

$$\frac{2.1 \times 10^3 \times 100 \times 4/2}{3600} = K (4/2) \times 10^{-2}$$

$$\frac{7 \times 100}{12} = K \times 10^{-2}$$

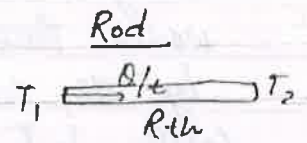
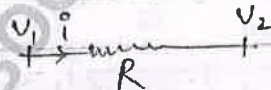
$$K = \frac{7 \times 100 \times 10^2}{12}$$

$$K = 4.67 \times 10^3 \text{ W m-K} \quad (\text{Ans.})$$

• Thermal Resistance ( $R_{th}$  or  $R$ )

→ Property of material to resist heat flow through it.

$$R_{th} \text{ or } R = \frac{L}{KA}$$



$$\therefore \frac{Q}{t} = \frac{KAL(T_1 - T_2)}{L}$$

$$\frac{Q}{t} = \frac{T_1 - T_2}{\frac{L}{KA}}$$

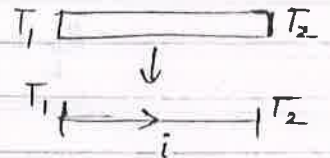
$$\left[ \frac{Q}{t} = \frac{T_1 - T_2}{R} \right]$$

$$\therefore i = \frac{V_1 - V_2}{R}$$

$$\therefore \left( \frac{Q}{t} \right) = \frac{T_1 - T_2}{R_{th}}$$

$i_r = \text{Heat current}$

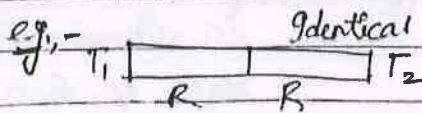
$$i = \frac{T_1 - T_2}{R}$$



Note: ⇒ (1) If two rods are identical

$L, A, K \rightarrow \text{same}$

$R \rightarrow \text{same}$



$\therefore$  series  $\rightarrow R_{eq} = 2R$


②. If two rods are identical in dimension.

$A$  &  $l \rightarrow$  same.

$K \rightarrow$  Different.


$$\therefore R = \frac{l}{KA} \Rightarrow R \propto \frac{l}{K}$$

Two rods same in dimension.

eg:-   $\therefore R \propto \frac{l}{K}$

$R_2 = 4R$      $R_1 = R$

Ques:- 3 identical rods are arranged in such a way that one end of rod is connected to another end of second rod. Heat flow through the combination is 40J in 9 minutes. If these rods would have been kept one over another then how long time taken by this combination to flow 80J of heat through it.

Case (i).   $\therefore \frac{Q}{t} = \frac{KA(T_1 - T_2)}{L}$

$$\frac{40}{9} = \frac{T_1 - T_2}{3R} \quad \text{--- (i)}$$

Case (ii).   $\therefore \frac{80}{t} = \frac{T_1 - T_2}{R/3} \quad \text{--- (ii)}$

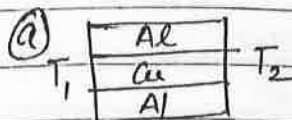
$$R_{eq} = \frac{R}{3}$$

$$\text{(i)} \div \text{(ii)}$$

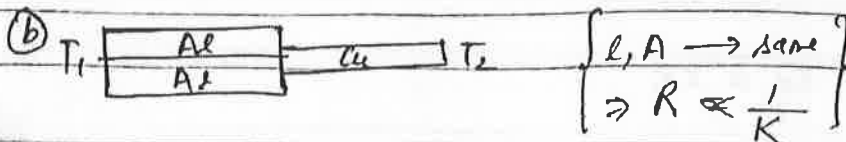
$$\frac{t}{10} = \frac{R}{9R}$$

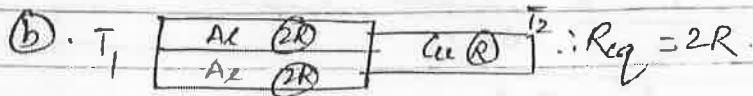
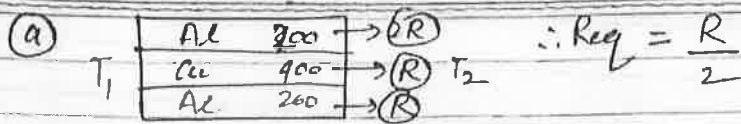
$$t = 2 \text{ min (Ans):}$$

Ques:-  $K_{Cu} = 400$  unit  
 $K_{Al} = 200$  unit



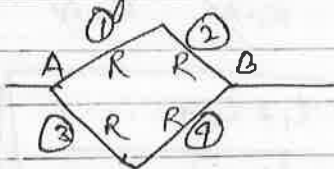
If rods are similar in dimension then ratio of heat currents through (a) & (b)





$$\therefore i \propto \frac{1}{R} \Rightarrow \frac{i_a}{i_b} = \frac{2R}{R/2} = \frac{4}{1}$$

Quey! - If two identical rods are arranged in series, 40 J of heat can flow through it in 1 min. If similar 4 rods are arranged as given in diagram then flow time taken by this combination to flow same amount of heat through it? [Temp. difference are same in both conditions]



$$\therefore R_{eq} = 2R$$

$$\therefore i_a = \frac{40}{60 \text{ sec}} = \frac{T_1 - T_2}{2R} \quad \text{--- (1)}$$

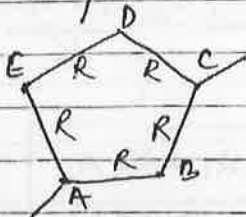
(b)  $\therefore R_{eq} = R$

$$\therefore i_b = \frac{40}{t} = \frac{T_1 - T_2}{R} \quad \text{--- (2)}$$

$$\text{(1)} \div \text{(2)}$$

$$t = 30 \text{ sec.}$$

Quey! - 5 identical rods are arranged in Pentagon frame and heat current through this combination is 'i' if temp. are maintained at point A & C of frame. P/o value of heat current through combination of given temp. would have been kept at point A and B.



Case (1)  $\frac{1}{R_{eq}} = \frac{1}{3R} + \frac{1}{2R}$

$$R_{eq} = \frac{6R}{5}$$

Case (2)  $R_{eq} = \frac{4R \times R}{4R + R} = \frac{4R}{5}$

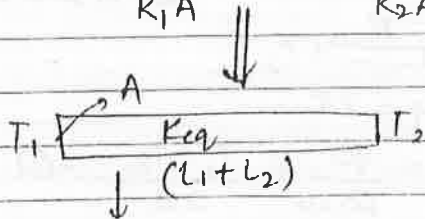
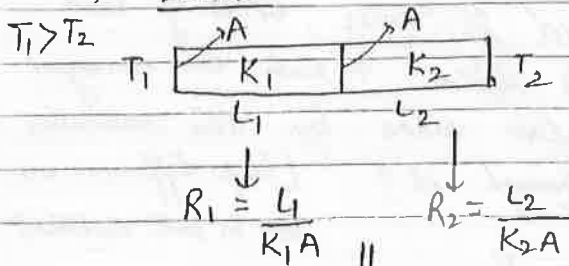
$$\therefore i = \frac{T_1 - T_2}{R}$$

$$i \propto \frac{1}{R} \Rightarrow \frac{i}{i_2} = \frac{4R/5}{6R/5} = \frac{2}{3}$$

$$i_2 = \frac{3}{2} i$$

• Different Combination of Rods.

(1). Series



$$\therefore R_{eq} = \frac{L_1 + L_2}{K_{eq} \cdot A}$$

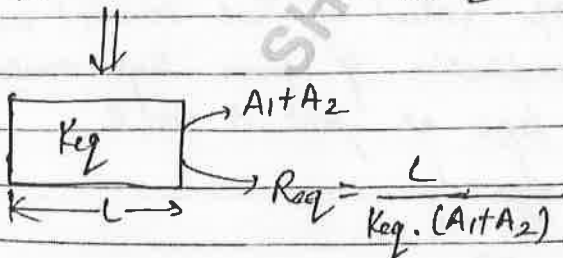
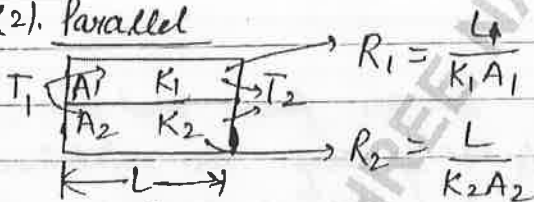
Series

$$\therefore R_{eq} = R_1 + R_2$$

$$\frac{L_1 + L_2}{(K_{eq}) \cdot A} = \frac{L_1}{K_1 \cdot A} + \frac{L_2}{K_2 \cdot A}$$

$$K_{eq} = \frac{L_1 + L_2 + \dots}{\frac{L_1}{K_1} + \frac{L_2}{K_2} + \dots}$$

(2). Parallel



Parallel

$$\therefore \frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

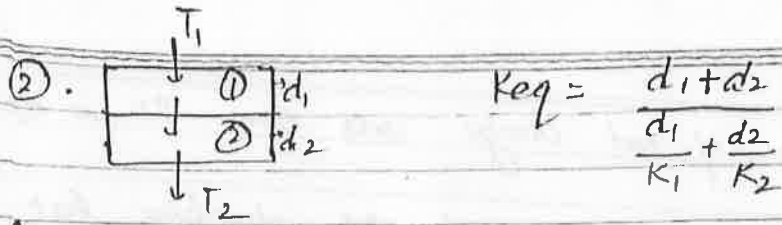
$$\frac{K_{eq} (A_1 + A_2)}{L} = \frac{K_1 A_1}{L} + \frac{K_2 A_2}{L}$$

$$K_{eq} = \frac{K_1 A_1 + K_2 A_2}{A_1 + A_2}$$



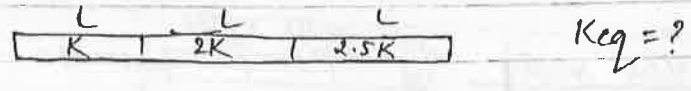
Parallel

$$K_{eq} = \frac{K_1 A_1 + K_2 A_2}{A_1 + A_2}$$



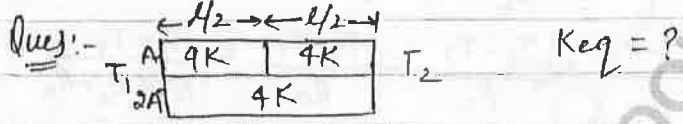
③. If temp. are not mentioned then take series as preference.

Que:- 3 rods similar in dimension



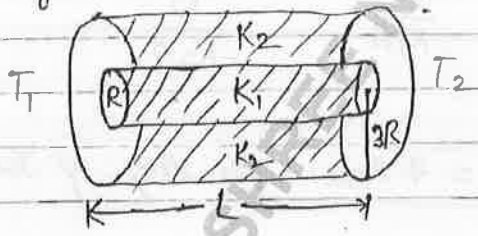
$$\therefore K_{eq} = \frac{L + L + L}{\frac{L}{K} + \frac{L}{2K} + \frac{L}{2.5K}} = \frac{3K \times 5K}{K(5 + 2.5 + 2)}$$

$$K_{eq} = \frac{15K}{9.5}$$



$\therefore K_{eq} = 4K$   $\rightarrow$  same material hai.

Que:- 2 concentric cylinders of radius R & 3R of material  $K_1$  &  $K_2$  conductivity are arranged as shown. F/o equivalent conductivity of combination?



Parallel combination:-

$$\therefore K_{eq} = \frac{K_1 A_1 + K_2 A_2}{A_1 + A_2}$$

$$K_{eq} = \frac{K_1 (\pi R^2) + K_2 [\pi (3R)^2 - \pi R^2]}{[\pi R^2] + [\pi (3R)^2 - \pi R^2]}$$

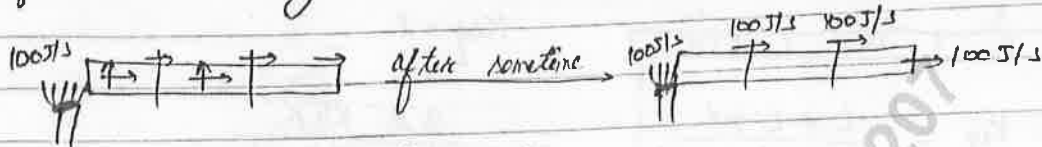
$$K_{eq} = \frac{K_1 + 8K_2}{9}$$

• Steady state

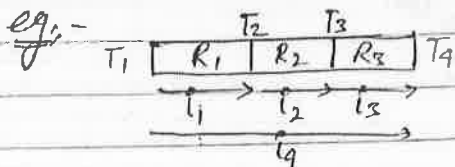
$\rightarrow$  When a rod is heated, each & every section of rod absorbs heat, increase its internal energy and remaining part is transferred to next section. That's why temp. of

each and every section of rod change with time. It is k/m as variable state.

After some time different sections of rod stop absorbing heat and completely transfer into next section that's why temp. of each and every section remains constant (steady) with time.



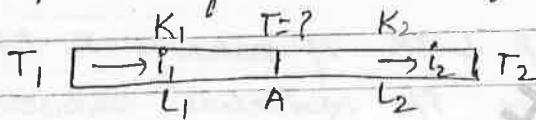
At steady state; -  
Heat current [ $i_{input} = i_{output}$ ]



$\therefore$  steady state,  
 $i_1 = i_2 = i_3 = i_4$

$$\frac{T_1 - T_2}{R_1} = \frac{T_2 - T_3}{R_2} = \frac{T_3 - T_4}{R_3} = \frac{T_1 - T_4}{R_1 + R_2 + R_3}$$

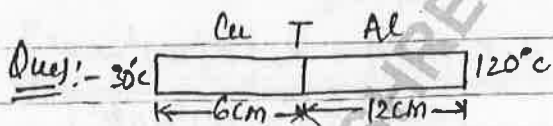
# Temperature of Junction Interface; -



Temp. of Junction = ?

Steady state at (A)  
 $i_1 = i_2$

$$\frac{T_1 - T}{\frac{L_1}{K_1 A}} = \frac{T_2 - T}{\frac{L_2}{K_2 A}}$$



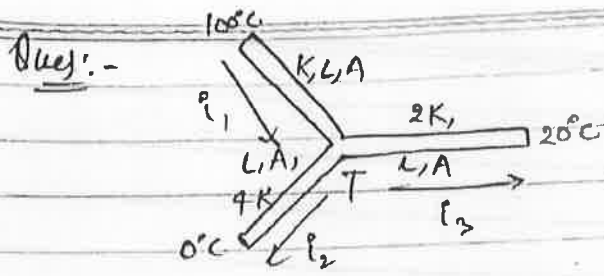
If  $K_{cu} = 4 K_{Al}$  find temp. of Junction.

$$\therefore \frac{120 - T}{\frac{12}{K_{Al} \cdot A}} = \frac{T - 30}{\frac{6}{4 K_{Al} \cdot A}}$$

$$\frac{120 - T}{12/4} = \frac{(T - 30) \times 2}{2}$$

$$120 - T = 8T - 240$$

$$9T = 360 \Rightarrow T = 40^\circ\text{C}$$



F/o temp of Junction?

$$\therefore i_1 = i_2 + i_3$$

$$\frac{100-T}{\frac{L}{KA}} = \frac{T-20}{\frac{L}{2KA}} + \frac{T-0}{\frac{L}{4KA}}$$

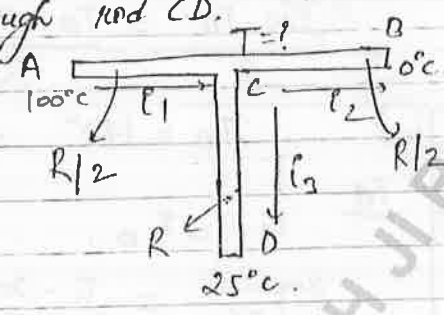
$$100-T = (T-20) \times 2 + 4T$$

$$100-T = 2T-40+4T$$

$$7T = 140$$

$$T = 20^\circ\text{C}$$

Quey:- 2 rods AB & CD are similar and CD connected to mid point of AB. If resistance of CD is  $(4K\text{W}^{-1})$  then f/o heat current through rod CD.



Steady at (C).

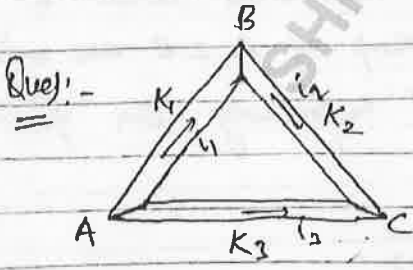
$$\therefore i_1 = i_2 + i_3$$

$$\frac{100-T}{\frac{R}{2}} = \frac{T-0}{\frac{R}{2}} + \frac{T-25}{R}$$

$$\Rightarrow T = 45^\circ\text{C}$$

Now,  $i_2 = \frac{45-25}{4}$

$$= \frac{20}{4} = 5 \text{ watt}$$



If rods are identical in dimension arranged in equilateral triangle. If heat current through ABC is equal to that of AC then

- (A)  $K_3 = K_1 + K_2$
- (B)  $K_3 = \frac{K_1 K_2}{K_1 + K_2}$
- (C)  $K_3 = \frac{2 K_1 K_2}{K_1 + K_2}$
- (D) NOTA

$$i_{ABC} = i_{AC}$$

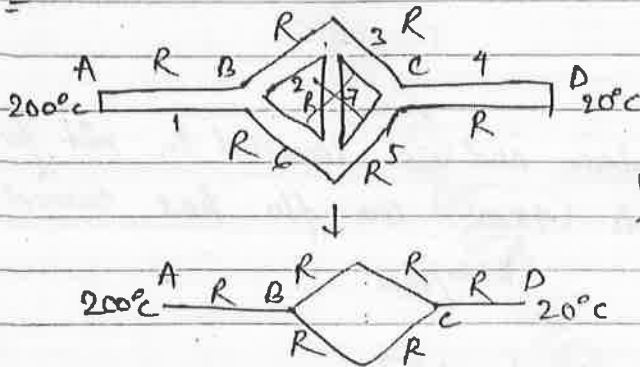
$$\frac{I_A - T_c}{R_{AB} + R_{BC}} = \frac{I_A - T_c}{R_{AC}}$$

$$\Rightarrow R_{AB} + R_{BC} = R_{AC}$$

$$\frac{L}{K_1 A} + \frac{L}{K_2 A} = \frac{L}{K_3 A}$$

$$\Rightarrow K_3 = \frac{K_1 K_2}{K_1 + K_2}$$

Ans! - 7 Identical rods.



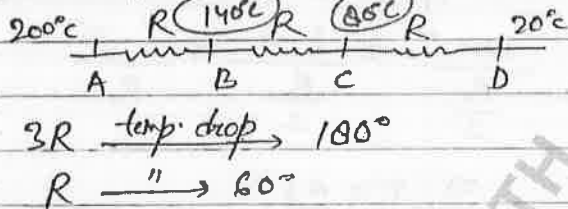
f/o temp. at point B & C.

$$\frac{M}{D} \quad \frac{T_B}{R} \quad i_{AB} = i_{BD}$$

$$\frac{200 - T_B}{R} = \frac{T_B - 20}{2R}$$

$$T_B = 140^\circ$$

M-2) Temp. Drop Method

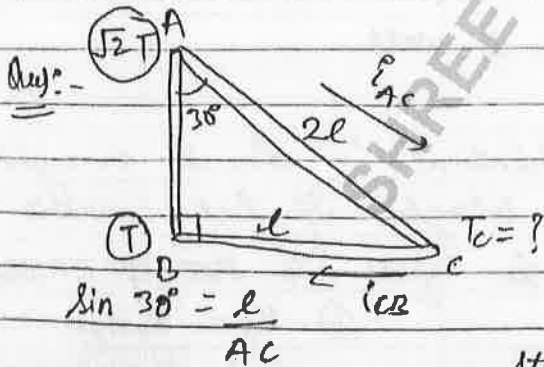


$$\frac{T_C}{R} \quad i_{AC} = i_{CD}$$

$$\frac{200 - T_C}{2R} = \frac{T_C - 20}{R}$$

$$T_C = 80^\circ C.$$

3R temp. drop  $\rightarrow$  180°  
R "  $\rightarrow$  60°



3 rods having same cross-section & material. If temp. of A & B are maintained at  $52T$  &  $T$  respectively then f/o temp at pt 'c'.

Steady state at 'c'.

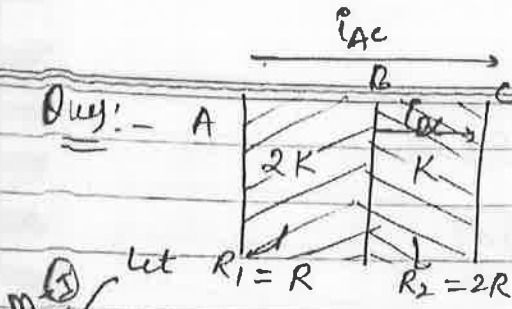
$$i_{AC} = i_{CB}$$

$$\frac{52T - T_c}{2l} = \frac{T_c - T}{l}$$

$$\frac{2l}{KA} = \frac{l}{KA}$$

$$\Rightarrow 52T - T_c = 2T_c - 2T \Rightarrow T_c = \frac{(2+52)T}{3}$$





If given wall made up of layer AB & BC of same <sup>cross-section</sup> ~~material~~ & thickness. If temp. difference across layer BC is  $21^\circ\text{C}$ . Then

m (i)  $\therefore R_1 = R$   $R_2 = 2R$   
 $\therefore l \ \& \ A = \text{same} \ \therefore R \propto \frac{1}{K}$

$\therefore R_{eq} = 3R$

$\therefore \frac{2R}{R} \xrightarrow{\text{temp. drop}} \frac{21^\circ\text{C}}{10.5^\circ\text{C}}$

So,  $3R \longrightarrow 10.5 \times 3 = 31.5^\circ\text{C}$

(OR)

m (ii) Steady state

$$i_{BC} = i_{AC}$$

$$\frac{(\Delta T)_{BC}}{2R} = \frac{(\Delta T)_{AC}}{3R}$$

$$(\Delta T)_{AC} = \frac{3}{2} \times 21^\circ\text{C} = 31.5^\circ\text{C}$$

• Temperature Gradient  $(dT/dx)$  {T.G.}

$\rightarrow$  change in temp. of rod in per unit length.



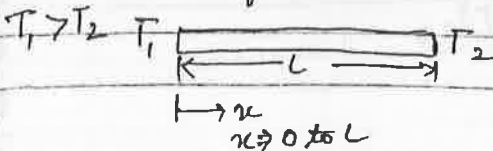
$$\therefore \text{T.G.} = \frac{dT}{dx} = \frac{T_2 - T_1}{L}$$

$$\therefore \frac{Q}{t} = i = \frac{KA(T_1 - T_2)}{L} = - \frac{KA(T_2 - T_1)}{L}$$

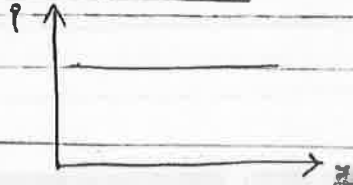
$$\Rightarrow \boxed{\frac{Q}{t} = -KA \frac{dT}{dx}}$$

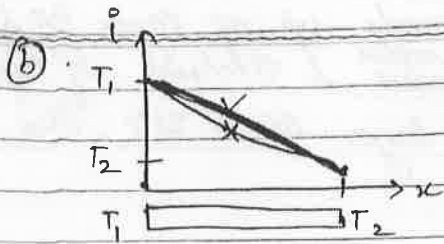
# Variation of temp. across the length of Rod:

(A) Uniform Area Rod



(a) Heat current v/s x





Slope,  $\frac{dT}{dx}$  (T.G.)

$$\therefore \left(\frac{Q}{t}\right) = -KA \frac{dT}{dx}$$

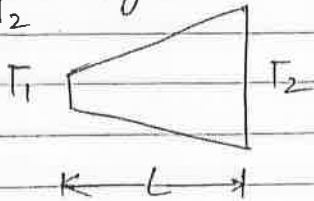
$$\frac{dT}{dx} = -ve$$

(Slope)

(B) Variable Area Rod

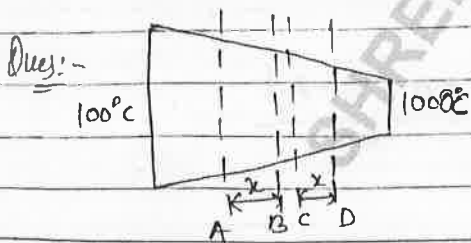
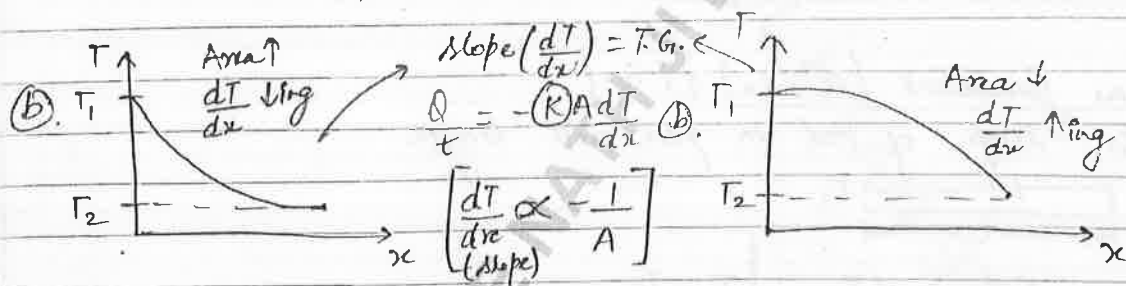
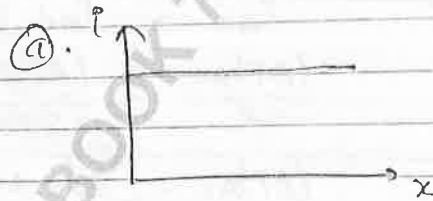
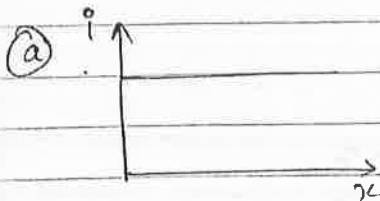
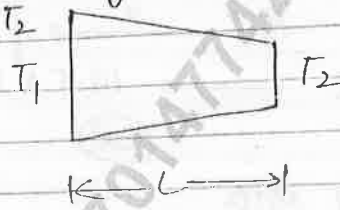
(i). Increasing area rod

$$T_1 > T_2$$



(ii). Decreasing area rod.

$$T_1 > T_2$$



Here, ①  $|T_A - T_B| = |T_C - T_D|$

②  $|T_A - T_B| > |T_C - T_D|$

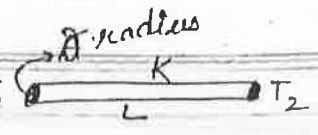
③  $|T_A - T_B| < |T_C - T_D|$

④ NOTA

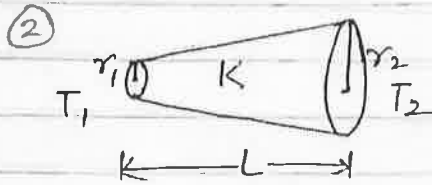
$\therefore$  Area of CD  $<$  AB

So,  $\left(\frac{dT}{dx}\right)_{CD} > \left(\frac{dT}{dx}\right)_{AB}$

$$(\Delta T)_{CD} > (\Delta T)_{AB}$$

Note:  $\Rightarrow$  ①  $T_1$    $T_1 > T_2 \quad \therefore \frac{Q}{t} = \frac{KA(T_1 - T_2)}{L}$

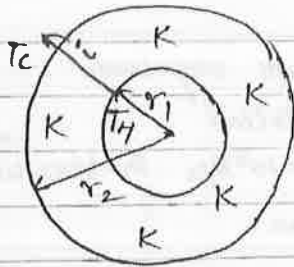
$$\frac{Q}{t} = \frac{K(\pi r^2)(T_1 - T_2)}{L}$$



$$\therefore \frac{Q}{t} = \frac{KA(T_1 - T_2)}{L} = \frac{K(\pi r_1 r_2)(T_1 - T_2)}{L}$$

Illu-28

③ 2 concentric hollow sphere of radius  $r_1$  &  $r_2$  with a material of 'K' conductivity filled in b/w them. If temp.  $T_H$  (hotter) &  $T_C$  (colder) are maintained at their surface respectively. Then  $\frac{Q}{t} = ?$



$$\therefore \frac{Q}{t} = \frac{KA(T_1 - T_2)}{L}$$

$$\frac{Q}{t} = \frac{K(4\pi r_1 r_2)(T_H - T_C)}{r_2 - r_1}$$

• Formation of Ice on Water bodies

$\rightarrow$  Anomalous Behaviour of Water:— Due to anomalous behaviour of water, it solidifies from top to bottom dir<sup>n</sup> while rest of liquids solidify (melted wax) from bottom to top dir<sup>n</sup>.

water ( $0^\circ\text{C} \rightarrow 4^\circ\text{C}$ )  $\Rightarrow T \uparrow \Rightarrow V \downarrow \Rightarrow \rho \uparrow$   
 ( $4^\circ\text{C} \rightarrow 0^\circ\text{C}$ )  $\Rightarrow T \downarrow \Rightarrow V \uparrow \Rightarrow \rho \downarrow$

$\rightarrow$  Formation of Ice from  $x_1$  thickness to  $x_2$  thickness from surface of water body in time 't' is given by —

$$t = \frac{\rho \cdot L}{2K\theta} [x_2^2 - x_1^2]$$

$$\Rightarrow t \propto (x_2^2 - x_1^2)$$

Here,  $\theta$  = temp. of Atmosphere  
 $\rho$  = density  
 $L$  = latent heat of fusion  
 $K$  = conductivity of Ice.

