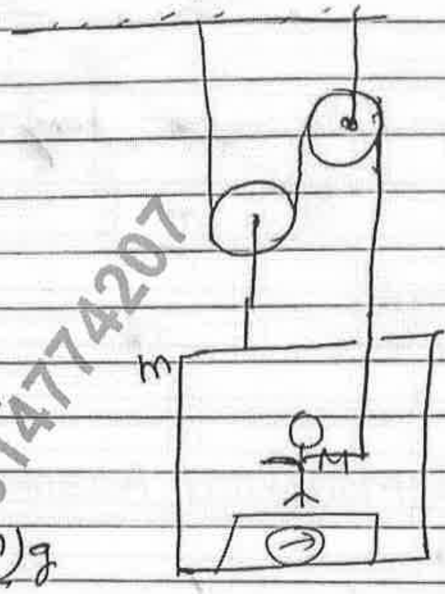
* Mass of Man → 70kg
" " lift → 50kg
Find the force by which he pulls the string & Normal Force on man.







$3T = Mg + mg$

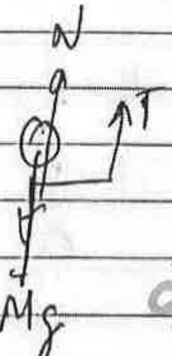
~~3T (N~~

$T = \frac{(M+m)g}{3}$

$T = \frac{\cancel{125\times10}}{3} = \frac{(\cancel{120})^{4}\,10}{\cancel{3}}$

$T = 400N$

Reading

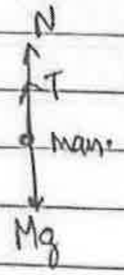


$N + T = Mg$

$N = Mg - T$

$= 700 - 400$

$N = 300N$



Reading of Weight Machine → $N = 30$ kg-wt

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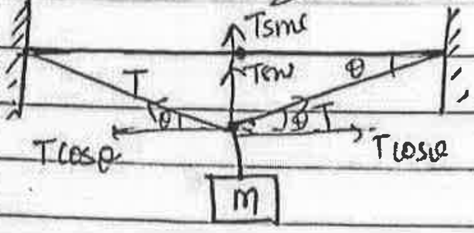
Brackets/Purchasing

mid point

* 

$$2T\sin\theta = mg$$

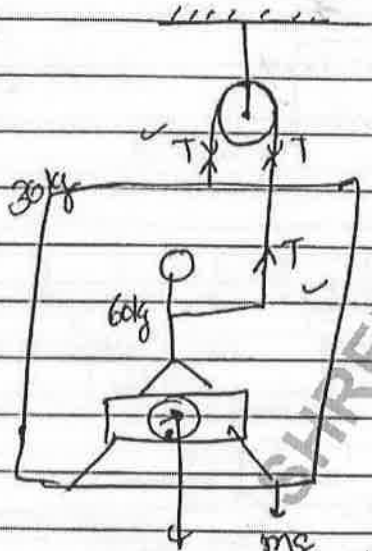
$$T = \frac{mg}{2\sin\theta}$$

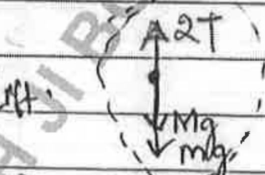
* Tension in the string so that string becomes straight

$\theta = 0$, $\sin 0 = 0$

$T = \frac{mg}{0} = \infty$ (Not possible) → $T = \infty$ (String break)

* In the given fig. mass of the man is 60kg & mass of lift 30kg. To keep the system in Eqb^m with what force he should pull the string and what is the force exerted by lift on the man or reading of spring balance.