Ques:- Time taken in formation of Ice upto 2cm from surface of water body is 5 hrs then find time taken in formation

of ice in further 1 cm.

$$\therefore t \propto (x_2^2 - x_1^2)$$

$$\Rightarrow \frac{5}{t_2} = \frac{2^2 - 0^2}{3^2 - 2^2}$$

$$\frac{5}{t_2} = \frac{4}{5}$$

$$t_2 = \frac{25}{4} = 6.25 \text{ hrs.}$$

• Radiation

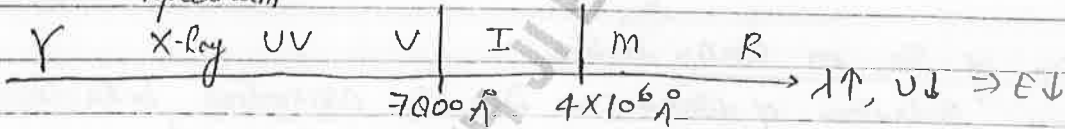
→ If  $T > 0$  Kelvin ; then body emits radiation

→ These radiations are EM waves.   
 → No medium required   
 → Fastest method

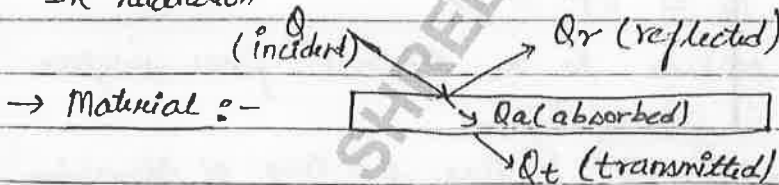
$c = 3 \times 10^8 \text{ m/s}$  in Vacuum.

→ Wave hai toh, ' $\lambda$ ' or  $\nu$  (frequency) bhi hoga.

→ EM wave spectrum



→ Radiation emitted from a body comes under IR region of EM wave spectrum that's why thermal radiation are also k/n as IR radiation



$$\therefore Q = Q_r + Q_a + Q_t$$

$$\frac{Q}{Q} = \frac{Q_r}{Q} + \frac{Q_a}{Q} + \frac{Q_t}{Q}$$

$$1 = \underbrace{(\rho)}_{\text{Reflective Power}} + \underbrace{(a)}_{\text{Absorptive Power/Coefficient}} + \underbrace{(t)}_{\text{Transmittive Power}}$$

## # Prevost Theory of Heat Exchange

→ A/c to it each and every body emits radiation at all possible temp. and absorbs as well.

$T \rightarrow$  temp. of body

$T_0 \rightarrow$  temp. of surrounding

①  $T > T_0$  [Hum in AC room]

Radiation emitted  $>$  absorbed

By body  $\Rightarrow$  Net emission  $\Rightarrow$  I.E.  $\downarrow \Rightarrow$  cooling

②  $T < T_0$  [Hum in heater room]

Radiation emitted  $<$  absorbed (by body)

$\rightarrow$  Net absorbed  $\Rightarrow$  I.E.  $\uparrow \Rightarrow$  heating

③  $T = T_0$

If two bodies are at thermal eqm then they will exchange equal amount of heat by radiation but net will be zero.

## # Stefan's Law

→ A/c to it emissive power of an ideal black body is directly proportional to fourth power of its absolute temp.

$$\therefore E \propto T^4$$

$$E = \sigma T^4$$

$\rightarrow$  Stefan's constant  $\Rightarrow \sigma = 5.67 \times 10^{-8} \frac{W}{m^2-K^4}$

$$\frac{Q}{At} = \sigma T^4$$

Radiation emitted per sec (OR) Radiating Power

$$\left(\frac{Q}{t}\right)_{IBB} = \sigma AT^4$$

$$\Rightarrow \left(\frac{Q}{t}\right)_{G.B} = e(\sigma AT^4)$$

→ If IBB kept in  $T_0$  (IBB) surrounding.

Stefan's for surrounding

$$E_0 = \sigma T_0^4$$

$$\left(\frac{Q}{t}\right)_{\text{absorbed by body}} = (\sigma T_0^4) \times A$$

$$\left(\frac{Q}{t}\right)_{\text{absorbed}} = \sigma AT_0^4$$

## # Ideal Black Body (IBB)

→ A hypothetical body that absorbs all the radiations incident on it, and emits as well.

$$\text{For IBB} \Rightarrow a_{\text{IBB}} = \frac{Q_a}{Q} = \frac{Q}{Q} = 1$$

$$a_{\text{IBB}} = 1 ; r = 0 = t \text{ for IBB.}$$

→ For Grey bodies (GB) {Real bodies}

$$\therefore a = \frac{Q_a}{Q} \Rightarrow Q_a < Q$$

$$a_{\text{GB}} < 1$$

eg:- Sun (Assumed)

↳ IBB

## # Important Definitions

(1). Absorptive Power ( $a$ )

$$\left[ a = \frac{Q_a}{Q} \right] \text{ (fraction)}$$

$$a_{\text{IBB}} = 1$$

$$a_{\text{GB}} < 1$$

(2). Spectral Absorptive Power ( $a_\lambda$ )

$$\left[ a_\lambda = \frac{Q_{a,\lambda}}{Q_\lambda} \right] \text{ (fraction)}$$

$$a_{\lambda, \text{IBB}} \rightarrow a_{\lambda, \text{IBB}} = 1$$

$$a_{\lambda, \text{GB}} \rightarrow a_{\lambda, \text{GB}} < 1$$

(3). Emissive Power ( $E$ )

→ Amount of radiation emitted from per unit area of body in per unit time at a given temp. is kn as Emissive power of body at that temp.

$$E = \frac{Q}{At} \Rightarrow \text{Unit} \rightarrow \frac{\text{J}}{\text{m}^2 \cdot \text{sec}} \text{ (or)} \frac{\text{W}}{\text{m}^2}$$

(4). Relative Emissive Power / Emissivity ( $e$  or  $e_r$ ) → (fraction)

$$(e \text{ or } e_r)_{\text{GB}} = \frac{E_{\text{GB}}}{E_{\text{IBB}}} = \frac{Q_{\text{GB}}/At}{Q_{\text{IBB}}/At} = \frac{Q_{\text{GB}}}{Q_{\text{IBB}}}$$

$$\left[ Q_{\text{GB}} = e Q_{\text{IBB}} \right]$$

$$\text{For } E_{\text{IBB}} = \frac{E_{\text{IBB}}}{E_{\text{IBB}}} \Rightarrow e_{\text{IBB}} = 1 \text{ so } e_{\text{GB}} < 1 \text{ (}\because Q_{\text{GB}} < Q_{\text{IBB}}\text{)}$$

If  $T > T_0$

$$\left(\frac{Q}{t}\right) = \underbrace{\sigma A T^4}_{\text{emitted}} - \underbrace{\sigma A T_0^4}_{\text{absorbed}}$$

$$\boxed{\frac{Q}{t} = \sigma A [T^4 - T_0^4]}$$

↳ Radiation emitted for net (Net)

∴  $T > T_0 \Rightarrow$  Net emission  $\Rightarrow$  Heat loss.

$$\frac{Q}{t} = \frac{\text{Heat loss}}{\text{time}} \Rightarrow \text{Rate of heat loss (R}_H)$$

$$\therefore \boxed{R_H = \frac{Q}{t} = \sigma A [T^4 - T_0^4]}$$

∴ Heat loss  $\Rightarrow$  Temp.  $\downarrow \Rightarrow$  Cooling

$$R_H = \frac{Q}{t} = \frac{msdT}{dt} = \sigma A [T^4 - T_0^4]$$

$$\boxed{R_f = \frac{dT}{dt} = \frac{\sigma A [T^4 - T_0^4]}{ms} = \frac{R_H}{R_c}}$$

↳ Rate of fall in temp. (or) Rate of cooling.

Note: - ①. Different shapes of same mass & material

Area: - Plate  $>$  Cube  $>$  sphere

②. If  $T$  &  $T_0$  of two bodies same: -

$$R_H = \sigma A [T^4 - T_0^4]$$

$$R_H \propto A$$

$$\therefore R_f = \frac{\sigma A [T^4 - T_0^4]}{ms}$$

$$R_f \propto \frac{A}{ms} \propto \frac{A}{\rho V}$$

→ Same shape (sphere)  
 $\frac{r^2}{r^3 \rho s} \propto \frac{1}{r \rho s}$   
→ Different shape  
 $\frac{A}{\rho V}$

Ques:- 2 solid spheres of same material having mass in ratio of  $1/3$  are heated upto same temp. and kept in same surrounding. F/o ratio of their initial rate of cooling?

$$R_f = \frac{\sigma A [T^4 - T_0^4]}{ms}$$

$$R_f \propto \frac{A}{m} \propto \frac{A}{m}$$

$$\Rightarrow \frac{R_{f1}}{R_{f2}} = \frac{A_1}{A_2} \left( \frac{m_2}{m_1} \right)$$

$$\frac{R_{f1}}{R_{f2}} = \left( \frac{r_1}{r_2} \right)^2 \left( \frac{3}{1} \right) = \left( \frac{1}{3} \right)^{2/3} \left( \frac{3}{1} \right) = \left( \frac{3}{3} \right)^{1/3} = 1$$

$$\left. \begin{aligned} \therefore \frac{m_1}{m_2} &= \frac{1}{3} \\ \frac{V_1 \rho}{V_2 \rho} &= \frac{1}{3} \\ \frac{4/3 \pi r_1^3}{4/3 \pi r_2^3} &= \frac{1}{3} \\ \frac{r_1}{r_2} &= \left( \frac{1}{3} \right)^{1/3} \end{aligned} \right\}$$

Ques:- A solid sphere and a cube of same vol<sup>m</sup> and surface finishing are heated upto same temp. and kept in a surrounding. F/o ratio of their initial rate of heat loss?

$$\therefore \frac{R_{H \text{ sphere}}}{R_{H \text{ cube}}} = \frac{4\pi r^2}{6a^2}$$

$$= \frac{4\pi}{6} \left\{ \frac{3}{4\pi} \right\}^{2/3}$$

$$= \frac{4\pi^{1/2} \cdot 2^{2/3}}{2 \times 3}$$

$$= \frac{1}{2} \left( \frac{4\pi}{3} \right)^{1/3}$$

$$= \left( \frac{4\pi}{8 \times 3} \right)^{1/3} = \left( \frac{\pi}{6} \right)^{1/3}$$

$$\left. \begin{aligned} V_{\text{sphere}} &= V_{\text{cube}} \\ \frac{4}{3} \pi r^3 &= a^3 \\ \frac{r}{a} &= \frac{3}{4} \left( \frac{4}{3\pi} \right)^{1/3} \end{aligned} \right\}$$

Ques:- A sphere, a cube and a circular plate of same mass and material heated upto same temp. and kept in a surrounding then which one will have -

① Higher rate of heat loss

② More time to cool down

③  $R_H \propto A$



Area  $\Rightarrow$  Plate  $>$  Cube  $>$  Sphere  
 $\downarrow$   
 $A_{\max}$   
 So,  $R_H$  max for Plate.

$$(2) R_f \propto \frac{A}{m(S)} \propto A.$$

$R_{f \min} \rightarrow$  Sphere  
 $\therefore$  Sphere will take more time to cool down.

Quej. - A solid sphere and a hollow sphere of same material and dimension heated upto  $500^\circ\text{C}$  temp. and kept in a surrounding then which one will have - (1) Higher rate of heat loss  
 (2) Higher rate of fall in temp.

$$(1) R_H \propto A.$$

Since both the spheres (solid & hollow) have same surface area, both will have same rate of heat loss.

$$(2) R_f \propto \frac{A}{m(S)} \propto \frac{1}{m}$$

We know,  $m_{\text{solid}} > m_{\text{hollow}} \Rightarrow \{ R_{f \text{solid}} < R_{f \text{hollow}} \}$

# Curve b/w  $\log_e E$  vs  $\log_e T$

$$\therefore E = \sigma T^4$$

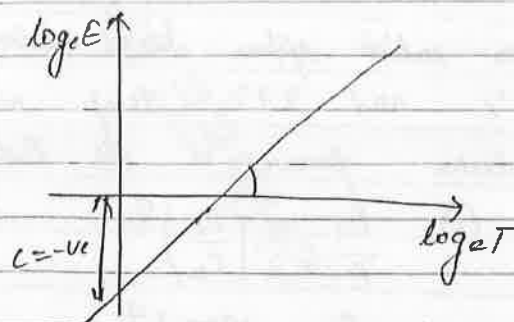
$$\log_e E = \log_e \sigma + 4 \log_e T$$

$$\therefore m = 4(+ve).$$

$$\therefore c = \log_e \sigma$$

$$\Rightarrow c = \log_e (5.67 \times 10^{-8})$$

$$\Rightarrow c = -ve$$



Quej. - F/o %age change in emissive power of body if it's temp. is red by - (1) 2% (2) 20%.

$$(1) E \propto T^4$$

$$\text{change} < 5\% \Rightarrow \frac{\Delta E}{E_i} \times 100 = 4 \left( \frac{\Delta T}{T_i} \times 100 \right) = 4 \times 2 = 8\%$$

$$\textcircled{2} \quad \frac{E_1}{E_2} = \left(\frac{T}{1.2T}\right)^4 \Rightarrow \frac{E_1}{E_2} = \frac{1}{2.07}$$

$$\Rightarrow E_2 = 2.07 E_1$$

$$\text{Now, } \frac{\Delta E}{E_1} \times 100 = \frac{2.07E_1 - E_1}{E_1} \times 100$$

$$= 107\%$$

Ques:- An ideal black body emits radiation at  $27^\circ\text{C}$  temp.  $\frac{3}{10}$  temp at which its emissive power becomes double.

$$\therefore E \propto T^4$$

$$\Rightarrow \frac{E}{2E} = \left(\frac{T}{300}\right)^4$$

$$T = 300(2)^{1/4}$$

$$T = 300 \times (1.414)^{1/4}$$

$$\textcircled{2} T = 300 \times 1.2$$

$$T = 360 \text{ K}$$

Ques:- Two solid sphere ideal black body of radius  $R$  &  $3R$  are at  $627^\circ\text{C}$  and  $27^\circ\text{C}$  temp. respectively. Then find ratio of their -  
 ① emissive power      ② radiation emitted per sec.

$$\textcircled{1} \quad \frac{E_1}{E_2} = \left(\frac{T_1}{T_2}\right)^4$$

$$\frac{E_1}{E_2} = \left(\frac{900}{300}\right)^4$$

$$\frac{E_1}{E_2} = \frac{81}{2}$$

$$\textcircled{2} \quad \frac{\left(\frac{R}{r}\right)_1}{\left(\frac{R}{r}\right)_2} = \frac{A_1}{A_2} \left(\frac{T_1}{T_2}\right)^4$$

$$= \frac{R^2}{9R^2} \left( \frac{A}{1} \right)$$

$$= \frac{9}{1}$$

Ques!:- A sphere at  $527^\circ\text{C}$  temp. kept in surrounding of  $127^\circ\text{C}$  emits 60 Watt radiation. If temp. of body is increased upto  $927^\circ\text{C}$  then find radiation emitted by it per sec.

$$\frac{Q}{t} = 60 = e\sigma A [800^4 - 400^4] \quad \text{--- (1)}$$

$$\frac{Q}{t} = P = e\sigma A [1200^4 - 400^4] \quad \text{--- (2)}$$

$$\text{(1)} \div \text{(2)}$$

$$\frac{60}{90} = \frac{800^4 [2^4 - 1^4]}{1200^4 [3^4 - 1^4]}$$

$$\frac{60}{P} = \frac{15}{80} \Rightarrow P = 320 \text{ Watt.}$$

Ques!:- A Cu-sphere of  $500^\circ\text{C}$  temp kept in surrounding of room temp. of  $27^\circ\text{C}$ . To maintain its temp. a heater of 70 Watt is required. If this sphere is now coloured black on half of its surface area then a heater of 210 Watt is required to maintain its temp. Then find emissivity of Cu-sphere.

$$\frac{Q}{t} = 70 \text{ W} = e\sigma A [T^4 - T_0^4] \quad \text{--- (1)}$$

$$\frac{Q}{t} = 210 \text{ W} = e\sigma \left( \frac{A}{2} \right) [T^4 - T_0^4] e + (1) \sigma \frac{A}{2} [T^4 - T_0^4] \quad \text{--- (2)}$$

$$\text{(1)} \div \text{(2)}$$

$$\frac{1}{3} = \frac{e}{\frac{e}{2} + 1}$$

$$\frac{e}{2} + \frac{1}{2} = 3e$$

$$\frac{1}{2} = \frac{5}{2}e \Rightarrow e = \frac{1}{5} = 0.2$$

• Newton's Law of Cooling (NLC)

From Stefan's law,

$$R_f = \frac{dT}{dt} = \frac{e\sigma A [T^4 - T_0^4]}{ms}$$

If  $T = T_0 + \Delta T$ ; when  $\Delta T$  very small  $\Delta T = T - T_0$

$$\begin{aligned} \therefore T^4 - T_0^4 &= (T_0 + \Delta T)^4 - T_0^4 \\ &= T_0^4 \left[ \left(1 + \frac{\Delta T}{T_0}\right)^4 - 1 \right] \\ &= T_0^4 \left[ 1 + 4 \frac{\Delta T}{T_0} - 1 \right] \\ &= 4T_0^3 \Delta T \end{aligned}$$

$$R_f = \frac{dT}{dt} = \left( \frac{4e\sigma A T_0^3}{ms} \right) \Delta T \Rightarrow \text{It is NLC.}$$

Acc. to NLC, -

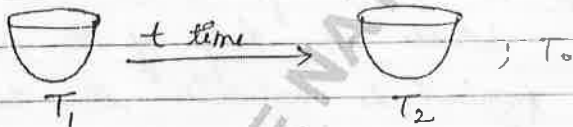
$$R_f \text{ (Rate of cooling)} \propto (T - T_0)$$

$$\hookrightarrow \frac{dT}{dt} = -K'(T - T_0)$$

$\hookrightarrow$  -ve shows rate of cooling dec with time.

$$\boxed{\frac{dT}{dt} = -K'(T - T_0)}$$

eg: -

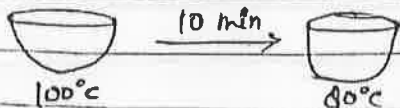


$$\frac{dT}{dt} = -K'(T - T_0)$$

$$\therefore \frac{T_2 - T_1}{t} = -K' \left[ \frac{T_1 + T_2}{2} - T_0 \right]$$

$$\boxed{\frac{T_1 - T_2}{t} = K' \left[ \frac{T_1 + T_2}{2} - T_0 \right]}$$

Que: -



$T_0 = 30^\circ\text{C}$ . then temp. of liquid after next 10 min.

$$\frac{100 - 40}{10} = K' \left[ \frac{(100 + 40)}{2} - 30 \right] \oplus$$

$$2 = K'(60) \quad \text{--- (1)}$$

$$\frac{80-T}{2 \times 10} = K' \left[ \frac{80+T}{2} - 30^\circ \right]$$

$$\frac{80-T}{10} = K' \left[ \frac{T+20}{2} \right] \quad \text{--- (2)}$$

$$\textcircled{1} + \textcircled{2}$$

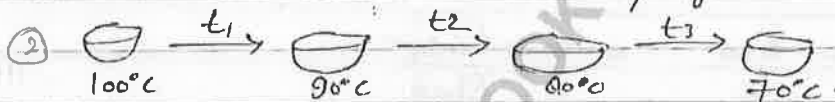
$$\frac{20}{80-T} = \frac{60 \times 2}{T+20}$$

$$T+20 = 480 - 6T$$

$$7T = 460$$

$$T = 65.7^\circ\text{C}$$

Note!  $\Rightarrow$   $\textcircled{1}$  NLC is used to determine specific heat of liquids.



$t_1 < t_2 < t_3$  [Rate of cooling  $\downarrow$  with time]

# Graphs

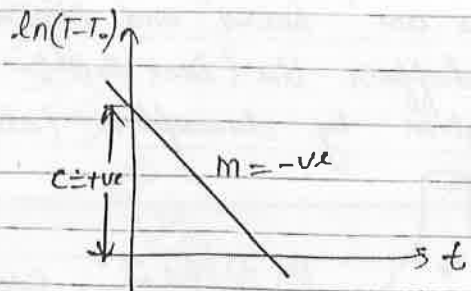
$\textcircled{1}$ .  $\ln(T-T_0)$  v/s time (t)

$$\therefore \text{NLC} \quad \frac{dT}{dt} = -K'(T-T_0)$$

$$\int \frac{dT}{(T-T_0)} = \int -K' dt$$

$$\ln(T-T_0) = -K't + c$$

$\begin{matrix} y \\ \hline m \\ \hline x + c \end{matrix}$



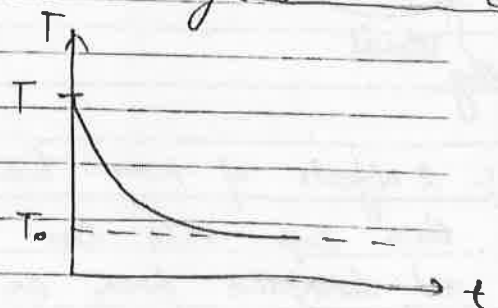
$\textcircled{2}$ . Variation of temp. (T) of body kept in surrounding ( $T_0$ ) with time (t)

$$\ln(T-T_0) = -K't + c$$

$$T-T_0 = e^{-K't + c}$$

$$T = T_0 + e^{-K't + c} \quad (\text{Exponential } \downarrow)$$

$$\text{If } t \rightarrow \infty \Rightarrow T = T_0$$



$\textcircled{3}$   
Slope  $\rightarrow \frac{dT}{dt} \Rightarrow R_f(\text{slope}) \rightarrow \downarrow$  es with time

• Kirchhoff's law

→ A/c to it, emissive power (E) is proportional to its absorptive power (a).  
 $E \propto a$

If  $E \uparrow \Rightarrow a \uparrow$  [Good absorber is good emitter].

⇒ Good emitter is bad reflector.

∴  $E \uparrow \Rightarrow a \uparrow \Rightarrow r \downarrow$  &  $t \downarrow$  [∵  $1 = r + a + t$ ]

⇒ ∴  $E \propto a$

$\frac{E}{a} = \text{const.}$  → For any body at same temp.

Let have G<sub>B</sub> and I<sub>B</sub> at same temp.

$$\left(\frac{E}{a}\right)_{I_B} = \left(\frac{E}{a}\right)_{G_B}$$

$$\frac{E_{I_B}}{1} = \frac{E_{G_B}}{a} \Rightarrow a = \frac{E_{G_B}}{E_{I_B}}$$

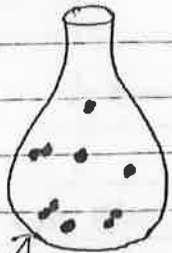
$$\boxed{a = e}$$

# Application of Kirchhoff's law

①. Days are hotter and Nights are colder in deserts.

②. Fraunhofer's line (Dark line) :- Produced due to absorption of Sun radiation by chromosphere (atmosphere) of sun.

③.



On taking it to dark room, Black dots emit more and appear brighter.  
 ∴ (Good absorber is a good emitter)

Ques:- 2 objects of same heat capacity and cross-section area change their temp with time as given in graph. If their emissivity and absorptive power are respectively e and a then -

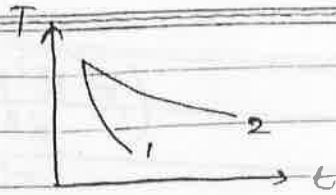
①  $e_1 = e_2 ; a_1 = a_2$

②  $e_1 > e_2 ; a_1 > a_2$

Ⓒ  $e_1 < e_2 ; a_1 < a_2$

Ⓓ  $e_1 < e_2 ; a_1 > a_2$

Slope  $\rightarrow \frac{dT}{dt} \Rightarrow$  Rate of Cooling



Slope of 1 > 2  $\Rightarrow R_f$  of 1 > 2

More emission 1 > 2

$e_1 > e_2 \Rightarrow a_1 > a_2$  ( $e = a$ )

or

$R_f = \epsilon \sigma A (T^4 - T_0^4)$

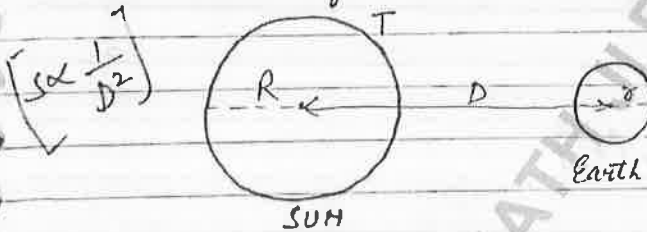
(ms)

$R_{f1} > 2$

$e_1 > e_2$

• Solar Constant (Star constant) / 'S'

$\rightarrow$  Amount of Radiation incident on per unit area of Earth in per unit time. from Sun is k/n as Solar constant of Earth.



$S = \frac{\sigma R^2 T^4}{D^2}$  Incident on per unit area of Planet in per sec.

For different Planets:-

R & T  $\rightarrow$  same

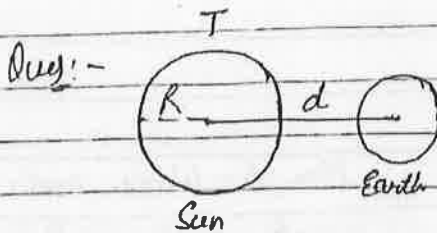
D  $\rightarrow$  Different

$[S \propto \frac{1}{D^2}]$

$\frac{Q}{t}$  Incident on Earth at any instant

$\frac{Q}{t} = (S) \times [\text{Area of Projection}]$

$\frac{Q}{t} = \left( \frac{\sigma R^2 T^4}{D^2} \right) \times \pi r^2$



F/o temp. of Earth at eq<sup>m</sup>.

At eq<sup>m</sup>

$\left( \frac{Q}{t} \right)_{\text{absorbed by Earth}} = \left( \frac{Q}{t} \right)_{\text{emitted by Earth}}$

$\left( \frac{Q}{t} \right)_{\text{Incident}} \times d = \epsilon \sigma (4\pi r^2) T^4$  ( $\because d = r$ )

$$\left( \frac{R^2 T^4}{d^2} \right) \times \pi r^2 = (4\pi r^2) T'^4$$

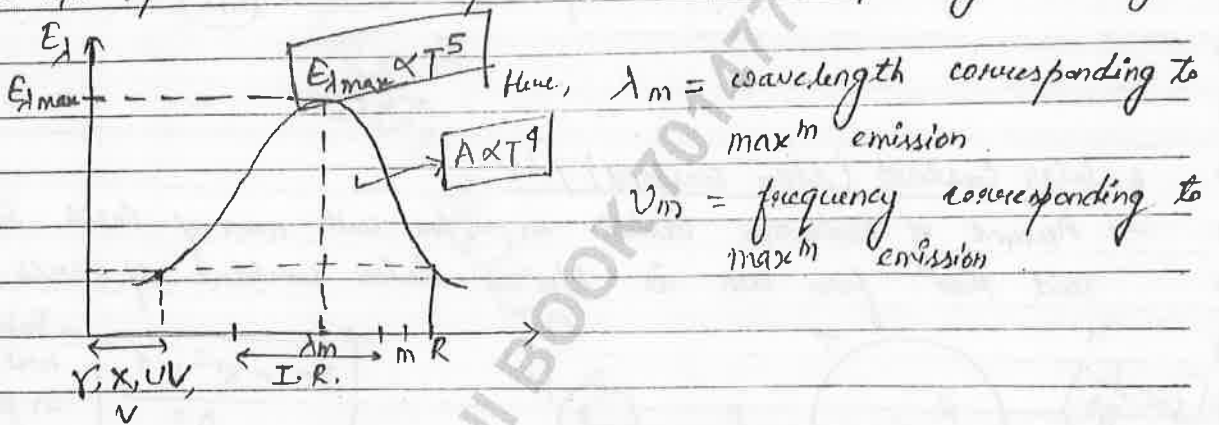
$$T'^4 = \frac{R^2 T^4}{4d^2}$$

$$T' = T \sqrt{\frac{R}{2d}}$$

### • Maxwell's Energy Distribution Curve

→ A/c to it, spectrum of IBB is continuous.

Graph b/w spectral emissive power  $[E_\lambda]$  v/s corresponding wavelength  $[\lambda]$



$\nu_m$  = frequency corresponding to max<sup>m</sup> emission

∴ Total emissive power  $[E]$

$$E = \int_0^\infty E_\lambda d\lambda$$

$$E = \text{Area of Curve}$$

∴  $E \propto T^4$  (Stefan)

[Area of curve  $\propto T^4$ ]

If  $T \uparrow \Rightarrow E \uparrow \Rightarrow \text{Area of Curve}$

### # Wien's Displacement Law: -

A/c to it, for a IBB: -

$$\lambda_m \propto \frac{1}{T}$$

$$\Rightarrow \lambda_m = \frac{b}{T}$$

Here,  $b$  = Wien's const.

$$b = 2.89 \times 10^{-3} \text{ m-K}$$

(or)

$$b = 2.89 \times 10^6 \text{ nm-K}$$

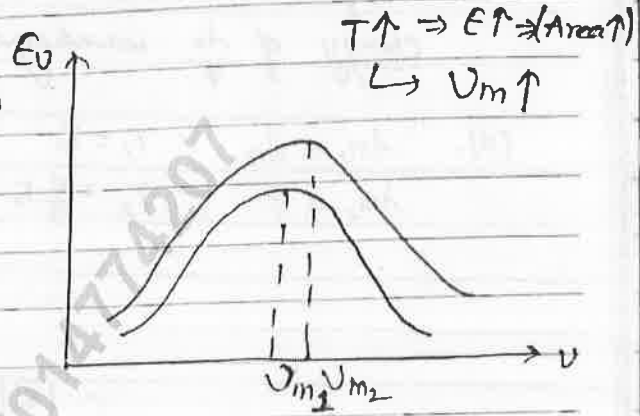
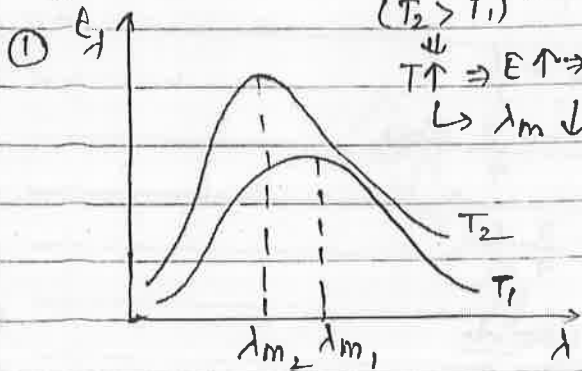
3



$$\therefore \lambda_m \propto \frac{1}{\nu_m} \propto \frac{1}{T} \Rightarrow \nu_m \propto T$$

If temp.  $\uparrow$   $\left\{ \begin{array}{l} \rightarrow \lambda_m \downarrow \Rightarrow \nu_m \uparrow \\ \rightarrow E \uparrow \Rightarrow (\text{Area of curve } \uparrow) \\ \quad \quad \quad \{E \propto T^4\} \end{array} \right.$

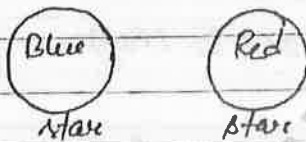
eg:- If IBB is heated from  $T_1$  to  $T_2$



Notes:- Colour of stars (hot objects) is indication of their temp.

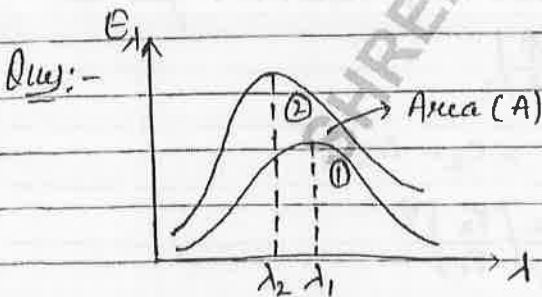
UV	V	IR	Microwaves	Radio waves $\rightarrow \lambda$
	VIBGYOR			

eg:-



$$\lambda_{\text{Blue}} < \lambda_{\text{Red}}$$

$$T_{\text{Blue}} > T_{\text{Red}} \quad \left( \lambda_m \propto \frac{1}{T} \right)$$



If  $16 A_2 = 81 A_1$  then  $f/o \frac{\lambda_2}{\lambda_1} = ?$

$$\therefore \frac{A_2}{A_1} = \frac{81}{16}$$

$$\therefore \text{Area} = E \propto T^4$$

$$\Rightarrow \left( \frac{T_2}{T_1} \right)^4 = \frac{81}{16} \Rightarrow \frac{T_2}{T_1} = \frac{3}{2}$$

$$\therefore T \propto \frac{1}{\lambda_m}$$

$$\frac{\lambda_{m1}}{\lambda_{m2}} = \frac{3}{2} \Rightarrow \frac{\lambda_2}{\lambda_1} = \frac{2}{3}$$

08/01/2020

Ques!: An IBB emits max<sup>m</sup> radiation corresponding to  $\lambda_0$  wavelength at  $T_0$  temp. - (a). If temp. of IBB is  $\uparrow$ ed upto  $4/3$  times of initial then displacement of wavelength corresponding to max<sup>m</sup> emission. (b). If it is heated in such a way that IBB emits max<sup>m</sup> energy of  $\frac{\lambda_0}{4}$  wavelength then it's emissive Power? (If initial is  $E_0$ )

(a).  $\lambda_{m1} = \lambda_0$  ;  $T_1 = T_0$   $\therefore \lambda_m \propto \frac{1}{T}$   
 $\lambda_{m2} = ?$  ;  $T_2 = \frac{4}{3}T_0$   $\lambda_m \rightarrow \frac{3}{4}$  times

$$\lambda_{m2} = \frac{3}{4} \lambda_0$$

$$\therefore |\Delta\lambda| = \frac{3}{4} \lambda_0 - \lambda_0 = -\frac{\lambda_0}{4}$$

(b).  $\lambda_{m1} = \lambda_0$  ;  $T_1 = T_0$   $\therefore T \rightarrow 4$  times  
 $\downarrow$   $\downarrow \lambda_m \propto \frac{1}{T}$   $E \propto T^4$   
 $\lambda_{m2} = \frac{\lambda_0}{4}$   $T_2 = 4T_0$   $E \rightarrow 256$  times  
 $\Rightarrow E = 256 E_0$  (Ans)!

Ques!- 2 solid spheres of emissivity 0.01 and 0.01 having wavelength corresponding to max<sup>m</sup> emission in ratio of  $2/3$ . Then f/o ratio of their radius if they emit same amount of radiation per sec.

$$\therefore \frac{Q}{t} = \epsilon \sigma A T^4 \quad \therefore \left(\frac{Q}{t}\right)_1 = \left(\frac{Q}{t}\right)_2$$

$$\Rightarrow \epsilon_1 \sigma A_1 T_1^4 = \epsilon_2 \sigma A_2 T_2^4$$

$$\frac{A_1}{A_2} = \frac{\epsilon_2}{\epsilon_1} \left(\frac{T_2}{T_1}\right)^4$$

$$\left(\frac{r_1}{r_2}\right)^2 = \frac{0.01}{0.01} \left(\frac{\lambda_{m1}}{\lambda_{m2}}\right)^4$$

$$\left(\frac{r_1}{r_2}\right)^2 = \frac{0.1}{1} \times \frac{16}{0.1}$$

$$\frac{r_1}{r_2} = \frac{4}{1}$$

Ques:- Temp. of Ideal black body is  $2617^\circ\text{C}$ . It emits radiation of  $U_1$  energy of  $500\text{ nm}$  wavelength, radiation of  $U_2$  energy of  $1000\text{ nm}$  wavelength and  $U_3$  energy of  $1500\text{ nm}$  wavelength. If Wien's constant ( $b$ ) =  $2.09 \times 10^6\text{ nm-K}$  then -

①  $U_1 > U_2 > U_3$

②  $U_1 < U_2 < U_3$

③  $U_2$  is max<sup>m</sup>

④  $U_1 > U_3 > U_2$

$$\lambda_m = \frac{b}{T} = \frac{2.09 \times 10^6\text{ nm-K}}{2690\text{ K}}$$

$$\Rightarrow \lambda_m = 1000\text{ nm}$$

↓

1000 nm emits (max<sup>m</sup>)

↓

energy  $\rightarrow U_2 \rightarrow$  max<sup>m</sup>.

SHREE NATHJI BOOK 70177207

## Calorimetry

### • Heat (Q)

→ Unit: → Joule, or Cal.  
↳ S.I. unit

$$1 \text{ Cal} = 4.18 \text{ J} \approx 4.2 \text{ J}$$

↳ Joule's const. (J).

$$\rightarrow 1 \text{ J} = \frac{1}{4.2} \text{ Cal} = 0.24 \text{ Cal}$$

⊙ Mechanical equivalent of Heat.

So,  $100 \text{ J} = 24 \text{ Cal}.$

→ Heat (Q) = Substance

Temp. change Sp. heat (s)	$Q = ms\Delta T$
Phase change (const. T & P)	$Q = mL$ Here, $L = \text{latent heat}$

### • Specific Heat (S)

→ Amount of heat required to change the temp. of unit mass by  $1^\circ\text{C}$   
⊙  $1 \text{ K}$ .

$$\therefore Q = ms\Delta T$$

$$\left[ S = \frac{Q}{m\Delta T} \right] \quad \text{Unit:} \rightarrow \frac{\text{J}}{\text{Kg-K}} \quad (\text{S.I. unit})$$

→ Other units: -  $\frac{\text{J}}{\text{g-K}}$  ⊙  $\frac{\text{Cal}}{\text{g-K}}$  ⊙  $\frac{\text{Cal}}{\text{Kg-K}}$

→ If  $S \uparrow \Rightarrow$  Dese se garam & Dese se thande hote hai.  
eg: - water.

→  $S \downarrow \Rightarrow$  air

$$\rightarrow S_{\text{water}} = \frac{1 \text{ Cal}}{\text{g-K}} \quad \text{⊙} \quad \frac{4.2 \text{ J}}{\text{g-K}} \quad \text{⊙} \quad \frac{4200 \text{ J}}{\text{Kg-K}} \rightarrow \text{SI unit} \quad \text{⊙} \quad \frac{1 \text{ KCal}}{\text{Kg-K}}$$

$$\rightarrow S_{\text{ice}} = \frac{0.5 \text{ Cal}}{\text{g-K}} \quad \text{⊙} \quad \frac{500 \text{ Cal}}{\text{Kg-K}} \quad \text{⊙} \quad \frac{2.1 \text{ J}}{\text{g-K}} \quad \text{⊙} \quad \frac{2100 \text{ J}}{\text{Kg-K}}$$

$$\rightarrow S_{\text{steam}} = 0.47 \approx 0.5 \frac{\text{Cal}}{\text{g-K}}$$

### • Latent Heat (L)

$$\therefore Q = mL$$

$$L = \frac{Q}{m} \Rightarrow \text{Unit:} \rightarrow \frac{\text{J}}{\text{Kg}} \quad \Rightarrow (\text{S.I.})$$

→ Other unit: →  $\frac{\text{Cal}}{\text{g}}$

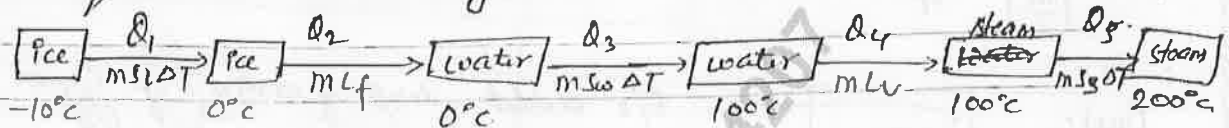
(1). ice  $\xrightarrow[\text{(}L_f\text{)}]{\text{melting}}$  water  
 $0^\circ\text{C}$   $0^\circ\text{C}$

Latent heat of fusion =  $80 \frac{\text{Cal}}{\text{g}}$

(2). water  $\xrightarrow[\text{(}L_v\text{)}]{\text{vaporisation}}$  steam  
 $100^\circ\text{C}$   $100^\circ\text{C}$

Latent heat of vaporisation ( $L_v$ )  
 $= 536 \approx 540 \frac{\text{Cal}}{\text{g}}$

Que:- Heat required to convert 1gm ice at  $-10^\circ\text{C}$  into steam of  $200^\circ\text{C}$ .



$$\therefore Q = Q_1 + Q_2 + Q_3 + Q_4 + Q_5$$

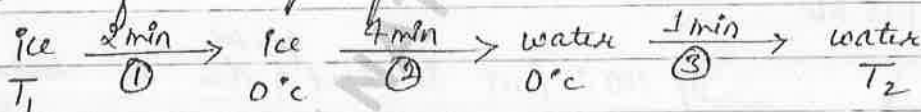
$$Q = 1 \times 0.5 \times 10 + (1 \times 80) + (1 \times 1 \times 100) + (1 \times 540) + (1 \times 0.5 \times 100)$$

$$Q = 5 + 80 + 100 + 540 + 50$$

$$Q = 775 \text{ Cal}$$

Imp.

Que:- Some amount of ice is being heated by uniform heating source for 7 minutes. In first 2 min. of heating, its temp. increases uniformly; In next 4 min. it temp. remains const.; and during remaining time its temp. again increases uniformly. Then find initial and final temp. of substance.



Let uniform source  $\Rightarrow 100 \frac{\text{Cal}}{\text{min}}$

$$\therefore Q_1 = 100 \times 2 = mS_i(0 - T_1) \quad \text{--- (1)}$$

$$\therefore Q_2 = 100 \times 4 = mL_f \quad \text{--- (2)}$$

$$\therefore Q_3 = 100 \times 1 = mS_w(T_2 - 0) \quad \text{--- (3)}$$

(1)  $\div$  (2)

$$\frac{2}{4} = \frac{mS_i(0 - T_1)}{mL_f}$$

$$\frac{1}{2} = \frac{0.5}{80} (-T_1)$$

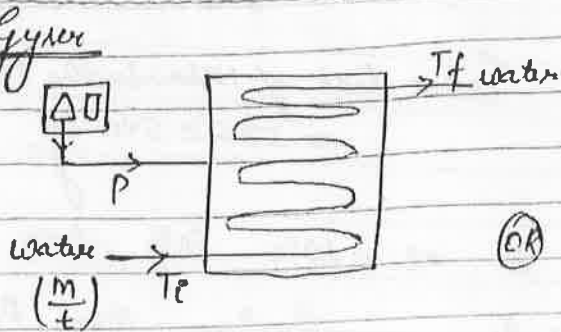
$$T_1 = -80^\circ\text{C}$$

(1)  $\div$  (3)

$$\frac{2}{1} = \frac{80}{1 \times (T_2)}$$

$$T_2 = 20^\circ\text{C}$$

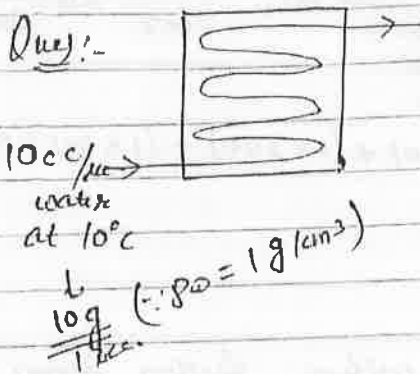
• Geyser



$$\therefore P_{\text{out}} \Rightarrow \frac{P \text{ Joule}}{\text{sec}} = \left(\frac{m}{t}\right) S \Delta T \quad \text{Joule}$$

$$\text{(OR)} \quad \left(\frac{P}{4.2}\right) \frac{\text{Cal}}{\text{sec}} = \left(\frac{m}{t}\right) S \Delta T \quad \text{Cal}$$

Que:-



f/o outlet temp. of water if Geyser is having 50% efficiency with 2.1 KW power.

$$\therefore \frac{2.1 \times 1000 \times 50}{100} = \left(\frac{10g}{1s}\right) \times \frac{1 \text{ Cal}}{g \cdot ^\circ C} \times (T_f - 10)$$

$$\Rightarrow \frac{2100}{2 \times 4.2} = 10 T_f - 100$$

$$250 = 10 T_f - 100$$

$$10 T_f = 350$$

$$T_f = 35^\circ C. \quad (\text{Ans}) :-$$

Que:-



If 100 J heat is lost per sec due to leakage from bucket then f/o time taken to increase temp. of water upto 50°C.

$$P = 1.15 \text{ KW} = \frac{1150 \text{ J}}{\text{sec}}$$

$$\therefore \text{Remaining } (1150 - 100) = 1050 \frac{\text{J}}{\text{sec}} = \frac{m S \Delta T}{t}$$

$$\Rightarrow 1050 \frac{\text{J}}{\text{sec}} = \frac{10 \times 10^3 \text{ g} \times 4.2 \text{ J}}{g \cdot ^\circ C} \times (30)$$

$$t = 1200 \text{ sec}$$

$$t = 20 \text{ min.}$$

Ex-1 = 19, 24, 25, 33 to 37  
 Ex-2 = all except 8, 26, 43, 74

09/01/2020

• Heat Capacity (Hc)

→ Amount of heat required to change temp. of body 1°C or 1 K.

Unit mass → specific heat (s)

m mass →  $Hc = ms$

→ depends on mass & material.

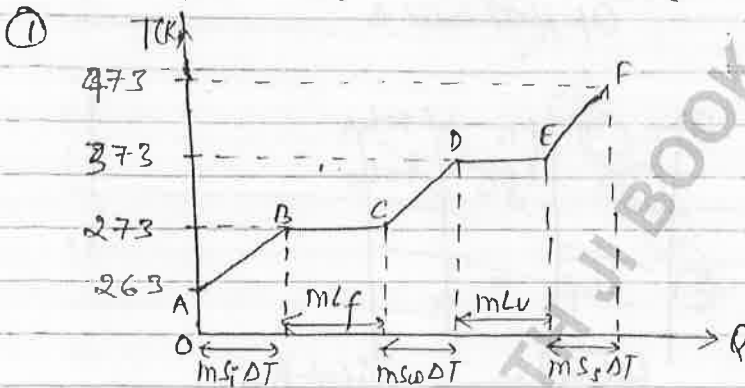
$$\therefore Q = \underbrace{ms}_{Hc} \Delta T \Rightarrow Hc = \frac{Q}{\Delta T}$$

If  $Hc \uparrow \Rightarrow$  Des is gain & Des is Thanda

• Heating Curve

→ Curve b/w temp. of body v/s Heat (Q) supplied to it (or time)

eg. → ice → ice → water → water → steam → steam  
 -10°C    0°C    0°C    100°C    100°C    200°C

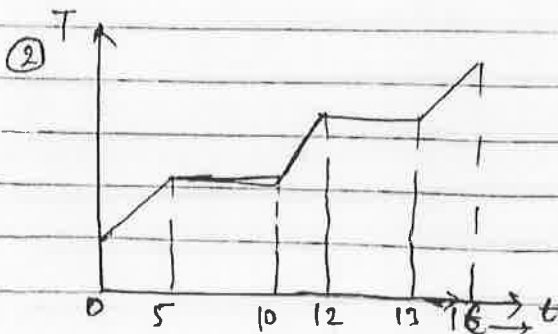


Slope  $\Rightarrow T$  v/s  $Q$   
 $\hookrightarrow \frac{\Delta T}{\Delta Q}$

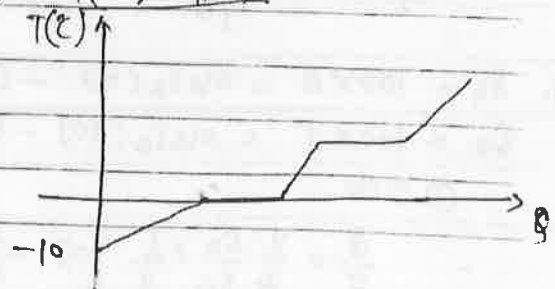
$$\underline{AB} \Rightarrow \text{slope}_{AB} = \frac{\Delta T}{\Delta Q} = \frac{\Delta T}{m s_i \Delta T} = \frac{1}{(Hc)_{ice}}$$

$$\text{slope}_{CD} = \frac{\Delta T}{\Delta Q} = \frac{\Delta T}{m s_w \Delta T} = \frac{1}{(Hc)_{water}}$$

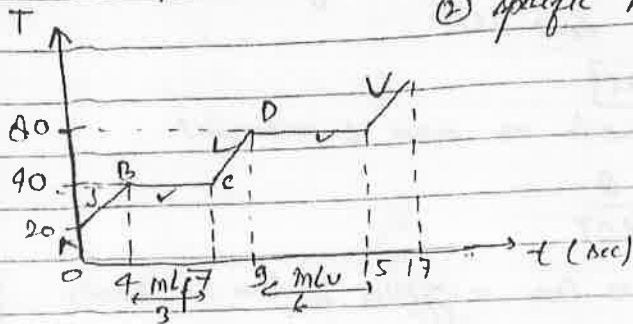
$$\text{slope}_{EF} = \frac{1}{(Hc)_{steam}}$$



③ T(°C) v/s Q



Ques:- An unknown substance being heated by uniform heating source.  
 Compare it's - ① latent heat,  $L_v$  &  $L_f$   
 ② specific heat in solid phase & liquid phase.