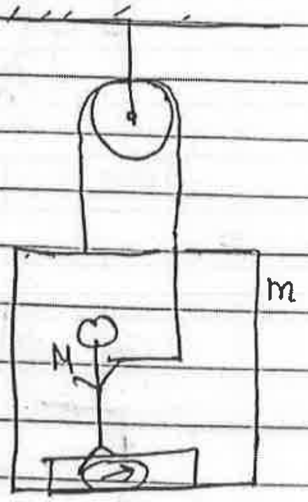


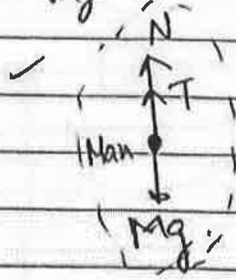


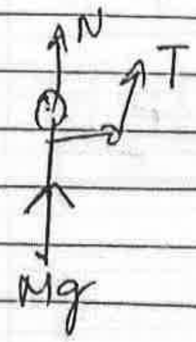
$$2T = Mg + mg$$

$$T = \frac{(M+m)}{2}g$$

$$T = \frac{(60+30)}{2}\times 10 = 450N$$







$$N + T = Mg$$

$$N = Mg - 400$$

$$= 150N$$

$$N = 15\ kg\text{-}wt$$

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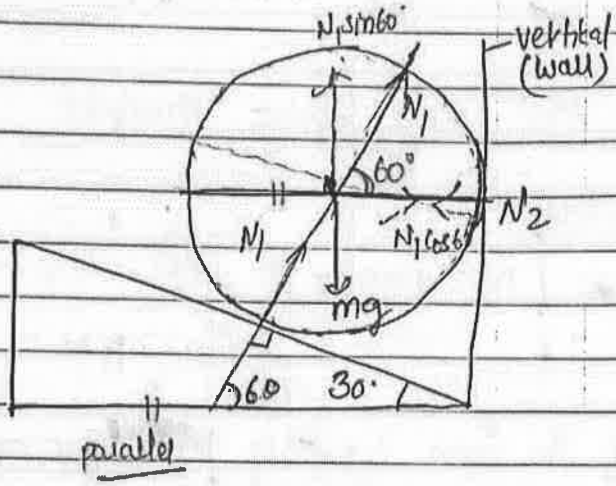
Q. Mass of sphere 'M' Find Normal Force exerted by Inclined plane & wall ?

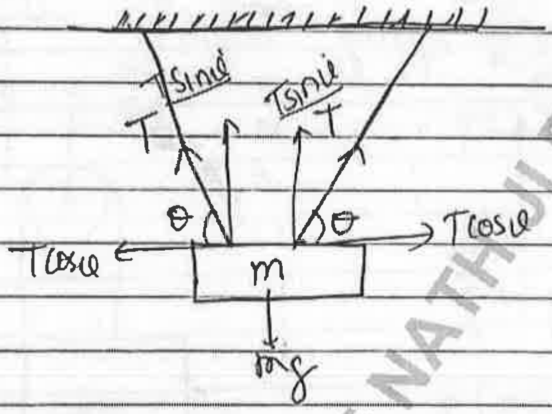


$N_1 \sin 60 = mg$ $\quad N_1 = \frac{2}{\sqrt{3}} mg$

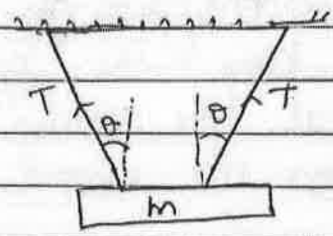
$N_1 \cos 60° = N_2$

$N_2 = \frac{mg}{\sqrt{3}}$

Q $T = ?$



also,



$2T\sin\theta = mg = w$

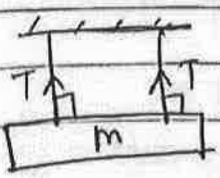
$$\boxed{T = \frac{w}{2\sin\theta}}$$

For minimum tension,

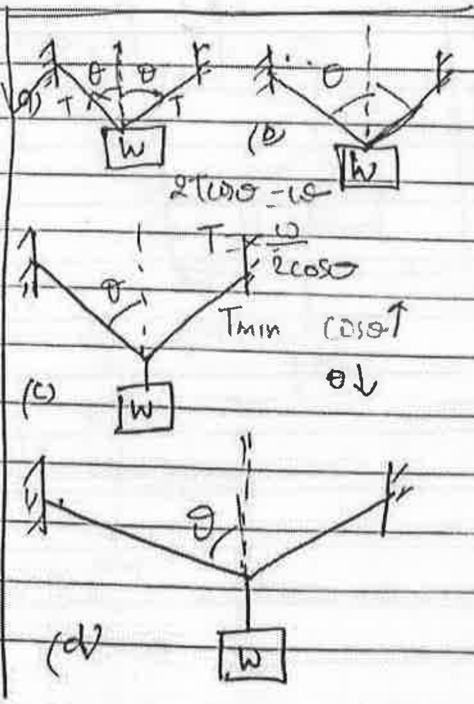
$\downarrow T \propto \frac{1}{(\sin\theta)}\uparrow$ $\quad$ $\sin\theta = +1$ (max)

$\theta = 90°$

string should be parallel



$\Leftarrow T_{min}$



$2T\cos\theta = w$

$T = \frac{w}{2\cos\theta}$

T_{min} $\cos\theta\uparrow$

$\theta\downarrow$

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