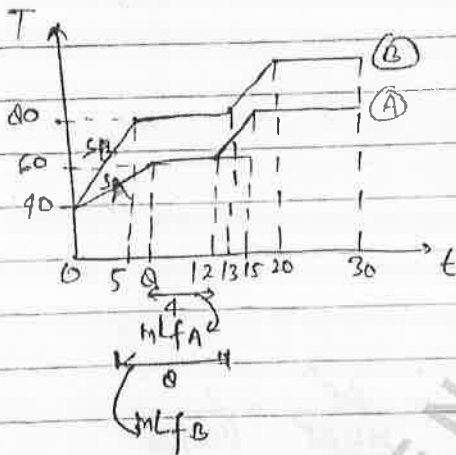
①  $mL_v > mL_f$   
 $\Rightarrow L_v > L_f$

② Slope  $\propto \frac{1}{H_c}$   
 $\text{Slope}_{AB} < \text{Slope}_{CD}$   
 $\left(\frac{20}{4}\right) < \left(\frac{40}{9}\right)$   
 $(H_c)_{AB} > (H_c)_{CD}$   
 $m_{\text{solid}} > m_{\text{liq}}$   
 $s_{\text{solid}} > s_{\text{liq}}$

Ques:- 2 substances A and B of mass in ratio of 3/2 are being heated by same uniform source as shown. Then find -

①  $\frac{L_f A}{L_f B}$

②  $\frac{(\text{Sp. heat})_{\text{solid A}}}{(\text{Sp. heat})_{\text{solid B}}}$



①  $mL_f B > mL_f A$   
 $\Rightarrow 2L_f B > mL_f A$

② Slope  $\propto \frac{1}{H_c}$   
 $(\text{Slope})_{SA} > (\text{Slope})_{SB}$   
 $\frac{(40-40)}{(9-0)} > \frac{(80-60)}{(13-0)}$

① Let source  $\Rightarrow 100 \text{ Cal/sec}$ .

$\therefore Q_A = 100 \times 9 = m_A L_f A$  - ①

$\therefore Q_B = 100 \times 9 = m_B L_f B$  - ②

①  $\div$  ②

$\frac{1}{2} = \frac{3}{2} \frac{L_f A}{L_f B} \Rightarrow \frac{L_f A}{L_f B} = \frac{1}{3}$

②  $Q_A = 100 \times 0 = m_A s_A (20)$  - ①

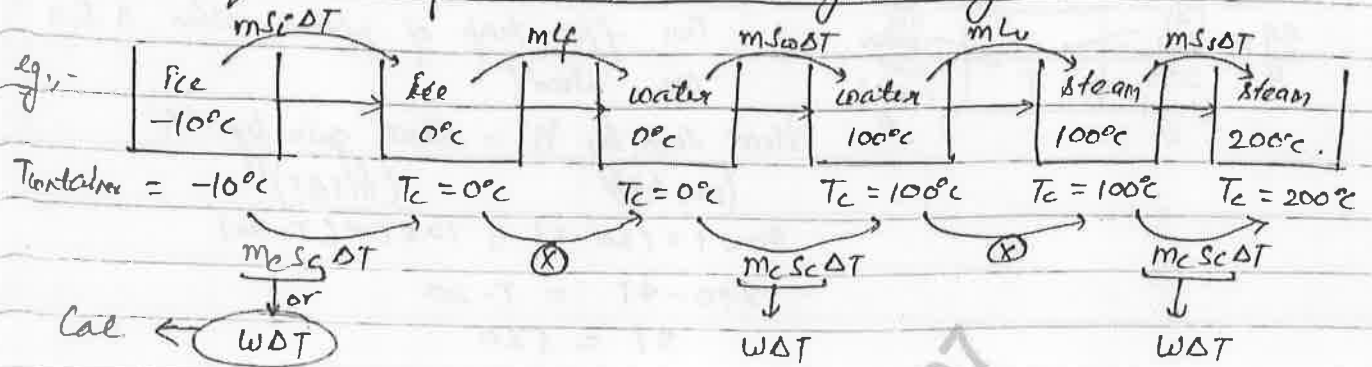
$Q_B = 100 \times 5 = m_B s_B (40)$  - ②

①  $\div$  ②

$\frac{0}{5} = \frac{3}{2} \frac{s_A}{s_B} \times \frac{1}{2} \Rightarrow \frac{s_A}{s_B} = \frac{32}{15}$



• Water Equivalent (of container) [W] in gm or Kg



→ Water equivalent means → container ko bhi thanda ya garam karo  
 →  $W = ?$

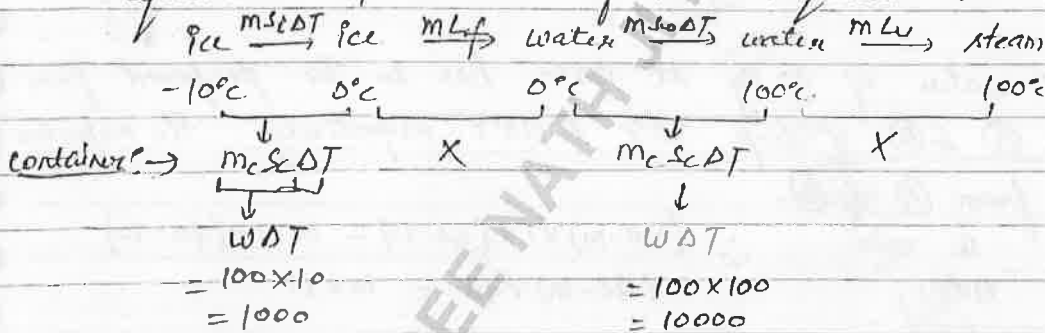
$$(H_c)_{\text{water}} = (H_c)_{\text{container}}$$

$$\downarrow m_s \quad \downarrow m_s$$

$$\text{gm} \leftarrow \text{W} \times 1 = m_c s_c$$

$$\Rightarrow \boxed{W = m_c s_c}$$

Ques:- 1 gm ice at  $-10^\circ\text{C}$  is kept in container of 0.1 Kg water-equivalent. F/o amount of heat required to vaporise ice?



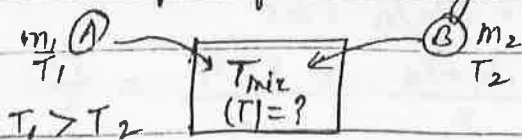
$$\therefore Q = Q_{\text{material}} + Q_{\text{container}}$$

$$Q = [(1 \times 0.5 \times 10) + (1000) + (1 \times 1 \times 10) + (1 \times 540)] + (1000 + 10,000)$$

$$Q = 725 \text{ Cal} + 11000 \text{ Cal}$$

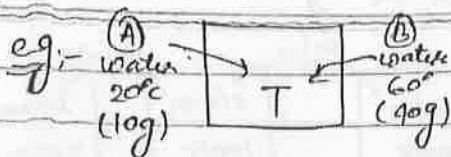
$$Q = 11725 \text{ Cal.}$$

• Principle of Calorimetry



Heat loss by A = Heat gain by B

$$\boxed{T_2 \leq T_{\text{mix}} \leq T_1}$$



Then f/o temp. of mixture when A & B are mixed?

$$\text{Heat loss by 'B'} = \text{Heat gain by 'A'}$$

$$(ms\Delta T)_B = (ms\Delta T)_A$$

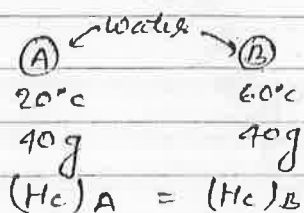
$$40 \times 1 \times (60 - T) = 10 \times 1 \times (T - 20)$$

$$240 - 4T = T - 20$$

$$5T = 560$$

$$T = 52^\circ\text{C.}$$

Note:  $\Rightarrow$  ①.



$T_{mix} \Rightarrow$  Avg. [Mean of temp. A & B]

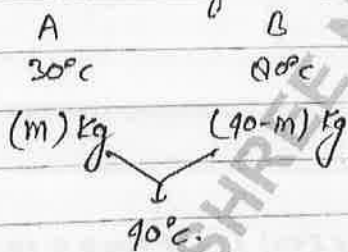
②. If substances A & B are  $(T_1)$   $(T_2)$

are mixed;-

$$T_{mix} = \frac{T_1 + T_2}{2}; \text{ only if}$$

they are having same heat capacity.

Que!:- A bucket of water of 40 kg at  $40^\circ\text{C}$  has to be prepared from 2 water tank A & B of temp.  $30^\circ\text{C}$  &  $80^\circ\text{C}$  respectively. f/o respective masses taken from A & B.



$$\therefore (40 - m) \times 1 \times (80 - 40) = m \times 1 \times (40 - 30)$$

$$(40 - m) \times 40 = m \times 10$$

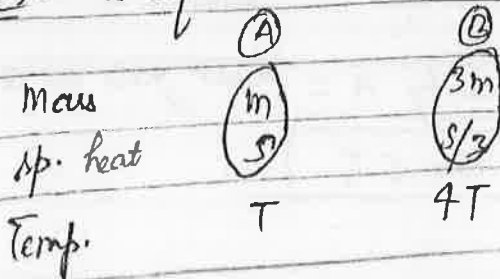
$$160 - 4m = m$$

$$5m = 160$$

$$m = 32 \text{ Kg. (from A).}$$

$$\therefore \text{From B} = 40 - 32 = 8 \text{ Kg.}$$

Que!:- 2 liquids



When A & B are mixed, f/o temp. of mixture

$$(Hc)_A = (Hc)_B = ms$$

$$\therefore T_{mix} = \frac{T_A + T_B}{2} = \frac{T + 4T}{2} = \frac{5T}{2}$$

Ques:- 170 gm water kept in cu-container of 100 gm mass at 30°C temp. A cu-ball of mass 100 gm and temp T is dropped into container, temp of mixture becomes 75°C. If specific heat of cu is 0.1 Cal/g-°C then find value of T=?

Loss in temp. = Gain in temp.

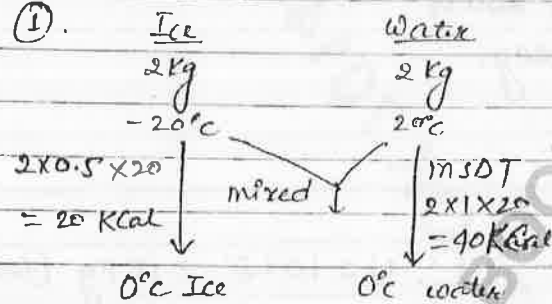
$$100 \times 0.1 \times (T - 75) = 100 \times 0.1 \times (75 - 30) + 170 \times 1 \times (75 - 30)$$

$$10T - 750 = 450 + 7650$$

$$10T = 750 + 450 + 7650$$

$$T = \frac{8850}{10} = 885^\circ\text{C}$$

Note! ⇒ (1).



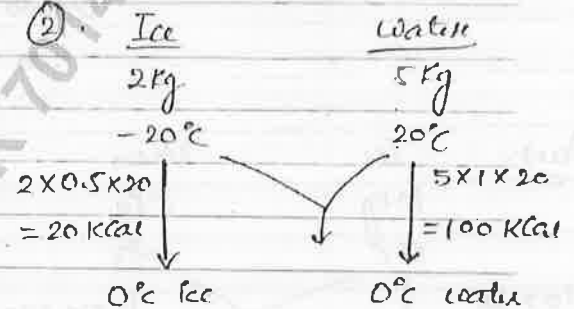
$$\therefore Q = 40 - (20) = 20 \text{ Kcal}$$

$$\Rightarrow \text{Ice} + \text{water} \\ 2 \text{ kg} \quad 2 \text{ kg} \\ 20 \text{ Kcal} = m L_f$$

$$\Rightarrow m = \frac{20}{80} = \frac{1}{4} = 0.25 \text{ kg (water)}$$

$$T = 0^\circ\text{C} \left\{ \begin{array}{l} W = 2 + 0.25 = 2.25 \text{ kg} \\ \text{Ice} = 2 - 0.25 = 1.75 \text{ kg} \end{array} \right.$$

(2).



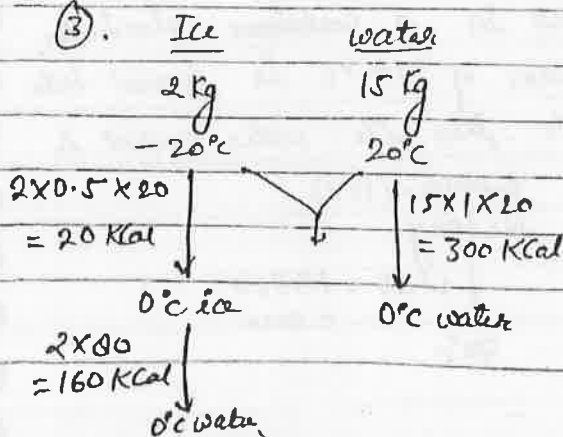
$$\therefore Q = 100 - 20 = 80 \text{ Kcal}$$

$$80 = m L_f$$

$$\Rightarrow m = \frac{80}{80} = 1 \text{ kg (water)}$$

$$\therefore \left. \begin{array}{l} W = 5 + 1 = 6 \text{ kg} \\ \text{Ice} = 2 - 1 = 1 \text{ kg} \end{array} \right\} T = 0^\circ\text{C}$$

(3).



$$\therefore \text{Water} = 2 + 15 = 17 \text{ kg}$$

$$\therefore Q = 300 - (20 + 160)$$

$$Q = 120 \text{ Kcal}$$

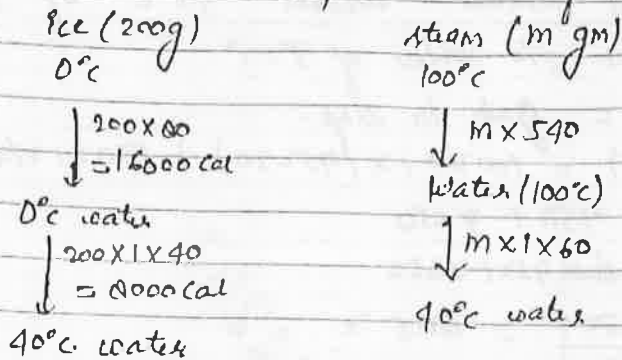
$$m s \Delta T = 120$$

$$17 \times 1 \times \Delta T = 120$$

$$\Delta T = 7.05^\circ\text{C}$$

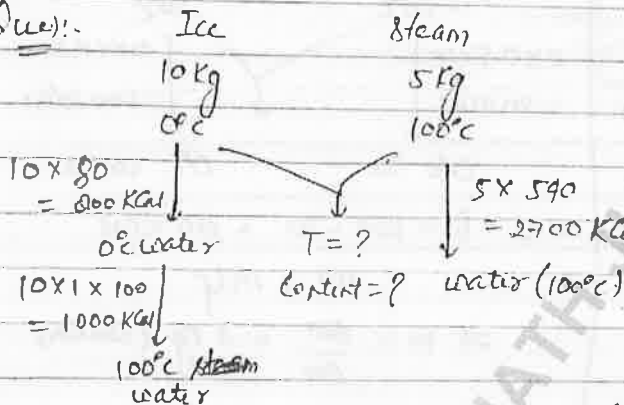
$$T_f - 0 = 7.05^\circ\text{C} \Rightarrow T_f = 7.05^\circ\text{C}$$

Ques:- Some steam at  $100^\circ\text{C}$  mixed with  $200\text{ g}$  ice at  $0^\circ\text{C}$ . If temp. of mixture is  $40^\circ\text{C}$  then find amount of steam used.



$\therefore \text{gain} = \text{loss}$   
 $24000 = 600m$   
 $m = 40\text{ g}$

Ques:-



$\therefore W = 10 + 5 = 15\text{ kg } (100^\circ\text{C})$

$\therefore Q = 2700 - (800 + 1000)$

$Q = 900\text{ KCal}$

Now,

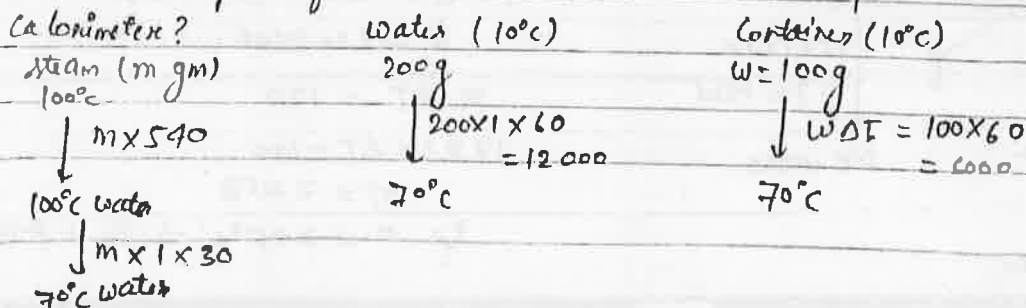
$Q = 900 = mLv$

Steam  $\rightarrow m = \frac{900}{540} = \frac{5}{3} = 1.67\text{ Kg (steam)}$

Water =  $15 - 1.67 = 13.33\text{ Kg (water)}$

$T_{\text{mix}} = 100^\circ\text{C}$

Ques:-  $200\text{ g}$  of water at  $10^\circ\text{C}$  temp. kept in a container calorimeter of  $0.1\text{ Kg}$  water equivalent. If steam of  $100^\circ\text{C}$  is passed into it then temp. of mix. becomes  $70^\circ\text{C}$ . Then find water content in



$$\therefore 570 \text{ m} = 12000 + 6000$$

$$m = 31.5 \text{ gm}$$

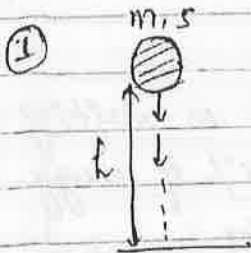
$$\therefore \text{Water} = 200 + m$$

$$= 200 + 31.5 = 231.5 \text{ gm.}$$

### # Question Based on Energy Conservation

$$\therefore Q = m\Delta T \text{ or } mL$$

$$Q = \underset{\substack{\downarrow \\ \text{cal}}}{m} \times \underset{\substack{\downarrow \\ \text{g}^\circ\text{C}}}{c} \text{ or } Q = \underset{\substack{\downarrow \\ \text{cal}}}{m} \times \underset{\substack{\downarrow \\ \text{g}}}{L}$$

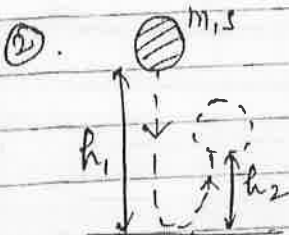


No rise in temp. of ball just after strike ; -

$$\text{Loss in PE} = m\Delta T$$

$$\frac{mgh}{J} = \frac{m\Delta T}{J} \times \frac{J}{\text{kg}^\circ\text{C}}$$

$$\frac{mgh}{4.2} = m\Delta T \times \frac{\text{cal}}{\text{kg}^\circ\text{C}}$$



$$\Delta T = ?$$

$$\therefore \text{Loss in PE} = m\Delta T$$

$$mg(h_1 - h_2) = m\Delta T \times \frac{J}{\text{kg}^\circ\text{C}}$$

③. In previous case-② if 60% of produced energy goes to surrounding

$$\frac{40}{100} \times [mg(h_1 - h_2)] = m\Delta T$$

④ In case-② 60% of energy converts into thermal energy

$$\frac{60}{100} \times mg(h_1 - h_2) = m\Delta T$$

Ques:- If a block of (mass 'm', specific heat  $\frac{c \text{ J}}{\text{kg}^\circ\text{K}}$ ) moving with  $v_0$  speed

strikes with an obstacle and rebounds with  $\frac{v_0}{2}$  speed. If energy

produced equally distributed b/w obstacle & block then find  
in temp. of block?

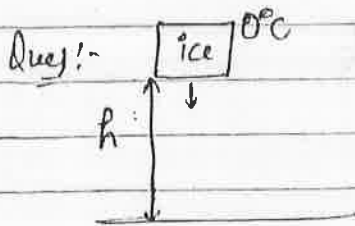
$$\therefore K_i = \frac{1}{2} m v_0^2 \quad \text{and} \quad K_f = \frac{1}{2} m \left(\frac{v_0}{2}\right)^2 = \frac{1}{2} m \frac{v_0^2}{4}$$

$$\therefore \Delta KE = \frac{1}{2} m v_0^2 \left(1 - \frac{1}{4}\right) = \frac{3}{8} m v_0^2$$

Since, 50% of  $\frac{3}{8} m v_0^2 = m s \Delta T$

$$\frac{1}{2} \times \frac{3}{8} m v_0^2 = m s \Delta T$$

$$\Rightarrow \Delta T = \frac{3 v_0^2}{16 c}$$



If  $1/4^{\text{th}}$  of ice gets melted on striking.  
Then find value of 'h' if 40% of energy  
produced lost in surrounding.

$$\frac{60}{100} \times mgh = \left(\frac{m}{4}\right) L_f \quad \leftarrow \frac{\text{cal}}{kg}$$

4.2

$$\frac{3}{5} \times 10 \times h = \frac{1}{4} \times 80,000$$

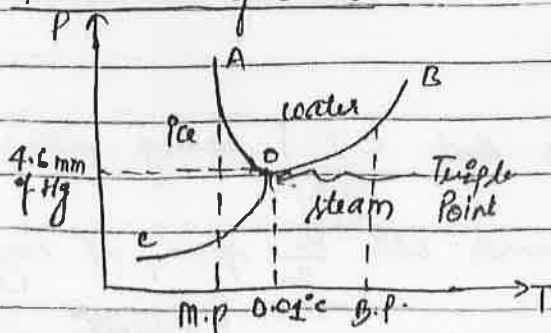
$$\Rightarrow h = 14000 \text{ m or } 14 \text{ Km}$$

• P-T Curve (Phase Diagram)

→ It shows phase of substance at given P & T.

→ It indicates dependency of B.P. & M.P. on pressure.

(1). P-T Curve of Water



OA → M.P.

OB → B.P.

OC → S.P.

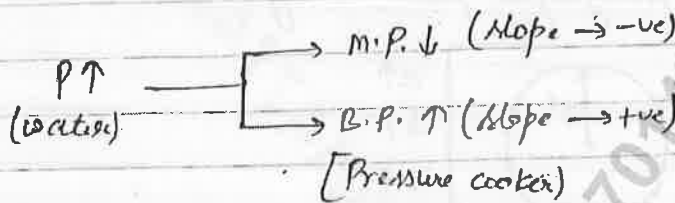
Slope  $\rightarrow \frac{\Delta P}{\Delta T}$

(OA) slope  $\Rightarrow \frac{\Delta P}{\Delta T} = -ve$

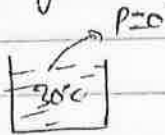
$P \uparrow \Rightarrow T \downarrow \Rightarrow M.P. \downarrow$  [Ice-skating].

(OB) slope  $\rightarrow \frac{\Delta P}{\Delta T} = +ve$

$P \uparrow \Rightarrow T \uparrow \Rightarrow B.P. \uparrow$

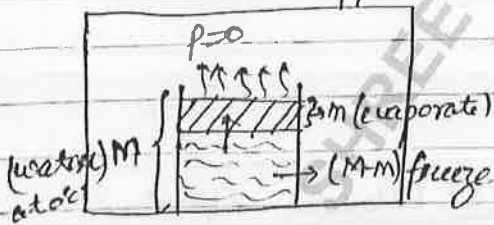


Ques:- If a glass of water at 30°C is taken to moon then what happens to water?



$P \downarrow \Rightarrow B.P. \downarrow$   
so, water evaporates quickly

Ques:- If a glass of water at 0°C kept in a small vacuum chamber then what happens to water?  $P=0$



$\therefore ML_v = (M-m) \times L_f$

$\left(\frac{m}{M}\right) = 0.132$  or 13.2%

$\rightarrow$  fraction of water evaporated.  
 $\Rightarrow$  Remaining  $\Rightarrow$  Ice = 86.8%

In case of pot (heat),

$mL_v = (M-m) s \Delta \theta$

## Thermal Expansion

→ If substance expands on heating or contracts on cooling then it is called Thermal Expansion.

→ There are two types of vibration: -

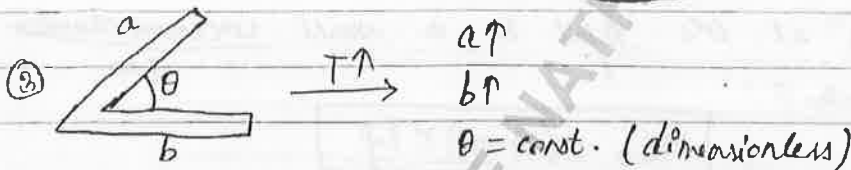
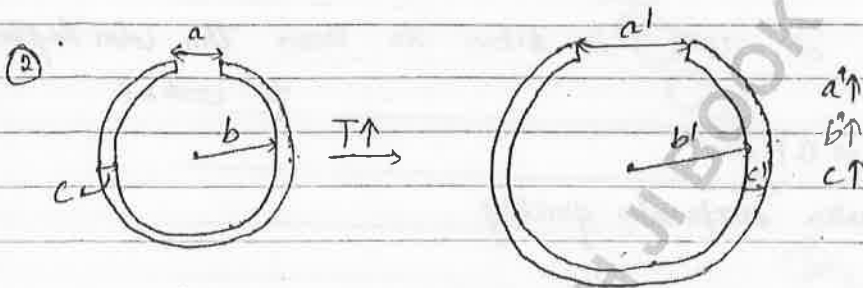
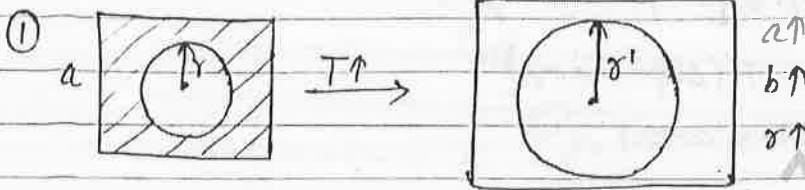
Longitudinal

→ If this vibration dominates on heating, substance expands.  
eg: → Most of substance

Transverse

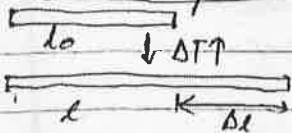
→ If this vibration dominates on heating, substance contracts.  
eg: → Polymer (rubber).

→ Thermal expansion like photographic plate enlargement.



### • Thermal Expansion in Solids

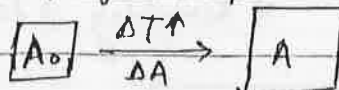
① Linear Expansion



$$\Delta l = l_0 \alpha \Delta T$$

↘ coefficient of linear expansion

② Areaal Expansion / (Superficial expansion)



$$\Delta A = A_0 \beta \Delta T$$

↘ coefficient of areaal expansion

③ Volumetric Expansion / (Cubic expansion)

$$\Delta V = V_0 \gamma \Delta T$$

↘ coefficient of volumetric expansion



$\rightarrow \alpha = \frac{\Delta L}{L_0 \Delta T} \Rightarrow \text{unit} = 1/^\circ\text{C}$ 
 $\rightarrow \beta = \frac{\Delta A}{A_0 \Delta T} \Rightarrow \text{unit} = 1/^\circ\text{C}$ 
 $\rightarrow \gamma = \frac{\Delta V}{V_0 \Delta T} \Rightarrow \text{unit} = 1/^\circ\text{C}$

$\rightarrow$  % change in length  $\frac{\Delta L}{L_0} \times 100 = \beta (\alpha \Delta T) \times 100$ 
 $\rightarrow$  % change in area  $\frac{\Delta A}{A_0} \times 100 = (\beta \Delta T) \times 100$ 
 $\rightarrow$  % Change in Vol<sup>m</sup>  $\frac{\Delta V}{V} \times 100 = (\gamma \Delta T) \times 100$

$\rightarrow L = L_0 + \Delta L$ 
 $\rightarrow A = A_0 + \Delta A$ 
 $\rightarrow V = V_0 + \Delta V$   
 $L = L_0 (1 + \alpha \Delta T)$ 
 $A = A_0 (1 + \beta \Delta T)$ 
 $V = V_0 (1 + \gamma \Delta T)$

Note  $\Rightarrow$  ① If  $\alpha \rightarrow$  higher  
 $\uparrow \alpha = \frac{\Delta L}{L_0 \Delta T} \uparrow$  then substance expands more on heating and contracts more on cooling.

②. Isotropic Material

$L \rightarrow \alpha$   $\therefore \gamma = 3\alpha$  ,  $\beta = 2\alpha$   
 $w \rightarrow \alpha$   
 $h \rightarrow \alpha$   $\therefore \alpha : \beta : \gamma = 1 : 2 : 3$

③. Anisotropic Material

$L \rightarrow \alpha_1$   $\therefore \gamma = \alpha_1 + \alpha_2 + \alpha_3$   
 $w \rightarrow \alpha_2$   
 $h \rightarrow \alpha_3$

④. Density  $\rightarrow \rho_0 = \frac{M}{V_0}$

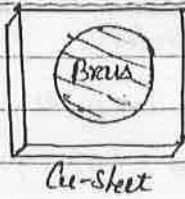
$\rho_0 \xrightarrow{\Delta T} \rho$   $\therefore \rho = \frac{M}{V} = \frac{M}{V_0 (1 + \gamma \Delta T)} = \frac{\rho_0}{1 + \gamma \Delta T}$

$\left( \rho = \frac{\rho_0}{1 + \gamma \Delta T} \right) \rightarrow$  For solids,  $\gamma \rightarrow$  very less  
 $L_0$

$\rho = \rho_0 (1 + \gamma \Delta T)^{-1}$   
 $\rho = \rho_0 (1 - \gamma \Delta T)$

$\Rightarrow$  % change in density  $\Rightarrow \frac{\Delta \rho}{\rho} = \gamma \Delta T \times 100$

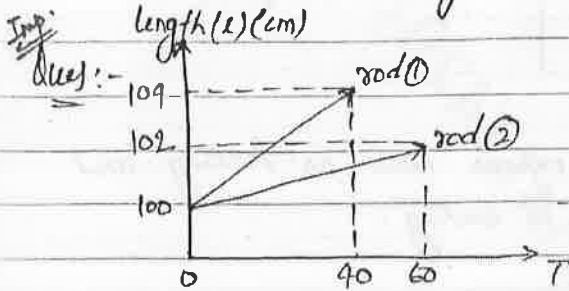
Quey:-



To remove Brass disc from 'Cu' sheet temp of system should - ①  $T \uparrow$  ②  $T \downarrow$  ③ first  $\uparrow$  then  $\downarrow$  ④ first  $\downarrow$  then  $\uparrow$

Learn  
 $\alpha_{\text{Brass}} > \alpha_{\text{Cu}}$

$\Rightarrow$  Brass will shrink more on cooling and ~~cut~~ comes out easily  $\rightarrow$  So,  $T \downarrow$



f/o coefficient of vol<sup>m</sup> expansion?

$$\therefore \frac{V_1}{V_2} = \frac{3\alpha_1}{3\alpha_2} = \frac{\alpha_1}{\alpha_2}$$

$$\frac{V_1}{V_2} = \frac{\Delta L_1}{L_0 \Delta T_1} = \frac{(104-100)}{100 \times 40}$$

$$\frac{V_1}{V_2} = \frac{\Delta L_2}{L_0 \Delta T_2} = \frac{(102-100)}{100 \times 60}$$

$$\frac{V_1}{V_2} = \frac{4}{40} \times \frac{60}{2} = \frac{3}{1}$$

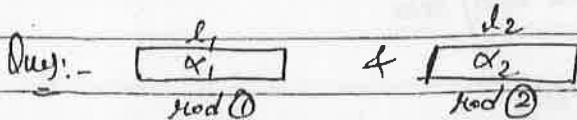
Quey:- If % age change in length of rod of  $l$  dimension is 2% on increasing its temp. by  $100^\circ\text{C}$  then f/o % change in vol<sup>m</sup> of rod of dimension  $(2l \times \frac{l}{3} \times 4l)$  of same material and same increase in temp.

$$\therefore \frac{\Delta l}{l_0} \times 100 = (\alpha \Delta T \times 100) = 2\%$$

$$\therefore \frac{\Delta V}{V_0} \times 100 = \frac{V_0 \gamma \Delta T \times 100}{V_0} = 3(\alpha \Delta T \times 100)$$

$$= 3 \times 2\%$$

$$= 6\%$$



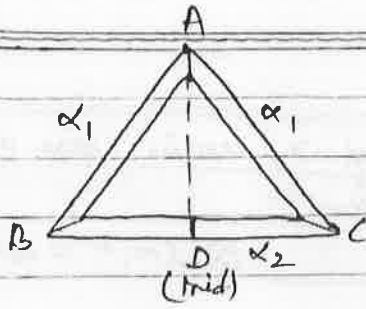
If rods are connected end to end then coefficient of linear expansion of new rod?

$$\therefore \Delta l_{\text{Total}} = \Delta l_1 + \Delta l_2$$

$$\alpha_{\text{eq}} (l_1 + l_2) \Delta T = \alpha_1 l_1 \Delta T + \alpha_2 l_2 \Delta T$$

$$\alpha_{\text{eq}} = \frac{\alpha_1 l_1 + \alpha_2 l_2}{l_1 + l_2}$$

Ques:-



If rods AB, BC & AC are identical in dimension and distance b/w AD remains same at all

temp. then -

(a)  $\alpha_1 = \alpha_2$

(c)  $\alpha_2 = 4\alpha_1$

(b)  $\alpha_1 = 4\alpha_2$

(d)  $\alpha_2 = 2\alpha_1$

$$AD^2 = AB^2 - BD^2$$

$$= l_0^2 - \frac{l_0^2}{4} = \frac{3l_0^2}{4}$$

↓ ΔT ↑

$$AD'^2 = AD^2 = AB'^2 - BD'^2$$

$$\frac{3l_0^2}{4} = [l_0(1 + \alpha_1 \Delta T)]^2 - \left[\frac{l_0}{2}(1 + \alpha_2 \Delta T)\right]^2$$

$$\frac{3}{4} \frac{l_0^2}{l_0^2} = \frac{l_0^2}{l_0^2} [1 + \alpha_1 \Delta T]^2 - \frac{l_0^2}{4} [1 + \alpha_2 \Delta T]^2$$

$$\frac{3}{4} = 1 + 2\alpha_1 \Delta T - \frac{1}{4} (1 + 2\alpha_2 \Delta T)$$

$$\frac{3}{4} = \frac{3}{4} + 2\alpha_1 \Delta T - \frac{1}{2} \alpha_2 \Delta T$$

$$2\alpha_1 \Delta T = \frac{\alpha_2 \Delta T}{2}$$

$$\alpha_2 = 4\alpha_1$$

Ques:- M.O.I. of sphere about geometric axis is  $I_0$ . If temp. of sphere rod by  $\Delta t$  then find fractional change in it's M.O.I. {take  $\alpha_{\text{sphere}} = \alpha$ }

m(1)  $\therefore I_0 = \frac{2}{5} MR_0^2$

↓ Δt ↑

$$I = \frac{2}{5} MR^2$$

$$I = \frac{2}{5} m \{R_0(1 + \alpha \Delta t)\}^2$$

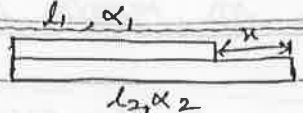
$$I = \frac{2}{5} MR_0^2 (1 + 2\alpha \Delta t)$$

$$I = I_0 (1 + 2\alpha \Delta t)$$

$$\frac{I - I_0}{I_0} = 2\alpha \Delta t$$

m(2)  $\frac{\Delta I}{I_0} = \frac{2 \Delta R}{R_0} \left\{ \because I \propto R^2 \right\}$

$$= 2\alpha \Delta t$$

Ques: -  2 rods. For value of x, remains same at all condition then: -

For  $x = \text{const.}$

$$\Delta l_1 = \Delta l_2$$

$$l_1 \alpha_1 \Delta T = l_2 \alpha_2 \Delta T$$

$$\frac{l_1}{l_2} = \frac{\alpha_2}{\alpha_1}$$

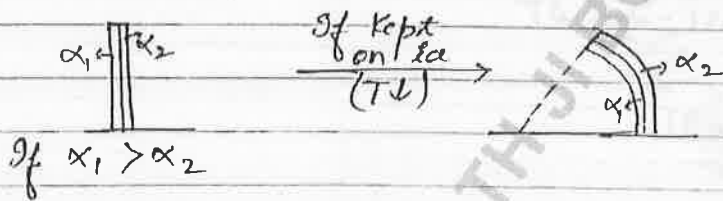
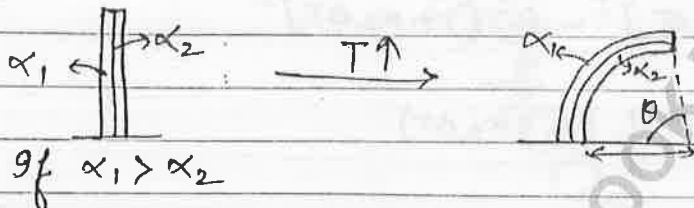
①  $\frac{l_1}{l_2} = \frac{\alpha_1}{\alpha_2}$       ②  $l_1 \alpha_1^2 = l_2 \alpha_2^2$

③  $\frac{l_1}{l_2} = \frac{\alpha_2}{\alpha_1}$       ④ NOTA

• Application of Thermal Expansion in Solids

①. Bimetallic Strip

→ Used as thermostat



②. Stress and Strain

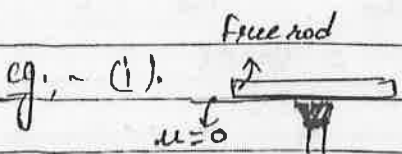
$$Y = \frac{F/A}{\Delta l/l_0}$$

Compressible stress

$$\frac{F}{A} = Y \left( \frac{\Delta l}{l_0} \right) \Rightarrow \left( \frac{F}{A} \right) = Y \times \Delta T \quad (\because \Delta l = l_0 \times \Delta T)$$

$$\Rightarrow F = Y A \times \Delta T$$

↳ Compressible stress force.



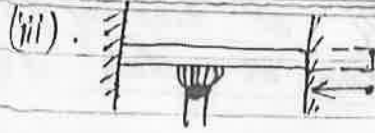
$\therefore$  if compressible stress & strain = 0



$\therefore T \uparrow \Rightarrow l \uparrow$

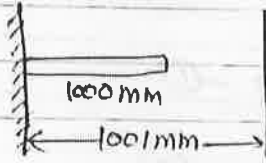


[Compressible stress & strain = 0]



∴ Agar rigid wall nahi hote, toh l↑  
but wall incompressible diya hai so  
compressive stress & strain.

Que:-



If  $\alpha_{rod} = 2 \times 10^{-6} / ^\circ C$ ,  $\gamma_{rod} = 10^{11} / m^2$   
If temp. of rod rcd by  $10^\circ C$  compressive  
stress produced in rod?

$$\begin{aligned} \therefore \Delta L &= L \alpha \Delta T \\ &= (1000) \times 2 \times 10^{-6} \times 10 \\ &= 2 \text{ mm} \end{aligned}$$

→ 1 mm [ $0^\circ \rightarrow 5^\circ$ ]  $\Rightarrow$  No Compressive stress  
→ 1  $\rightarrow$  2 mm [ $5^\circ \rightarrow 10^\circ$ ]  $\Rightarrow$  Compressive stress produced.

$$\therefore \frac{F}{A} = \gamma \alpha \Delta T$$

$$\begin{aligned} &= 10^{11} \times 2 \times 10^{-6} \times (5) \\ &= 10^9 \text{ N/m}^2 \end{aligned}$$

(3). Time Period ( $t$ ) of Clock Pendulum

∴ Temp.  $\uparrow \Rightarrow l \uparrow$  [Thermal expansion].

$$\therefore t = 2\pi \sqrt{\frac{l}{g}}$$

In Summer  $\rightarrow$  Temp.  $\uparrow \Rightarrow l \uparrow \Rightarrow t \uparrow \Rightarrow$  Clock slow  $\Rightarrow$  Time loss

In winter  $\rightarrow$   $T \downarrow \Rightarrow l \downarrow \Rightarrow t \downarrow \Rightarrow$  Clock fast  $\Rightarrow$  Time gain

$$\therefore t = 2\pi \sqrt{\frac{l}{g}}$$

$$\therefore \frac{\Delta t}{t} = \frac{1}{2} \frac{\Delta l}{l}$$

Time loss or gain  $\left( \Delta t = \frac{1}{2} \alpha \Delta T (t) \right) \rightarrow$  Time for which loss or gain calculated.

$\rightarrow$  ' $\alpha$ ' should be low for good Pendulum.

eg:- (i). Quartz

(ii). In-var (Fe-Ni alloys)

Ques:- Time loss of a clock pendulum in a day is 10 sec, if it is used at  $40^\circ\text{C}$ . If it would have used at temp. of  $5^\circ\text{C}$ , time gain in a day is 12 sec.  $\therefore$  temp. at which it will show accurate time.

$$5^\circ\text{C} \xrightarrow{\text{gain}} T \xrightarrow{\text{loss}} 40^\circ\text{C}$$

$$\Delta t = 12 \text{ sec} \quad \epsilon = 10 \text{ sec}$$

$$\therefore \text{loss} \Rightarrow \Delta t = 10 \pm \frac{1}{2} \times [40 - T] \times 1 \text{ day} \quad \text{--- (1)}$$

$$\therefore \text{gain} \Rightarrow \Delta t = 12 = \frac{1}{2} \times [T - 5] \times 1 \text{ day} \quad \text{--- (2)}$$

$$\text{(1) + (2)}$$

$$T = 19^\circ\text{C}$$

• Application of Thermal Energy in Gases.

$\rightarrow$  Only  $\gamma$  is defined.

$$\gamma = \frac{\Delta V}{V_0 \Delta T} \quad \text{OR} \quad \frac{\delta V}{V_0 \delta T}$$

Ques:- If an I.G. follows  $PT^5 = \text{const.}$  then find coefficient of vol<sup>m</sup> expansion of I.G. in process. [ $T_0 \rightarrow$  Initial temp.]

$$\gamma = \frac{\Delta V}{V_0 \Delta T}$$

$$\therefore (PT^5 = \text{const.})$$

$$\frac{nRT}{V} \cdot T^5 = \text{const.}$$

$$V \propto T^5$$

$$\frac{\Delta V}{V_0} = 5 \frac{\Delta T}{T_0}$$

$$\gamma = \frac{\Delta V}{V_0 \Delta T} = \frac{5}{T_0}$$

Ques:-  $\therefore$  coefficient of vol<sup>m</sup> expansion of I.G. in IB process.

$$\therefore \gamma = \frac{\Delta V}{V_0 \Delta T}$$

$$\text{IB} \rightarrow V \propto T$$

$$\frac{\Delta V}{V_0} = \frac{\Delta T}{T_0}$$

$$\gamma = \frac{\Delta V}{V_0 \Delta T} = \frac{1}{T_0}$$

15/01/2020

Ques:- If an I.G. follows  $PV^n = \text{const.}$  then find coefficient of vol<sup>m</sup> expansion of I.G. in process. ( $T_0 \rightarrow$  initial temp.)

$$\therefore \gamma = \frac{\Delta V}{V_0 \Delta T}$$

$$\therefore PV^n = \text{const.}$$

$$\frac{nRT}{V} \cdot V^n = \text{const.}$$

$$T \propto \frac{V}{V^n}$$

$$T \propto V^{1-n}$$

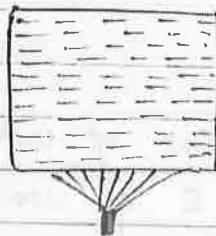
$$\frac{\Delta T}{T_0} = (1-n) \frac{\Delta V}{V_0}$$

$$\frac{\Delta V}{V_0 \Delta T} = \frac{1}{(1-n)} \times \frac{1}{T_0}$$

• Application of Thermal Energy in Liquids  
 $\rightarrow$  Only  $\gamma$  is defined.

$$\gamma = \frac{\Delta V}{V_0 \Delta T} \quad \text{or} \quad \Delta V = V_0 \gamma \Delta T$$

$V_0 \rightarrow$  Initial vol<sup>m</sup> of container & liquid



$\rightarrow$  First container expands,

$$(\Delta V)_c = V_0 \gamma_c \Delta T \quad [\text{liquid level } \downarrow]$$

then liquid expands,

$$(\Delta V)_L = V_0 \gamma_L \Delta T \quad [\text{lig. level } \uparrow]$$

$$\therefore \Delta V_{\text{app.}} = (\Delta V)_L - (\Delta V)_c$$

$$V_0 \gamma_{\text{app}} \Delta T = V_0 \gamma_L \Delta T - V_0 \gamma_c \Delta T$$

$$\gamma_{\text{app}} = \gamma_L - \gamma_c$$

Here,  $\gamma_c =$  vol<sup>m</sup> expansion coefficient of Container

$\gamma_L =$  Real / Pure / Absolute coefficient of vol<sup>m</sup> expansion of liquid.

$\gamma_{\text{app}} =$  Apperant / Relative coefficient of vol<sup>m</sup> expansion of liquid.

→ If liquid overflows  $\Rightarrow \gamma_L > \gamma_c$   
 $(\Delta V)_{\text{overflow}} = (\Delta V)_L - (\Delta V)_c = V_0 (\gamma_L - \gamma_c) \Delta T$

→ If level remains unchanged  $\Rightarrow \gamma_L = \gamma_c$   
 $\downarrow \quad \downarrow$   
 $\gamma_L = 3\alpha$

Que:- If coefficient of vol<sup>m</sup> expansion of liquid is  $\frac{4}{3}$  times of that of container then the value of  $\frac{\gamma_{\text{app}}}{\gamma_{\text{absolute}}} = ?$

$$\gamma_L = \frac{4}{3} \gamma_c$$

$$\frac{\gamma_{\text{app}}}{\gamma_L} = \frac{\gamma_L - \gamma_c}{\gamma_L} = 1 - \frac{\gamma_c}{\gamma_L}$$

$$= 1 - \frac{3}{4} = \frac{1}{4}$$

Que:- Apparent coefficient of vol<sup>m</sup> expansion of a liquid filled completely in vessel A & B of same vol<sup>m</sup> is  $\gamma_1$  &  $\gamma_2$ . If coefficient of linear expansion of A vessel is  $\alpha_1$ , then find that of vessel B.

$$\gamma_{\text{app}} = \gamma_L - \gamma_c$$

$$\text{(A)} \quad \gamma_1 = \gamma_L - 3\alpha_1 \quad \text{--- (1)}$$

$$\text{(B)} \quad \gamma_2 = \gamma_L - 3\alpha_2 \quad \text{--- (2)}$$

$$\text{(1)} - \text{(2)}$$

$$\gamma_1 - \gamma_2 = 3\alpha_2 - 3\alpha_1$$

$$\alpha_2 = \frac{\gamma_1 - \gamma_2 + 3\alpha_1}{3}$$

• Temperature scale

→ To design a scale, it's

$$\boxed{\frac{\text{Reading} - \text{M.P.}}{\text{B.P.} - \text{M.P.}} = \text{const.}}$$



Scale	M.P.	B.P.
$^{\circ}\text{C}$	0	100
K	273	373
$^{\circ}\text{F}$	32	212
$^{\circ}\text{X}$	A	B

$$\frac{^{\circ}\text{C} - 0}{100 - 0} = \frac{\text{K} - 273}{373 - 273} = \frac{^{\circ}\text{F} - 32}{212 - 32} = \frac{^{\circ}\text{X} - \text{A}}{\text{B} - \text{A}}$$

$$\Rightarrow \frac{^{\circ}\text{C}}{100} = \frac{\text{K} - 273}{100} = \frac{^{\circ}\text{F} - 32}{100}$$

$$\Rightarrow \frac{^{\circ}\text{C}}{5} = \frac{\text{K} - 273}{5} = \frac{^{\circ}\text{F} - 32}{9}$$

↓ change

$$\frac{\Delta\text{C}}{5} = \frac{\Delta\text{K}}{5} = \frac{\Delta\text{F}}{9} \Rightarrow$$

$$\Delta\text{F} = \frac{9}{5} \Delta\text{C} = 1.8 \Delta\text{C}$$

Ques! - Two unknown scales  $^{\circ}\text{A}$  &  $^{\circ}\text{B}$  having ice point and steam point of water respectively  $-25^{\circ}\text{A}$  and  $-5^{\circ}\text{A}$  and  $-45^{\circ}\text{B}$  &  $-25^{\circ}\text{B}$ .  $^{\circ}\text{F}$  temp. of block in  $^{\circ}\text{A}$  scale if it is at  $10^{\circ}\text{B}$  temp.

$$\frac{^{\circ}\text{A} - (-25)}{-5 - (-25)} = \frac{^{\circ}\text{B} - (-45)}{-25 - (-45)}$$

$$\Rightarrow \frac{^{\circ}\text{A} + 25}{20} = \frac{10 + 45}{20}$$

$$\Rightarrow ^{\circ}\text{A} = 30 \quad \text{So } A \rightarrow 30^{\circ}\text{A}$$

#  $^{\circ}\text{C}/\text{K}$  scale Calibrated with length [Mercury Thermometer]

$$\frac{^{\circ}\text{C} - 0}{100 - 0} = \frac{l_c - l_{0^{\circ}\text{C}}}{l_{100^{\circ}\text{C}} - l_{0^{\circ}\text{C}}} \quad \& \quad \frac{\text{K} - 273}{373 - 273} = \frac{l_K - l_{273}}{l_{373} - l_{273}}$$

# Platinum - wire Thermometer (Resistance)

$$\frac{^{\circ}\text{C} - 0}{100 - 0} = \frac{R_c - R_{0^{\circ}\text{C}}}{R_{100^{\circ}\text{C}} - R_{0^{\circ}\text{C}}}$$

Ques:- If in Hg-scale length of Hg;  $20^{\circ}\text{C} = 10\text{cm}$ ,  $100^{\circ}\text{C} = 60\text{cm}$ ,  
then find temp. at which Hg column length becomes 50cm.

$$\frac{^{\circ}\text{C} - 0}{100 - 0} = \frac{50 - 10}{60 - 10}$$

$$\frac{^{\circ}\text{C}}{100} = \frac{40}{50}$$

$$^{\circ}\text{C} = \frac{400}{5} = 80^{\circ}\text{C}$$

SHREE NATHJI BOOK 7014774207

- Elasticity.
- Surface Tension.

- Fluid Statics
- Fluid Dynamics.
- Viscosity.

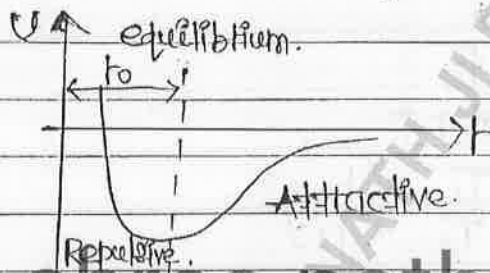
## ELASTICITY

After removal of deforming force, a body recovers or try to recover its original config<sup>n</sup>. This phenomena is elasticity.

Reason → Interatomic force of attract<sup>n</sup> or repulsion, due to internal restoring force.

Perfectly Elastic - GRF large, difficult to deform.  
eg. quartz, steel etc.

perfectly plastic - GRF negligible, & remains permanently deformed.  
eg. clay, putty.



3/9/19

shree nath ji 7014774207

$$\# \text{ Stress} = \frac{\text{Internal Restoring force}}{\text{Area}} = \frac{F_{\text{ext}} \cdot (\text{in eq. } \text{m}^3)}{A} \quad \text{N m}^{-2} \text{ or Pascal.}$$

### 1. Longitudinal Stress -



$$= \frac{F}{A}$$

Force per unit area

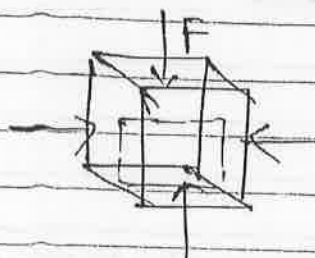
(a) ↑ ↑ ↑ tensile

(b) ↓ ↓ ↓ compressional

2. Hydraulic / volumetric

$= \Delta P$  (change in Pressure)

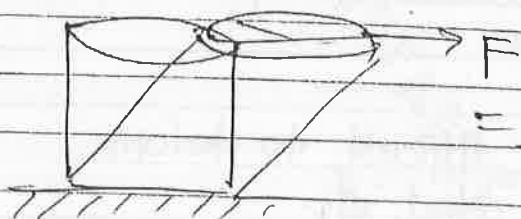
$= P_0 + P - P_0 = P$



$= P_0 + h\rho g - P_0$

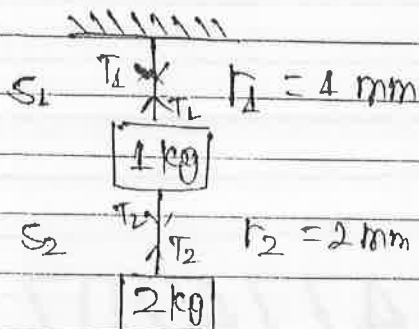
$= h\rho g$

3. Shear or Tangential stress.



$= \frac{F}{A}$  along which force is acting.

Q.



$S = \frac{F}{A r^2}$

$\frac{S_1}{S_2} = \frac{T_1}{T_2} \times \left(\frac{r_2}{r_1}\right)^2$

$= \frac{30}{20} \times \left(\frac{2}{1}\right)^2$

$= \frac{3}{2} \times 4 = 6$

Q.

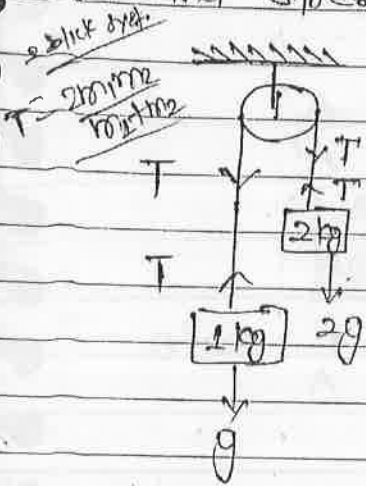


$W_1, L, \rho$ .

Wt. of rope is  $W_1$ , length is  $L$ , and area of cross-section ' $s$ ' find stress @ dist.  $L/4$  from lower end.

→ stress =  $\frac{W_L + W/4}{S}$

Q. Find stress in the string connecting the blocks.



$A = 2 \text{ mm}^2$

$a = \frac{(m_1 g - m_2 g)}{m_1 + m_2} = \frac{2g - g}{3}$

$= \frac{g}{3} = \frac{10}{3} \text{ m/s}^2$

$20 - T = 2 \times \frac{10}{3}$

$20 - \frac{20}{3} = T \Rightarrow T = \frac{40}{3} \text{ N}$

$\sigma = \frac{T}{A} = \frac{40/3}{2 \times 10^{-6}}$   
 $= \frac{20}{3} \times 10^6 \text{ Nm}^{-2}$

$1 \text{ mm} = 10^{-3} \text{ m}$   
 $1 \text{ mm}^2 = 10^{-6} \text{ m}^2$

Q. A ball is taken @ a depth of 20 cm in a liquid of relative density 0.8. Find stress.

→ Hydrostatic  $\Delta P = \rho_0 h g - P_0$  relative density =  $\frac{\rho}{\rho_w}$  (no unit)

$= h \rho g$

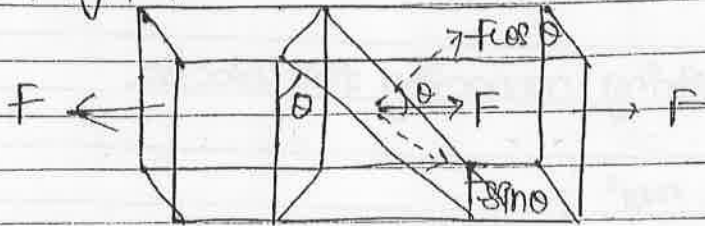
$= 20 \times 10^{-2} \times 0.8 \times 10^3 \times 10$  } or

$= 20 \times 10^{-2} \times 0.8 \times 10^3 \times 10$   
 $= 1.6 \times 10^3 \text{ Pascal}$

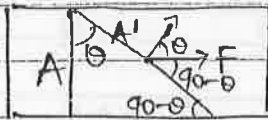
specific gravity  
 $\rho = \rho_0 \times \rho_w$

NCEERT Exemplar

Q. Find longitudinal & shear stress on the inclined area on the fig.



$$\cos \theta = \frac{A}{A'}$$



$$A' = \frac{A}{\cos \theta}$$

$$\text{Longitudinal stress} = \frac{F \cos \theta}{A / \cos \theta} = \frac{F}{A} \cos^2 \theta$$

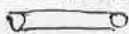
$$\text{Shear tangential} = \frac{F \sin \theta}{A / \cos \theta} = \frac{F \sin \theta \cos \theta \times 2}{2A} = \frac{F \sin 2\theta}{2A}$$

### # Breaking stress or ultimate strength -

Max<sup>m</sup> tolerated stress beyond which the body is actually broken or fractured.

$$B.S. = \frac{F}{A} \text{ Nm}^{-2}$$

Steel



depends only on nature of material.  
independent of dimension of wire.

Breaking force

$$F = B.S. \times A$$

same metal

$$F \propto A$$

$$F \propto r^2$$

Q. To break a wire of diameter 2 mm, req<sup>d</sup> force is 20 N. To break another wire of same metal of diam. 4 mm, req<sup>d</sup> force will be -

P-222  
BB-1 → 2 x All in  
56  
18/5/16

PAGE NO. \_\_\_\_\_

DATE: / /

$$\Rightarrow \frac{F_1}{F_2} = \left(\frac{l_1}{l_2}\right)^2$$

$$\Rightarrow \frac{20}{F_2} = \left(\frac{2}{42}\right)^2$$

$$\Rightarrow F_2 = 80 \text{ N.}$$

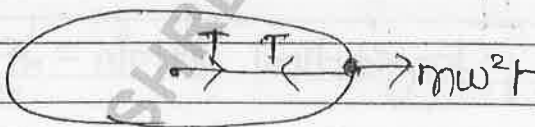
Q. A compressive force of  $5 \times 10^5 \text{ N}$  is subjected to a bone of length 20 cm & cross sect<sup>n</sup>  $4 \text{ cm}^2$ . If ultimate compressive strength of bone is  $1.7 \times 10^8 \text{ N/m}^2$ , Will the bone break or not?

$$\rightarrow \text{stress} = \frac{5 \times 10^5}{4 \times 10^{-4}} = 1.25 \times 10^9 \text{ N/m}^2$$

stress > B.S.  
will break.

Q. A stone of mass 1 kg is tied with one end of 2m long string & cross sect<sup>n</sup>  $1 \text{ mm}^2$ . Find <sup>max</sup> angular speed by which string can be revolve in the plane without breaking. (B.S. =  $8 \times 10^8 \text{ N/m}^2$ ).

→



$$T = m\omega^2 r$$

$$\text{stress} = \frac{T}{A}$$

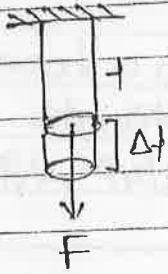
$$B. \text{ stress} = \frac{m\omega_{\text{max}}^2 r}{A}$$

$$8 \times 10^8 = \frac{1 \omega_{\text{max}}^2 \times 2}{10^{-6}}$$

$$\omega_{\text{max}} = 20 \text{ rad/s.}$$

# strain =  $\frac{\text{change in configuration}}{\text{original config}}$  dimensionless.

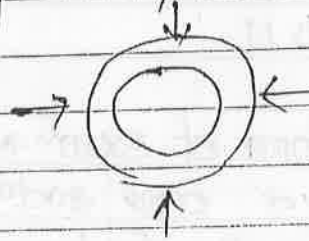
1. Longitudinal strain -



$$= \frac{\Delta l}{l}$$

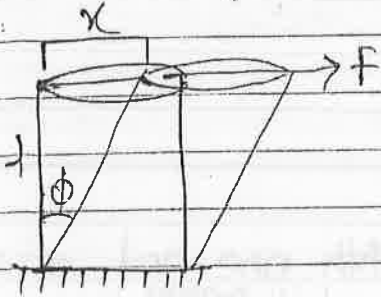
$$= \frac{l + \Delta l - l}{l}$$

2. Hydraulic / Volumetric



$$= \frac{\Delta V}{V}$$

3. Shear / Torsional.

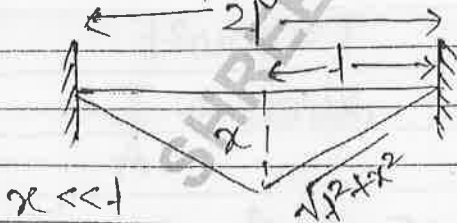


$$\tan \phi = \frac{x}{l}$$

φ is very small,  $\tan \phi \approx \phi$ .

$$\phi = \frac{x}{l}$$

Q. If wire is pulled from mid point through distance 'x'. Find longitudinal strain.



$$\text{longitudinal strain} = \frac{l_f - l_i}{l_i}$$

$$= \frac{2\sqrt{l^2 + x^2} - 2l}{2l}$$

$$= \frac{2\sqrt{l^2 + x^2}}{2l} - 1$$

$$= \frac{\sqrt{l^2 + x^2}}{l} - 1$$

$$= \left( \frac{l^2 + x^2}{l^2} \right)^{1/2} - 1$$

$$= \left( 1 + \frac{x^2}{l^2} \right)^{1/2} - 1$$

$$\left( 1 + \frac{x^2}{l^2} \right)^{1/2} \approx 1 + \frac{1}{2} \frac{x^2}{l^2}$$



Hooke's law -

Under small deformation,  
 Stress  $\propto$  Strain.

$$\text{Stress} = E \times \text{Strain}.$$

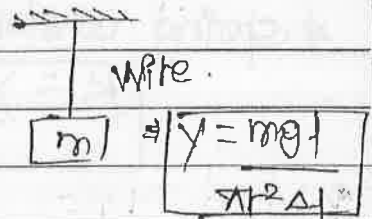
Modulus of Elasticity,  $E = \frac{\text{Stress}}{\text{Strain}} \text{ Nm}^{-2}$  or Pascal.

$\rightarrow$  depends only on nature of material.

1. Young's modulus -

$$Y = \frac{F/A}{\Delta l/l}$$

$$\Rightarrow Y = \frac{Fl}{A\Delta l}$$



Q. When tension in a wire is  $T_1$  its length is  $l_1$  & when tension is  $T_2$ , length  $l_2$ , find original length.

Area = const.

$\rightarrow$

$$Y = \frac{FA}{A\Delta l}$$

$$\Rightarrow F \propto \Delta l.$$

$$\frac{F_1}{F_2} = \frac{\Delta l_1}{\Delta l_2}$$

$$\frac{T_1}{T_2} = \frac{l_1 - l}{l_2 - l}$$

$$\Rightarrow T_1 l_2 - T_1 l = T_2 l_2 - T_2 l$$

$$\Rightarrow l(T_2 - T_1) = T_2 l_1 - T_1 l_2$$

$$l = \frac{T_2 l_1 - T_1 l_2}{T_2 - T_1}$$

Q. When tension in wire is 4 N, its length is 'a', when tension is 5 N, length is b, find the length of the wire when tension is 9 N.

4N  $\rightarrow$  a   
 5N  $\rightarrow$  b   
 9N  $\rightarrow$  x

Given

$$F_1 = \Delta l_1$$

$$F_2 = \Delta l_2$$

$$\frac{4}{5} = \frac{a-1}{b-1}$$

$$\frac{5}{9} = \frac{b-1}{x-1}$$

$$5x - 5 = 9b - 9$$

$$5x = 9b - 4$$

$$= 9b - 4(5a - 4b)$$

$$= 9b - 20a + 16b$$

$$4b - 4 = 5a - 5$$

$$4 = 5a - 4b$$

$$5x = 25b - 20a$$

$$x = 5b - 4a$$

$F = \frac{YA}{l} \Delta l$  (elasticity)  
 $F = kx$  (spring)

\* Spring constant of wire :-

$$k = \frac{YA}{l}$$

Q: Find force req<sup>d</sup> to stretch a wire by 0.2% diameter  
 $\phi$  2mm &  $\gamma = 2 \times 10^{11} \text{ N/m}^2$ .

$$\gamma = \frac{F l}{A \Delta l}$$

$$\Rightarrow F = \frac{\gamma A \Delta l}{l} = \frac{\gamma \pi r^2 \Delta l}{l}$$

$$= \frac{2 \times 10^{11} \times 3.14 \times (10^{-3})^2 \times 0.2}{100}$$

$$= 1.256 \times 10^3$$

$$= 1.256 \text{ KN}$$

let cylinder  
 wire

Q, from vol<sup>m</sup> 'V' of a wire of length 'l' is drawn for constant stretching force. Wof graph is st. line?

$\rightarrow$  1)  $\Delta l \propto l$

3)  $\Delta l \propto l^{1/2}$

2)  $\Delta l \propto l^{1/4}$

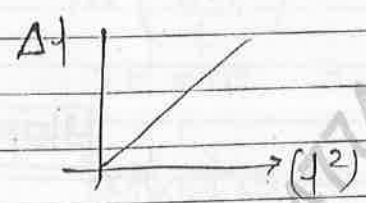
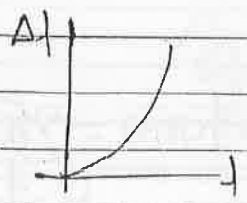
~~4)  $\Delta l \propto l^2$~~

$$V = A l$$

$A = \frac{V}{l}$  (var. 'l' में दिया है, so 'A' को eliminate करती). जो दिया है उसके opp. को eliminate करना ही formula से.

$$V = F l^2$$

$$V \propto l^2$$

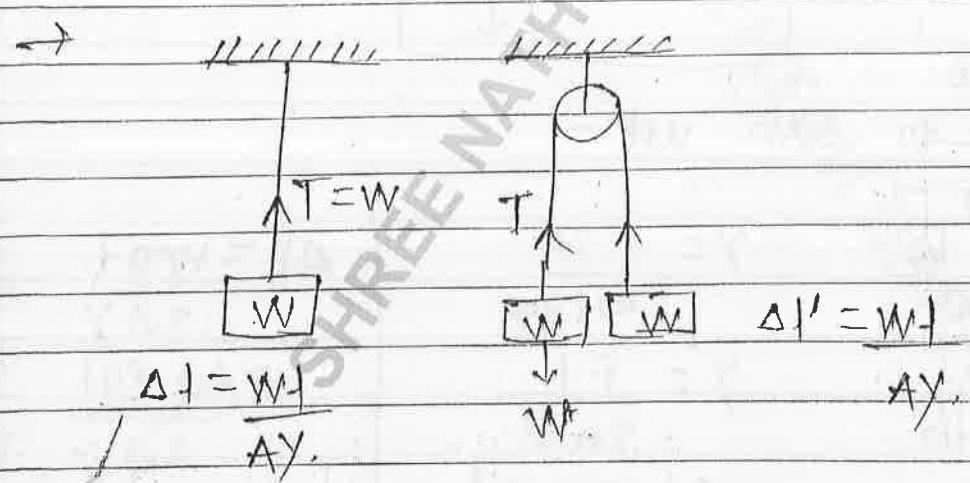


$$l = \frac{V}{A}$$

$$y = \frac{FV}{A^2 \Delta l}$$

$$\Delta l \propto \frac{1}{A^2}$$

Q. A block of wt. 'W' is suspended by a wire, extension produced is 'l'. Now the same wire passes over massless & frictionless pulley and blocks of each of weight 'W' are suspended by the both end. Find elongation.



only choose direction of tension.

Q. Four wires of same metal are given, for same stretching force, which will have largest elongation,

- a) length of wire = 300 cm, diam. = 3 mm
- b) " " = 50 cm, " = 0.5 mm
- c) " " = 100 cm, " = 1 mm
- d) " " = 200 cm, " = 2 mm

$$\gamma = \frac{F}{A \Delta l}$$

$$1) \frac{33}{1} \frac{300}{9} : \frac{5000 \times 100}{0.25} \cdot \frac{100}{4^2} \cdot \frac{200}{4} \frac{50}{1}$$

$$\Delta l \propto \frac{1}{l^2}$$

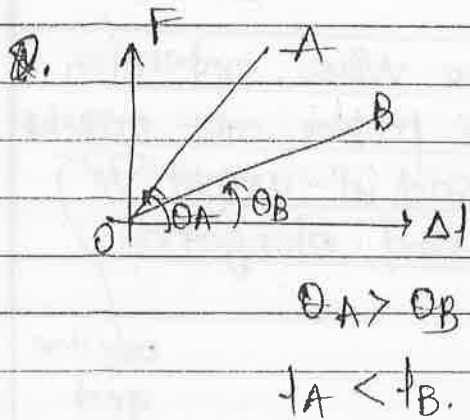
↓  
लंबाई बढ़ेगी



$$F = \left( \frac{\gamma A l^2}{l} \right) \Delta l$$

$$y = mx + c$$

$$\text{slope} = m = \tan \theta = \frac{\gamma A l^2}{l}$$

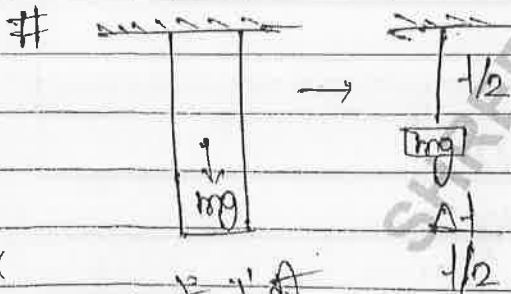


eg. Both of same metal, same thickness.

$$\tan \theta = \frac{\gamma A l^2}{l}$$

$$\tan \theta \propto \frac{1}{l}$$

# Extension due to own wt -



$$y = \frac{F}{A \Delta l} \Rightarrow \frac{mg}{2A \Delta l}$$

$$y = \frac{F}{2A \Delta l}$$

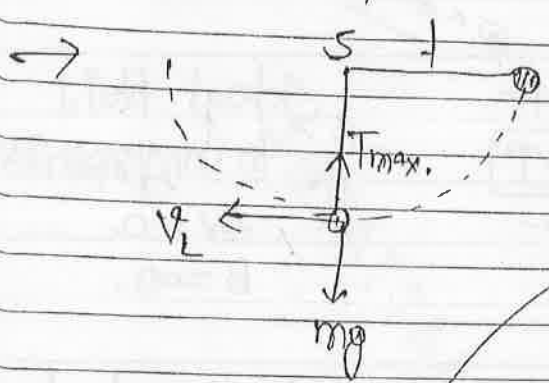
$$\Delta l = \frac{mg l}{2A y}$$

$$\Delta l = \frac{V \rho g l}{2A y} = \frac{A l \rho g l}{2A y}$$

$$\Delta l = \frac{l^2 \rho g}{2y}$$

# strain at 'l' is  $\frac{1}{2}$

Q. A simple pendulum is made by wire of length  $l$  & radius  $r$ . mass of bob is  $m$ . If pendulum is released from HZ position. Find maximum extension in the wire. Young's modulus  $Y$ .



$$T_{max} - mg = \frac{mv_L^2}{r}$$

$$T_{max} = \frac{mv_L^2}{r} + mg$$

com E,  
 Loss in P.E. = Gain in K.E.  
 $mg \cdot l = \frac{1}{2} mv_L^2$

$$v_L^2 = 2gl$$

$$T_{max} = \frac{m}{r} \times 2gl + mg$$

$$T_{max} = 3mg$$

$$\therefore \Delta l = \frac{(3mg)l}{(Yr^2)y}$$

5/9/19

# Bulk Modulus - [B or K]

$$B = \frac{\Delta P}{\frac{-\Delta V}{V}}$$

[P ↑, V ↓, While Bulk modulus always +ve].

$$B = - \frac{\Delta P V}{\Delta V}$$

Pascal or  $N \cdot m^{-2}$ .

compressibility  $[C]$   
 $C = \frac{1}{B}$  Pascal<sup>-1</sup>.

→ Fluid also possess Bulk modulus.

Gas

gradually

isothermal

$$B_T = P$$

Adiabatic

$$B_A = \gamma P$$

$$\gamma = \frac{C_P}{C_V}$$

@ sudden

Ideal fluid

In compressible.

$$\Delta V = 0.$$

$$B = \infty.$$

Q. A cube of cu of side 10 cm is subjected to a hydraulic stress of  $7 \times 10^5$  Pa. If bulk modulus of cu is  $140 \times 10^9$  Pa. Find vol<sup>m</sup> compression.

$$\rightarrow \Delta V = -\frac{\Delta P}{B} V = -\frac{7 \times 10^5}{140 \times 10^9} (10)^3 \text{ cm}^3.$$

$$\Delta V = -0.05 \text{ cm}^3.$$

vol<sup>m</sup> compressed by 0.05 cc.

Q. Bulk modulus of rubber ball is  $9 \times 10^8$  Pa. upto what depth it should be taken in water so that its vol<sup>m</sup> is compressed by 0.2%.

$$B = \frac{-h \rho g}{\Delta V/V}$$

$$h = \frac{B \times \Delta V}{V \rho g} \times \frac{1}{100} = \frac{9 \times 10^8 \times 0.2}{100 \times 10^3 \times 10} \times \frac{1}{100}$$

$$= 180 \text{ m.}$$

Q. A hydraulic stress of 'P' is applied on a ball, of bulk modulus is 'B'. Find - (i) fractional change in vol. (ii) " " radius. (iii) " " density.

→ (1)  $\frac{\Delta V}{V} = -\frac{P}{B}$  fractional compression =  $\frac{P}{B}$

(2)  $V = \frac{4}{3} \pi r^3$

$\frac{\Delta V}{V} = 3 \frac{\Delta r}{r}$

$3 \frac{\Delta r}{r} = -\frac{P}{B}$

$\frac{\Delta r}{r} = -\frac{P}{3B}$  fractional compression in radius =  $\frac{P}{3B}$

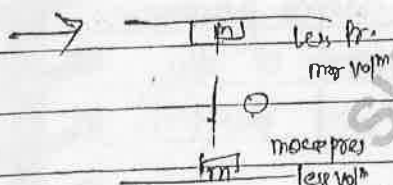
(3)  $\rho = \frac{m}{V}$

$\frac{\Delta \rho}{\rho} = -\frac{\Delta V}{V} \Rightarrow \frac{\Delta \rho}{\rho} = \frac{P}{B}$  (fractional increment) in density.

*Accept solved example pmt-10*

Q. An ocean is 3000 mt. deep. of compressibility of water is  $45.4 \times 10^{-11} \text{ Pa}^{-1}$ . find fractional compressional of water @ its wateres

$\frac{\Delta V}{V} = -\frac{P}{B}$

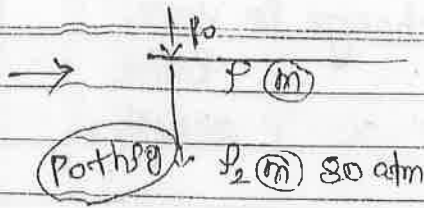


$= -h \rho g \times C$   
 $= 3000 \times 10^3 \times 10 \times 45.4 \times 10^{-11}$   
 $= 136.2 \times 10^{-4}$   
 $= 0.01362$

Q. find density of water @ a depth in an ocean. where Press. due to water column is 80 atm. Bulk mod. of water  $2.2 \times 10^9 \text{ Pa}$ .

1 atm = 10<sup>5</sup> Pascal.

PAGE NO. \_\_\_\_\_  
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$$\rho = \frac{m}{V}$$

$$B = \frac{\Delta P}{\frac{\Delta P}{\rho}}$$

$$\frac{\Delta P}{\rho} = \frac{-\Delta V}{V}$$

$$\Delta P = \frac{\Delta P}{B} \rho$$

#  $\eta$  Modulus of

$$\eta = \frac{F/A}{\phi}$$

$$\eta = \frac{F/A}{x/l}$$

$$\eta = \frac{Fl}{Ax}$$

$$P_2 - P = \frac{\Delta P}{B} \rho$$

$$P_2 = P \left[ \frac{\Delta P}{B} + 1 \right]$$

$$= 10^3 \left[ \frac{80 \times 10^5}{2.2 \times 10^8} + 1 \right]$$

$$= 1.0036 \times 10^3 \text{ kg/m}^3$$

$$= 10^3 [0.0036 + 1]$$

# modulus of Rigidity -

Q. A sq. slab of side 5 cm & thickness 10 ml is rigidly clamp from the narrower surface. NOW a force of 8 kilo N is applied along the upper surface of  $\eta = 8 \times 10^8 \text{ N/m}^2$ . find displacement of the surface with respect to fixed surface.

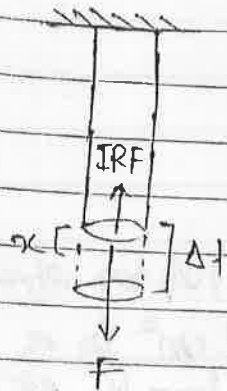
$$\rightarrow x = \frac{Fl}{A\eta} = \frac{8 \times 10^3 \times 5 \times 10^{-2}}{5 \times 1 \times 10^{-4} \times 8 \times 10^8}$$

$$= 10^{-3} \text{ m}$$

$$= 1 \text{ mm}$$



### # Elastic Potential Energy stored in a stretched wire:-



$$F = \frac{YA \cdot x}{l}$$

$$W = \int F dx$$

$$F_{av.} = \frac{0 + F}{2}$$

Work against IRF to stretch a wire,

$$W = F_{av} \cdot x$$

$$W = \frac{F \times \Delta l}{2}$$

$$U = \frac{1}{2} F \Delta l$$

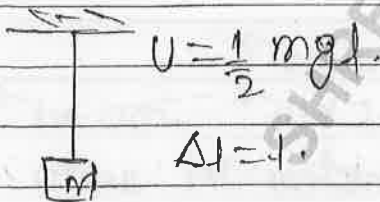
$$U = \frac{1}{2} \times \text{stress} \times \text{strain} \times \text{Vol}^m \text{ of wire} \quad \left| \begin{array}{l} \text{Joule} \\ \leftarrow U = \frac{1}{2} \times F \times \Delta l \times A \times l \end{array} \right.$$

Energy density on PE. per unit vol<sup>m</sup>.

$$u = \frac{U}{V} = \frac{1}{2} \times \text{stress} \times \text{strain} \quad \left| \begin{array}{l} \text{Joule/m}^3 \\ \text{or N-m}^{-2} \end{array} \right.$$

$$y = \frac{\text{stress}}{\text{strain}} \quad \text{or} \quad \text{strain} = \frac{\text{stress}}{y}$$

$$u = \frac{(\text{stress})^2}{2y}$$



$$U = \frac{1}{2} mgl$$

$$\Delta l = l$$

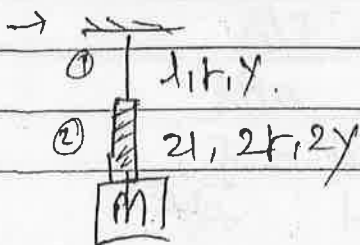
store =  $\frac{1}{2} mgl$  (gain in PE).

Loss in Gravitational PE =  $mgl$ ,

$\frac{1}{2} mgl$  = Loss in Heat

Q. Find Ratio of Elastic Potential energy stored in the wires.

$$U = \frac{1}{2} \times F \times \Delta l$$



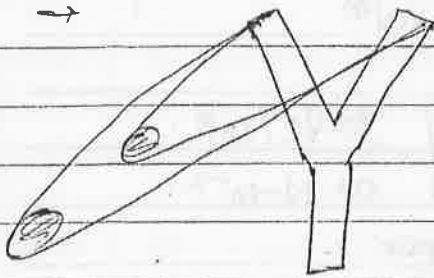
$$= \frac{1}{2} \times F \times \Delta l$$

$$\frac{U_1}{U_2} = \frac{l_1}{l_2} \times \left(\frac{r_2}{r_1}\right)^2 \times \left(\frac{v_2}{v_1}\right)^2$$

$$= \frac{1}{2} \times \left(\frac{2r}{r}\right)^2 \times \frac{2v}{v}$$

$$= 4/1$$

Q. A stone of mass 5 gm is fixed with a rubber string of length 10 cm & cross-section  $1 \text{ cm}^2$  in a catapult. Now the string is pulled upto length 45 cm & released. Find speed of projection of stone. ( $\gamma = 8 \times 10^7$ .)



$$\frac{1}{2} F \Delta l = \frac{1}{2} m v^2$$

$$\gamma \frac{A \Delta l}{l} \times \Delta l = m v^2$$

$$v = \sqrt{\frac{\gamma A}{m l} \Delta l}$$

$$= \sqrt{\frac{8 \times 10^7 \times 1 \times 10^{-4}}{5 \times 10^{-3} \times 10 \times 10^{-2}} \times 5 \times 10^{-2}}$$

$$= \sqrt{16 \times 10^6 \times 5 \times 10^{-2}}$$

$$= 4 \times 10^3 \times 5 \times 10^{-2} = 200 \text{ ms}^{-1}$$

Q. 2 wires of same metal have diameters in ratio 1:3. Both are stretched by same force. Find ratio of elastic P.E. stored per unit vol<sup>n</sup> in them

→

$$U = \frac{1}{2} \times \frac{F}{A} \times \frac{\Delta l}{l}$$

$$= \frac{1}{2} \times \frac{F \times F}{A \times A l}$$

$$\gamma = \frac{F}{A}$$

$$\frac{\Delta l}{l} = \frac{F}{A \gamma}$$

$$\frac{\Delta l}{l} = \frac{F}{A \gamma}$$



EX-1-6, 9, 14, 23, 25, III-

17, 19, 21, 23 leave

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EX-1-1-16   
 28

$$\psi \propto \frac{1}{A^2} \quad [A = \pi r^2]$$

$$\psi \propto \frac{1}{r^4} \quad \Rightarrow \left( \frac{\psi_1}{\psi_2} \right) = \left( \frac{r_2}{r_1} \right)^4$$

$$\Rightarrow \left( \frac{3}{4} \right)^4 = \frac{81}{16}$$

6/9/19

\* Poisson's Ratio ( $\sigma$ ) -



$$\text{Lateral strain} = \frac{\Delta D}{D} = \frac{\Delta r}{r}$$

$$\text{Longitudinal strain} = \frac{\Delta l}{l}$$

$$\sigma = - \frac{\Delta D/D}{\Delta l/l} = - \frac{\Delta r/r}{\Delta l/l} \quad \text{dimensionless.}$$

$$\text{Theoretical value } -1 \leq \sigma \leq 0.5$$

$$\text{Practical value } = 0.2 \text{ to } 0.4.$$

Q. If a wire is stretched & there is no change in its vol<sup>m</sup>. find value of Poisson's ratio.

$$\rightarrow V = \pi r^2 l$$

$$\frac{\Delta V}{V} = \frac{2\Delta r}{r} + \frac{\Delta l}{l}$$

$$- \frac{2\Delta r}{r} = \frac{\Delta l}{l} \quad \Rightarrow \quad \frac{-\Delta r/r}{\Delta l/l} = \frac{1}{2} = 0.5$$

Relation -

$$\gamma, \beta, \eta, \sigma$$

$$* \gamma = 3\beta(1 - 2\sigma)$$

Interatomic force consto (K)

$$* \gamma = 2\eta(1 + \sigma)$$

$$K = \gamma / r_0 \text{ Nm}^{-1}$$

$$* \frac{\sigma}{\gamma} = \frac{3}{2} + \frac{1}{\beta}$$

$r_0$  = interatomic  
separation.

Q.  $\gamma$  of a material is  $\frac{2.4}{3}$  times of  $\eta$ . Find  $\sigma$ .

$$\Rightarrow \frac{2.4}{3}\eta = 2\eta(1 + \sigma)$$

$$\Rightarrow 1.2 = 1 + \sigma$$

$$\sigma = 0.2$$

Q.  $\beta$  and  $\sigma$  of a mat. are  $10^{10}$  Pa. &  $0.2$ . If  $r_0 = 1.2 \text{ \AA}$ , find  $K$ .

$$\gamma = 3 \times 10^{10} (1 - 2 \times 0.2)$$

$$\gamma = 1.8 \times 10^{10} \text{ Nm}^{-2}$$

$$K = 1.8 \times 10^{10} \times 1.2 \times 10^{-10}$$

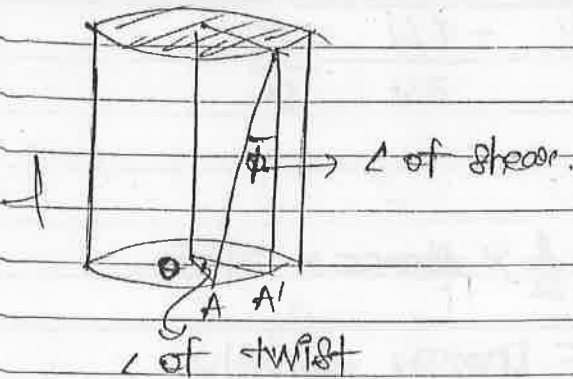
$$= 2.16 \text{ N/m}$$

all last 4 are

are so.

7/19, 21, 23.

# Relatn b/w  $\angle$  of twist &  $\angle$  of shear

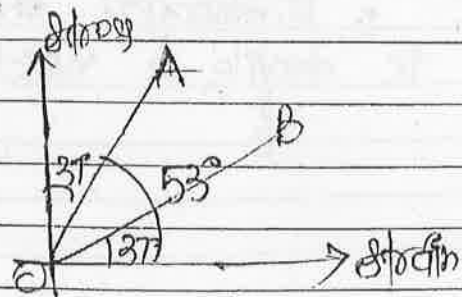
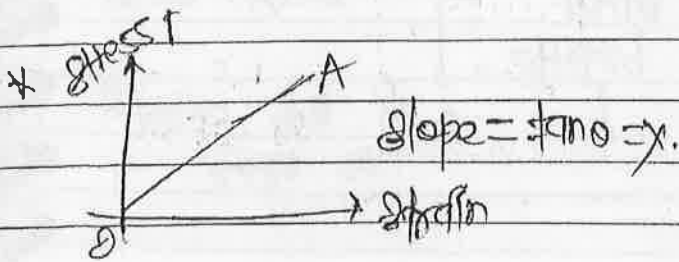
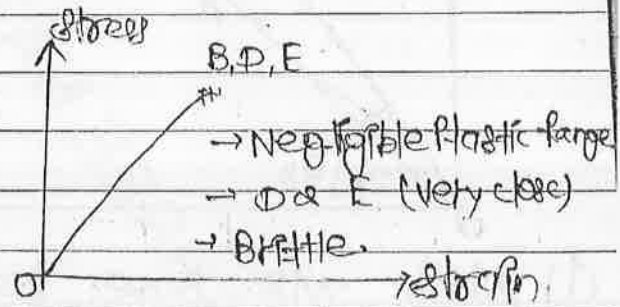
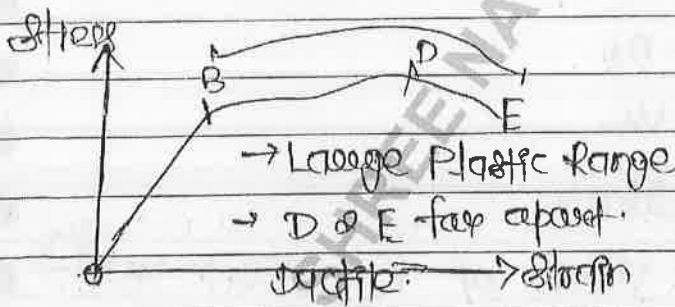
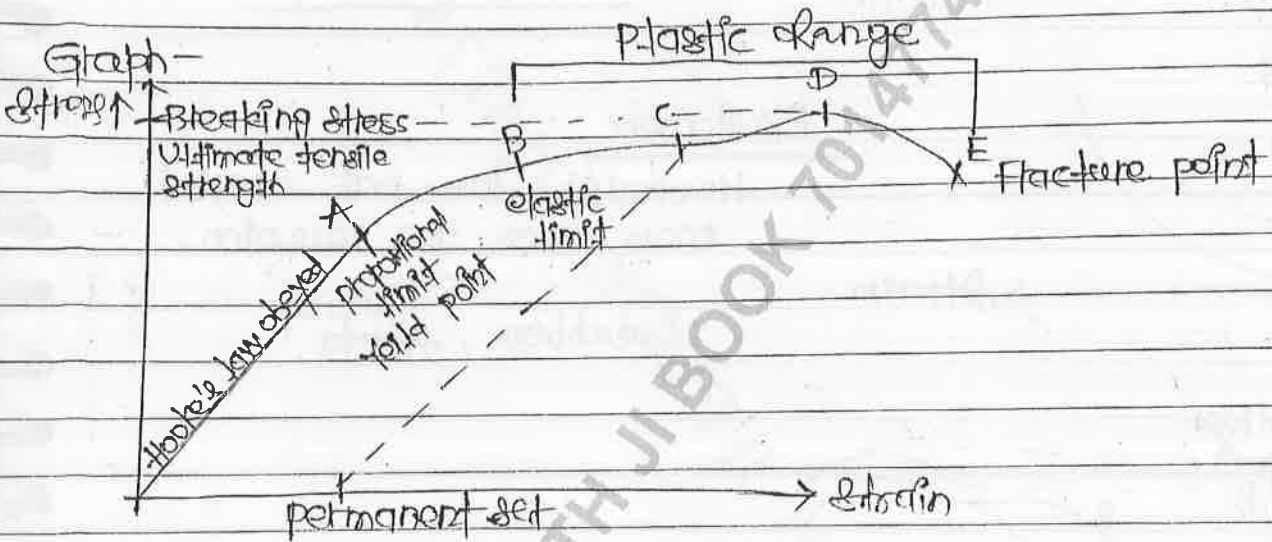
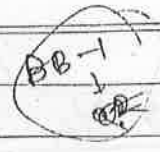


$$\theta = \frac{AA'}{r}$$

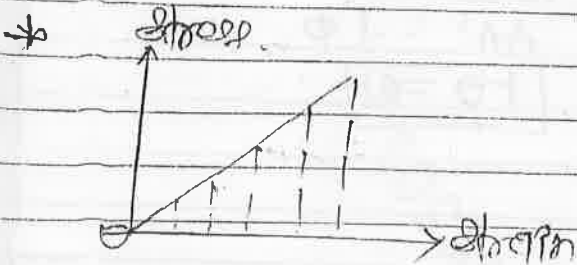
$$AA' = r\theta$$

$$r\theta = r\phi$$

$$\boxed{\theta = \phi}$$

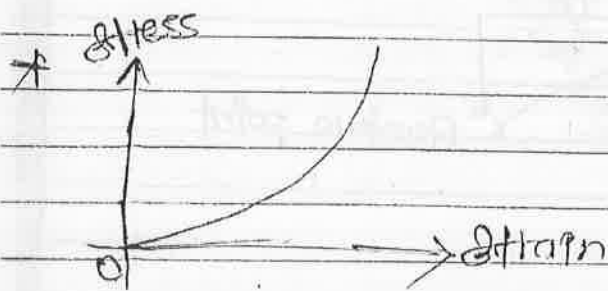


$$\Rightarrow \frac{Y_A}{Y_B} = \frac{\tan \theta_A}{\tan \theta_B} = \frac{\tan 53^\circ}{\tan 37^\circ} = \frac{4/3}{3/4} = \frac{16}{9}$$



$$\text{Area} = \frac{1}{2} \times \text{stress} \times \text{strain}$$

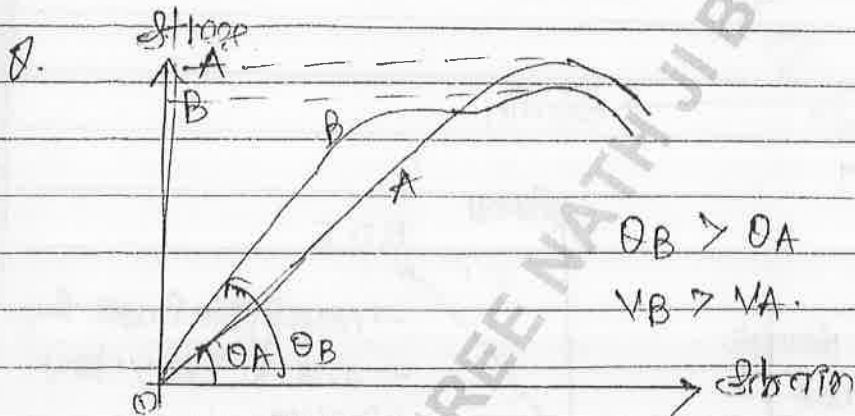
= Energy density.



Elastomers

Hooke's law not obeyed  
in the region.

→ rubber, fibre of nylon.



$$\theta_B > \theta_A$$

$$V_B > V_A$$

(i) Which wire have lesser value of  $\gamma$ .  $\theta_B > \theta_A \Rightarrow \frac{Y_B}{Y_A}$

(ii) " " is stronger than other.  $B.S.(A) > B.S.(B)$

(iii) Which is ductile & which is brittle.

B

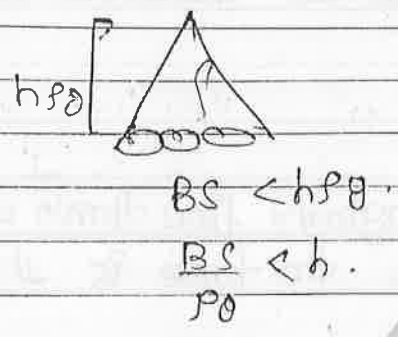
A

A is B.S. wire is  
is strong.

\* Thermal stress -

$\frac{F}{A} = \frac{Y \times \Delta l}{l}$	$l' = l(1 + \alpha \Delta \theta)$	$\alpha = \text{coefficient of linear expansion } ^\circ\text{C}^{-1}$
$\frac{F}{A} = Y \alpha \Delta \theta$ $\text{Nm}^{-2}$	$l' = l + l \alpha \Delta \theta$	
	$\frac{l' - l}{l} = \alpha \Delta \theta$	
Tension $\cdot$ $F = YA \alpha \Delta \theta$ N.	$\frac{\Delta l}{l} = \alpha \Delta \theta$	$\Delta \theta = \text{change in temp}$

can be  
 1. \* The max<sup>m</sup> ht of mountain on earth is 10 km. bcz, if ht exceeds this, then stress in the base rock, exceeds breaking stress.



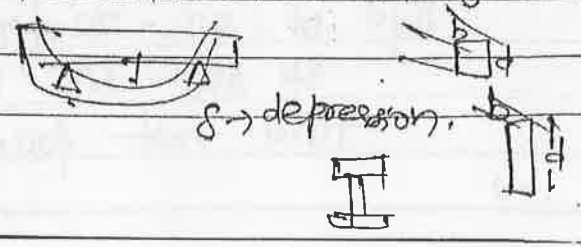
7/9/19

2. Thickness of cable of lift :-

$B.S. = \frac{W}{A}$   
 $A = \frac{W}{B.S.}$   
 Area of cross-section of lift

3. Girder - A rod having cross-section in shape of letter I this rod can tolerate wt. without bending.

$\delta = \frac{wl}{4bd^3y}$



Surface tension is a property of free surface of liquid due to which its free surface tries to minimise its surface area.

Reason → Intermolecular force of attract<sup>n</sup> or cohesive force.

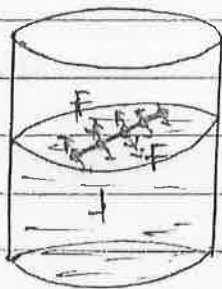
cohesive force → force of attract<sup>n</sup> b/w the molecules of same substance.

\* solid posses definite shape due to strong cohesive force.

\* Adhesive force → force of attract<sup>n</sup> b/w the molecules of different substance.

Eg. Ink sticks with paper.

# Force Acting on either side of an imaginary line drawn on the free surface of liquid, where the force is  $\perp$  to the line & along the surface.

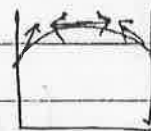


$$T = \frac{F}{l} \text{ N/m or dyne/cm.}$$

Scalar  $[M^1 L^{-1} T^{-2}]$

$$F = T \cdot l$$

• Nature of liquid.



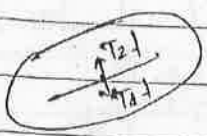
Q. A Matchstick of length 8 cm is placed on surface of a liquid of ST. 72 dyne/cm. by some process at 1 of its side S.T. is reduced upto 60 cm/dyne. Find net force on the stick

→



↓

$F = T_1$        $T_1 > T_2$ .



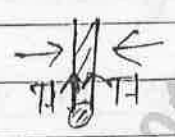
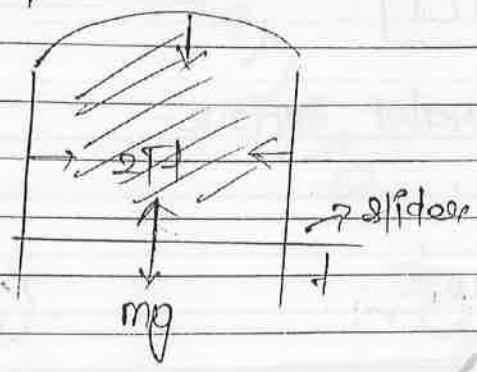
$$F_{net} = T_1 - T_2$$

$$= (71 - 60) \text{ N}$$

$$= (72 - 60) \text{ N}$$

$$= 12 \times 8 = 96 \text{ dyne}$$

Q. A soap solution film of surface tension  $5 \times 10^{-3} \text{ N/m}$  is made on a U-shape wire frame AC to figure. For eqbm of slider. Find its radius. Density of slider =  $5 \times 10^3 \text{ kg/m}^3$



$$m = \frac{2Tl}{g}$$

$$2Tl = mg$$

$$2T = \rho V g$$

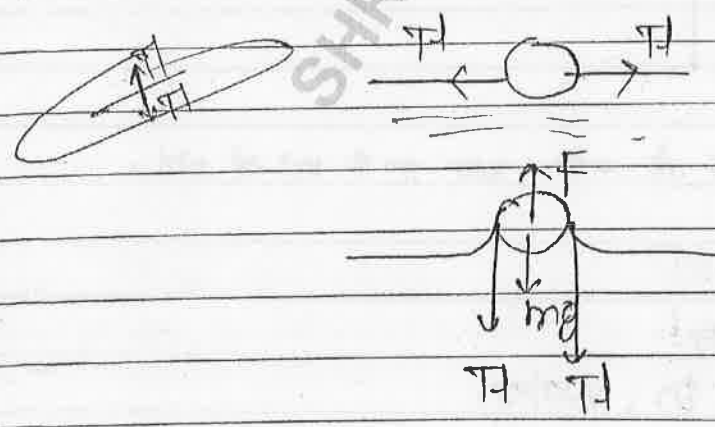
$$2T = \pi r^2 l \rho g$$

$$r = \sqrt{\frac{2T}{\pi \rho g}} = \sqrt{\frac{2 \times 5 \times 10^{-3}}{\pi \times 5 \times 10^3}}$$

$$= \sqrt{2 \times 10^{-4}} = 1.414 \times 10^{-2} \text{ m}$$

$$= 1.414 \text{ cm}$$

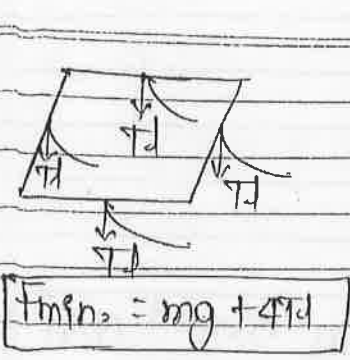
Minm force reqd to lift an object from liquid surface -



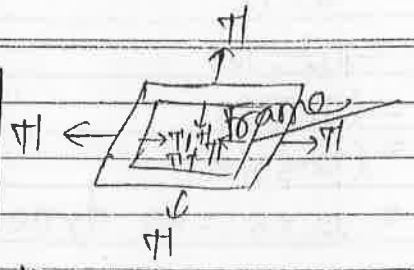
$$F_{min} = mg + 2T$$

On addition to weight,

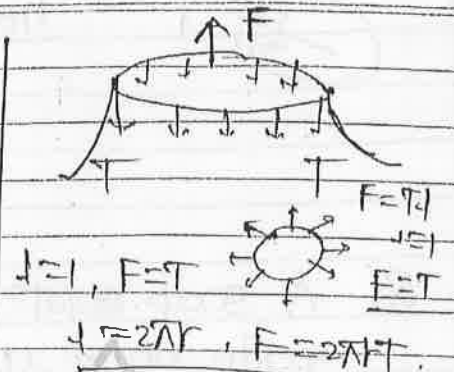
$$F_{ex} = 2T$$



$$F_{min} = mg + 4\pi d$$



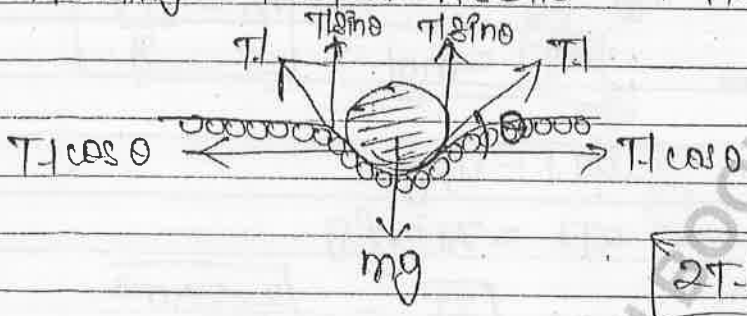
$$F_{min} = mg + 8\pi d$$



$$F_{min} = mg + 2\pi r \gamma$$

#  $F_{min} = mg + 2 \times 2\pi r \gamma$

# Floating of steel Needle on water surface -



$$2\pi l \sin \theta = mg$$

$$W_{max} = 2\pi l$$

if  $\theta = 90^\circ$   
 $\sin 90^\circ = 1$

S.T. depends on -

1. Temperature -

temp.  $\uparrow$ , S.T.  $\downarrow$

2. Boiling point - S.T. = 0

critical temp. - S.T. = 0.

$\rightarrow$  सतह तनाव जी बि जल को water हुआ गुण में 100° की जल .

2. Impurities -

(a) soluble - S.T. increases.

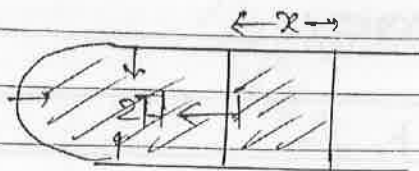
eg. NaCl in water.

(b) less soluble - decreases S.T.

eg. soap or detergent in water.

### # Surface Energy :-

To increase free surface area of a liq, work has to be done against force of surface tension. This work is stored as P.E. among the molecules in free surface of liq. So molecules on free surface possess additional energy than the molecules well inside the liq. This additional energy is called surface energy.



$$W = Fx$$

$$W = 2Tl \times x$$

$$W = T \times 2lx$$

$$\Delta A = A_2 - A_1$$

$$\Delta A = 2(lx)$$

$$= 2x(\Delta A)_{upper}$$

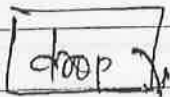
$$W = \Delta E = T \Delta A$$

$$T = \frac{\Delta E}{\Delta A} = \frac{W}{\Delta A} \quad \text{J/m}^2$$

- surface energy stored per unit area is called surface tension
- work done by to ↑ surface area by unit.

### Film over bubble

$$W = T \times 2(\Delta A)_{upper}$$



$$W = T(\Delta A)_{upper}$$

$$\Delta J = 10^7 \text{ erg}$$

$$\frac{1 \text{ J}}{10^7}$$

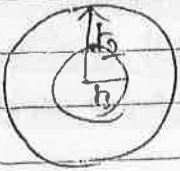
Q 10 Find work done to ↑ area of a soap sol<sup>n</sup> film from 40cm x 50 cm to 60 cm x 80 cm.  $T = 75 \text{ dyne/cm}$

$$\begin{aligned} W &= T \times 2(\Delta A)_{upper} \\ &= 75 \times 2(4800 - 2450) \\ &= 150 \times 2400 \\ &= 360000 \text{ erg} \\ &= 0.036 \text{ J} \end{aligned}$$

sp. cases :-

I. Drop :-

1. Work to increase radius of a drop from  $r_1$  to  $r_2$ .



$$W = T (\Delta A)_{\text{upper}}$$

$$= T [4\pi r_2^2 - 4\pi r_1^2]$$

$$W = 4\pi T [r_2^2 - r_1^2]$$

Increase in surface energy.

2. Work to form a drop of radius  $r$ .



$$W = T (\Delta A)_{\text{upper}}$$

$$= T [4\pi r^2 - 0]$$

$$W = 4\pi r^2 T$$

Total surface energy.

II. Bubble.

3. Work to increase radius of a bubble from  $r_1$  to  $r_2$ .

$$W = T \times 2 (\Delta A)_{\text{upper}}$$

$$= T \times 2 [4\pi r_2^2 - 4\pi r_1^2]$$

$$W = 8\pi T [r_2^2 - r_1^2]$$

4. Work to form a bubble of radius  $r$ .

$$W = T \times 2 \times (4\pi r^2 - 0)$$

$$W = 8\pi r^2 T$$

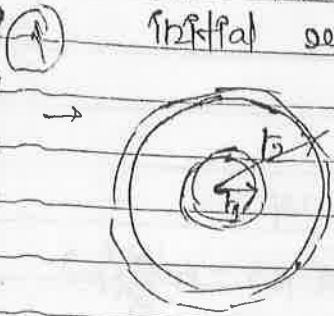
Q. Find work done to make radius of a bubble 3 times of that of that rad. =  $r$ . &  $8\pi T = T$

$$\rightarrow W = T \times 2 \times [4\pi r^2 - 4\pi r^2]$$

$$W = 8\pi T [(3r)^2 - r^2] = 8\pi T [9r^2 - r^2]$$

$$W = 8\pi T^2 (9 - 1) \Rightarrow = 64\pi r^2 T$$

Q. Find work done to ↑ vol<sup>m</sup> of soap bubble by 700%  
Initial radii,  $r_1$  to  $r_2$ .



$$W = T \times 2 \times [4\pi r_2^2 - 4\pi r_1^2]$$

$$W = 8\pi T [r_2^2 - r_1^2]$$

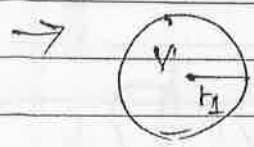
$$W = 8\pi T [(2r_1)^2 - r_1^2]$$

$$W = 24\pi T^2 r_1^2$$

$$V = \frac{4\pi r^3}{3}$$

$$\frac{V_1}{V_2} = \left(\frac{r_1}{r_2}\right)^3$$

Q. Work done to form a water drop of vol<sup>m</sup>  $V$  is  $W$ . Find work done to form a drop of vol<sup>m</sup>  $2V$ .



$$W = T [4\pi r_1^2 - 0]$$

$$W = 4\pi T r_1^2$$

$$\frac{100}{800} = \left[\frac{r_1}{r_2}\right]^3$$

$$\frac{1}{2} = \frac{r_1}{r_2}$$

$$r_2 = 2r_1$$



$$W_2 = T [4\pi r_2^2 - 0]$$

$$W_2 = 4\pi T r_2^2 = \left(\frac{r_2}{r_1}\right)^2 W$$

$$W = 4\pi T r_1^2$$

$$\frac{V_1}{V_2} = \left(\frac{r_1}{r_2}\right)^3$$

$$W_2 = (2^{1/3})^2 W$$

$$W_2 = 2^{2/3} W$$

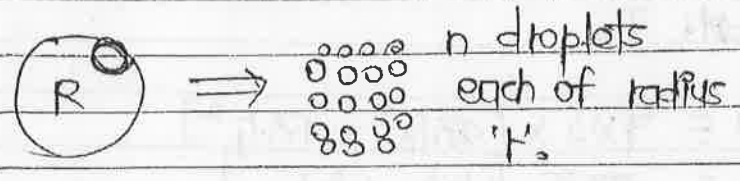
$$= (2^2)^{1/3} W$$

$$W_2 = 4^{1/3} W$$

$$\frac{V_1}{2V} = \left(\frac{r_1}{r_2}\right)^3$$

$$r_2 = \left(\frac{1}{2}\right)^{1/3} r_1$$

5.  $\Rightarrow$  Deep  $\Rightarrow$  drop of  $\Rightarrow$  change of  $\Rightarrow$   $\Rightarrow$



$$W = \Delta E = T \times (\Delta A)_{\text{upper}} = T [n \times 4\pi r^2 - 4\pi R^2]$$

$$W = \Delta E = 4\pi T [nr^2 - R^2] = 4\pi T [n \times R^2 - R^2] \quad n^{2/3}$$

$$W = \Delta E = 4\pi R^2 T [n^{1/3} - 1] \quad \text{change in SE}$$

energy absorbed from surrounding.

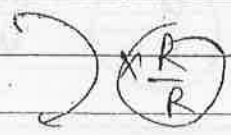
$$V_i = V_f \quad \frac{4}{3}\pi R^3 = n \frac{4}{3}\pi r^3$$

$$R = n^{1/3} r$$

$$r = \frac{R}{n^{1/3}}$$

Diff

$$m \Delta \theta J = 4\pi R^2 T \left[ \frac{R}{r} - 1 \right]$$

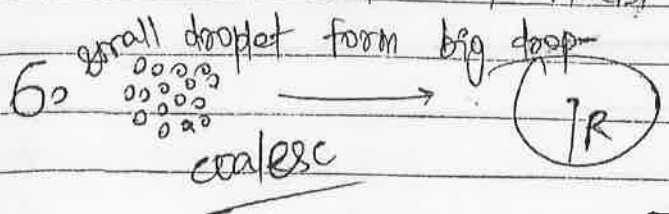


$$\Delta \theta = \frac{4\pi R^3 T}{\frac{4}{3}\pi R^3 \rho J} \left[ \frac{1}{r} - \frac{1}{R} \right]$$

$$\Delta \theta = \frac{3T}{\rho J} \left[ \frac{1}{r} - \frac{1}{R} \right]$$

$\star$  Surface area  $\uparrow$ , surface energy  $\uparrow$ , Internal energy  $\downarrow$ , temperature of droplet  $\downarrow$ .

$\Rightarrow$  energy absorbed from surrounding, that's why we feel cool near fountain.



$$W = -4\pi R^2 T [n^{1/3} - 1]$$

-ve surface area decrease.

BB → 6 completed.  
 I - 71, 88, 90, 92  
 93, 101, 103,  
 105, 107,  
 110.  
 II - 100,

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decrease in surface energy

$$W = 4\pi R^2 T [n^{1/3} - 1]$$

energy released to surroundings,

→ [T ↑]

surface area ↓, "

surface energy ↓, "

internal energy ↑, "

temp. of droplet ↑, so energy is released to surroundings.

Q. Some amount of droplets each of radius 'r' coalesce to form a drop of radius R. then WOF is correct -

(A)  $4\pi VT \left[ \frac{1}{r} - \frac{1}{R} \right]$  energy released.

(B)  $3\pi VT \left[ \frac{1}{r} + \frac{1}{R} \right]$  energy released.

(C)  $3\pi VT \left[ \frac{1}{r} - \frac{1}{R} \right]$  energy released.

(D) Energy neither released nor absorbed.

$W = 4\pi R^2 T [n^{1/3} - 1]$  energy released to surroundings.

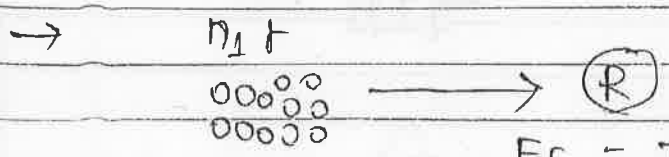
$= 4\pi R^2 T \left[ \frac{R}{r} - 1 \right]$

$= 4\pi R^3 T \left[ \frac{1}{r} - \frac{1}{R} \right]$

$3V = 4\pi R^3 = 3\pi VT \left[ \frac{1}{r} - \frac{1}{R} \right]$

Q. 125 identical droplets coalesce, find ratio of total final surface energy to total initial surface energy.

$$\frac{R}{r} = n^{1/3}$$



$$E_f = T [4\pi R^2 - 0]$$

$$E_i = T [4\pi r^2 - 0]$$

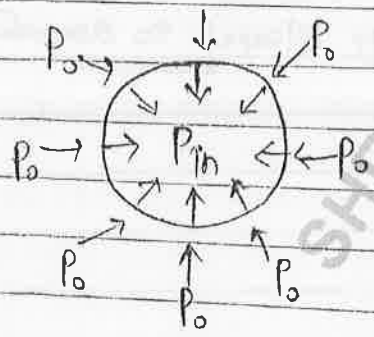
$$= \frac{1}{n} \times \left(\frac{R}{r}\right)^2$$

$$= \frac{1}{n} \times n^{2/3}$$

$$= \frac{1}{n^{1/3}} = \frac{1}{(125)^{1/3}} = \frac{1}{5}$$

### # EXCESS PRESSURE :-

The free surface of liquid drop tries to minimize its free surface area and compresses the matter enclosed, in eqbm the pressure inside the drop is more than the pressure outside & the difference of pressure inside & outside is called excess pressure.



$$P_{in} = P_0 + P_{ex}$$

$$P_{ex} = P_{in} - P_0$$

drop

$$P_{ex} = \frac{2T}{r}$$

Bubble

$$P_{ex} = \frac{4T}{r}$$



Q. Pressure inside two soap bubbles are 1.08 atm & 1.06 atm, find ratio of their surface areas.

$$\rightarrow \frac{P_1}{P_2} = \frac{r_2}{r_1} \Rightarrow \frac{1.08}{1.06} = \frac{r_2}{r_1}$$

$$\frac{r_1}{r_2} = \frac{3}{4}$$

$$\frac{A_1}{A_2} = \frac{2 \times 4\pi r_1^2}{2 \times 4\pi r_2^2}$$

$$\frac{A_1}{A_2} = \left(\frac{r_1}{r_2}\right)^2 = \left(\frac{3}{4}\right)^2 = \frac{9}{16}$$

$$P_{in} = P_0 + \frac{4T}{r}$$

$$P_{ex} = \frac{4T}{r}$$

$$1.08 - 1 = \frac{4T}{r_1}$$

$$0.08 = \frac{4T}{r_1}$$

Q. Excess pressure inside a soap bubble is 3 times that of another. Find ratio of their volume.

$$\rightarrow P_1 = 3P_2 \Rightarrow \frac{r_1}{r_2} = 3$$

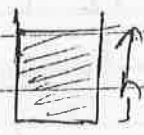
$$\frac{V_1}{V_2} = \left(\frac{r_1}{r_2}\right)^3 = \left(\frac{1}{3}\right)^3 = \frac{1}{27}$$

Q. Excess pressure inside a water drop is 27 pascal. If 27 such drops coalesce, find excess pressure in the new drop.

$$\rightarrow P \propto \frac{1}{r} \rightarrow (R)$$

$$\frac{P_1}{P_2} = \frac{r_2}{r_1} \Rightarrow \frac{27}{P_2} = \frac{R}{r} = 3r \Rightarrow P_2 = 9 \text{ Pascal}$$

Q. The difference of pressure inside & outside of a bubble of radius 2 millimeter is equal to 8 mm of water column. Find surface tension of soap sol<sup>n</sup>.



→  $R_{ext} = 2 \text{ mm}$

Diagram of a bubble with internal pressure  $P_{in}$  and external pressure  $P_{out}$ .

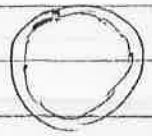
$$P_{in} - P_{out} = \frac{4T}{r} = h\rho g$$

$P_{out} + h\rho g$   
Water column =  $h\rho g$

$$T = \frac{h\rho g r}{4} = \frac{8 \times 10^{-3} \times 2 \times 10^{-3} \times 10^3 \times 10}{4} = 4 \times 10^{-2} \text{ Nm}^{-1}$$

Q. Pressure inside a bubble of radius 1 mm which is made by detergent sol<sup>n</sup> of S.T.  $2.5 \times 10^{-2} \text{ N/m}$  is equal to the pressure @ a pt.  $z_0$  depth below free water surface. Find value of  $z_0$ .

→



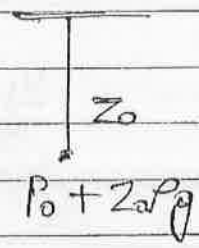
$$P_{in} = P_0 + \frac{4T}{r} = P_0 + z_0 \rho g$$

$$z_0 = \frac{4T}{r\rho g}$$

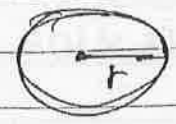
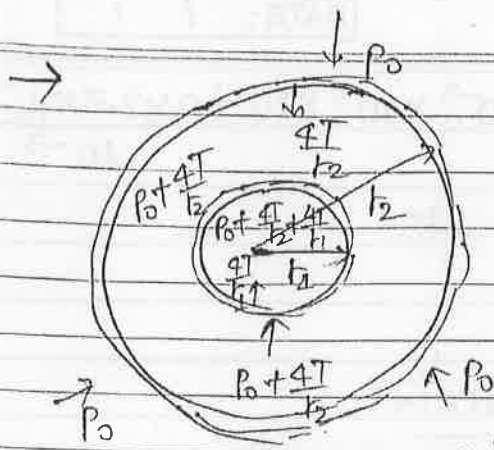
$$z_0 = \frac{4 \times 2.5 \times 10^{-2}}{10^{-3} \times 10^3 \times 10}$$

$$z_0 = 10.0 \times 10^{-3} \text{ m}$$

$$= 10 \text{ mm} = 1 \text{ cm}$$



Q. A soap bubble of radius 4 mm is held inside another soap bubble of rad. 3 mm. Find the rad. of a bubble which have excess pressure equal to the difference of pressure inside the smaller & outside the larger.

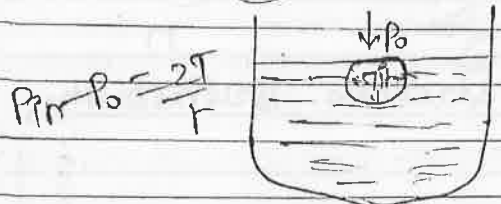


$$\frac{4T}{r} = \frac{P_0 + 4T}{r_2} + \frac{4T}{r_1} - P_0$$

$$\frac{1}{r} = \frac{r_1 + r_2}{r_1 r_2}$$

$$r = \frac{r_1 r_2}{r_1 + r_2} = \frac{1 \times 3}{1 + 3} = \frac{3}{4} = 0.75 \text{ mm}$$

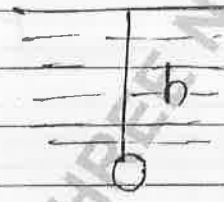
# (a) Bubble just below water surface



$$P_{in} - P_0 = \frac{2T}{r}$$

$$P_{in} = P_0 + \frac{2T}{r} \Rightarrow \boxed{P_{in} - P_0 = \frac{2T}{r}}$$

(b) 'h' depth below.



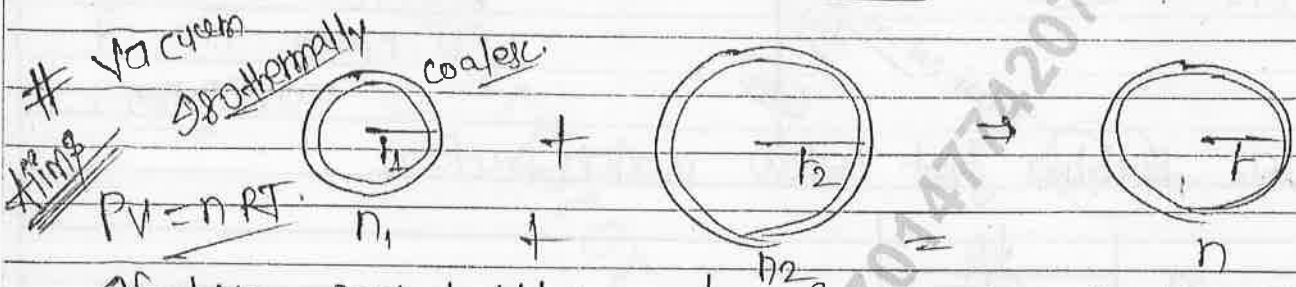
$$\boxed{P_{in} = P_0 + h\rho g + \frac{2T}{r}}$$

Q. Find pressure inside a bubble of radius 1 mm held @ a depth of 8 cm below water surface. surface tension of water,  $7.3 \times 10^{-2} \text{ N/m}$ , ( $P_0 = 1.013 \times 10^5 \text{ Pa}$ )

$$P_{in} = P_0 + h\rho g + \frac{2T}{r}$$

$$= 1.013 \times 10^5 + 8 \times 10^{-2} \times 1000 \times 9.8 + \frac{2 \times 7.3 \times 10^{-2}}{1 \times 10^{-3}}$$

$$\begin{aligned}
 &= 4.013 \times 10^5 + 8 \times 10^{-2} \times 10^3 \times 10 + 2 \times 7.3 \times 10^{-2} \times 10^{-3} \\
 &= 4.013 \times 10^5 + 800 + 146 \\
 &= 101300 + 946 \\
 &= 102246 \\
 &= 1.02246 \times 10^5 \text{ Pa}
 \end{aligned}$$



Of two soap bubbles coalesc in vacuum Isothermally, then radius of new bubble -

$$\begin{aligned}
 \frac{P_1 V_1}{RT} + \frac{P_2 V_2}{RT} &= \frac{PV}{RT} \\
 \Rightarrow \frac{4T}{r_1} \times \frac{4}{3} \pi r_1^3 + \frac{4T}{r_2} \times \frac{4}{3} \pi r_2^3 &= \frac{4T}{r} \times \frac{4}{3} \pi r^3 \\
 r_1^2 + r_2^2 &= r^2 \\
 \boxed{r} &= \sqrt{r_1^2 + r_2^2}
 \end{aligned}$$

17

Q. Two soap bubbles coalesc in air Isothermally. If in the process change in vol<sup>m</sup>  $\Delta V$  & change in surface area  $\Delta A$ . then prove

$$\boxed{3P\Delta V + 4T\Delta A = 0} \quad \Delta V, \Delta A.$$

BB → 7 → 3, 4, 5

Ex-I → 72, 73, 74, 91, 94, 99, 100, 102, 109

II - 2,  
III - 21, 22

PAGE NO.   
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→  $n_1 + n_2 = n$

$$\left( P_0 + \frac{4T}{r_1} \right) \frac{4}{3} \pi r_1^3 + \left( P_0 + \frac{4T}{r_2} \right) \frac{4}{3} \pi r_2^3 = \left( P_0 + \frac{4T}{r} \right) \frac{4}{3} \pi r^3$$

$$\Rightarrow P_0 \times \frac{4}{3} \pi r_1^3 + \frac{4T}{3} \times 4\pi r_1^2 + P_0 \times \frac{4}{3} \pi r_2^3 + \frac{4T}{3} \times 4\pi r_2^2 = P_0 \times \frac{4}{3} \pi r^3 + \frac{4T}{3} \times 4\pi r^2$$

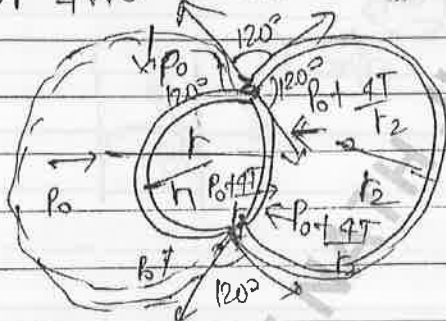
$$\Rightarrow P_0 \left( \frac{4}{3} \pi r_1^3 + \frac{4}{3} \pi r_2^3 - \frac{4}{3} \pi r^3 \right) + \frac{4T}{3} (4\pi r_1^2 + 4\pi r_2^2 - 4\pi r^2) = 0$$

$$\Rightarrow P_0 \Delta V + \frac{4T}{3} \Delta A = 0$$

$$\Rightarrow 3 P_0 \Delta V + 4T \Delta A = 0$$

Hence proved

# If two bubbles are kept in contact then radius of curvature of common interface

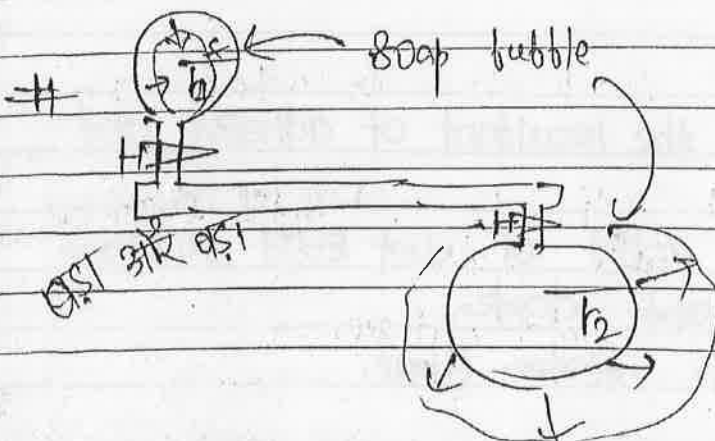


Three coplanar vectors  $\Rightarrow 120^\circ$

$$\frac{4T}{r} = P_0 + \frac{4T}{r_1} - \left[ P_0 + \frac{4T}{r_2} \right]$$

$$\Rightarrow \frac{4T}{r} = P_0 + \frac{4T}{r_1} - P_0 - \frac{4T}{r_2}$$

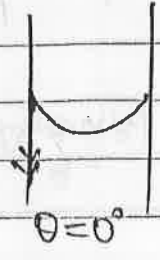
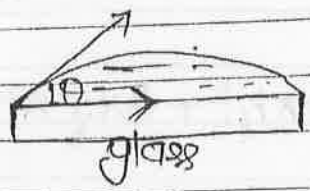
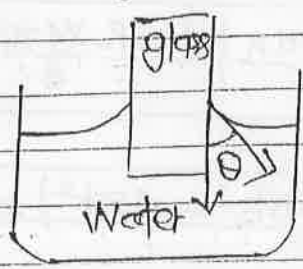
$$r = \frac{r_1 r_2}{r_2 - r_1}$$



$\uparrow P \propto \frac{1}{r}$   
 $r_1 < r_2$   
 $P_1 > P_2$   
 Largest expands while smallest contracts & ultimately collapse.

# # Angle of contact :-

## 1. water - glass

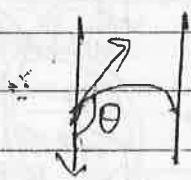
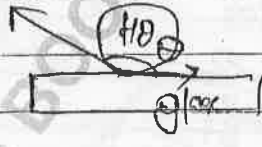
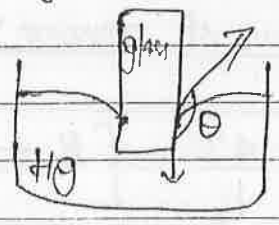


Water glass  $\sim 8^\circ$   
 pure water glass  $\sim 0^\circ$   
 $\theta < 90^\circ$  acute.

mercury  $\rightarrow$  Hemisphere

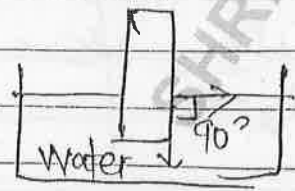


## 2. Hg - glass.



Hg - glass  $\theta \sim 135^\circ$   
 $\theta > 90^\circ$  obtuse.

## 3. Silver - Water.



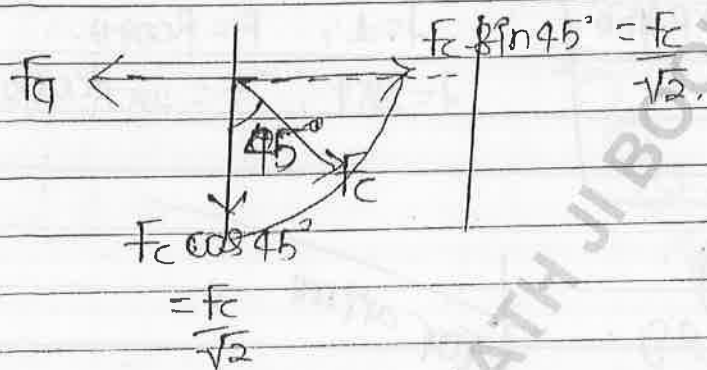
- \*  $\angle$  of contact depends on the resultant of adhesive and cohesive force.
- \* If a liq spreads over solid or wet solid surface then  $\angle$  of contact is ~~obt~~ acute.  
 eg. water, glass.

\* If liq. does not spread over solid; then  $\angle$  of contact  $\rightarrow$  obtused. eg. Hg & glass, Wax-water, Lotus leaf -water etc.

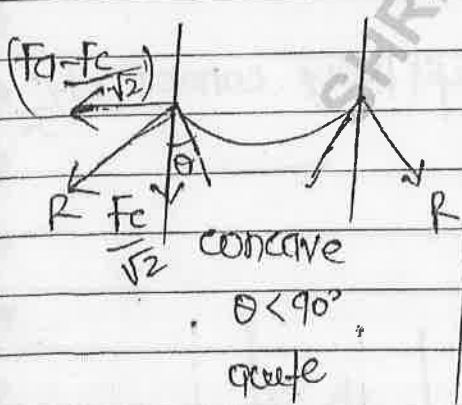
\* Water proofing agent converts  $\angle$  of contact from acute to obtuse.

\* Angle of contact is independent of inclin<sup>n</sup> of solid inside the liq.

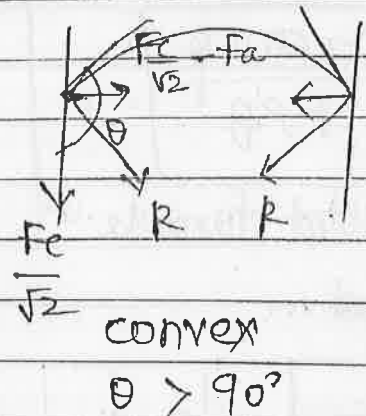
# Shape of meniscus.



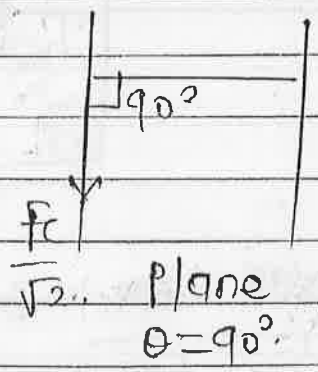
(a)  $F_a > \frac{F_c}{\sqrt{2}}$



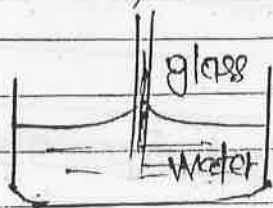
(b)  $F_a < \frac{F_c}{\sqrt{2}}$



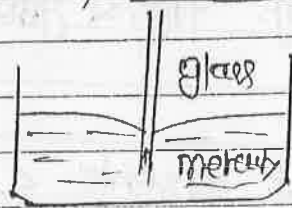
(c)  $F_a = \frac{F_c}{\sqrt{2}}$



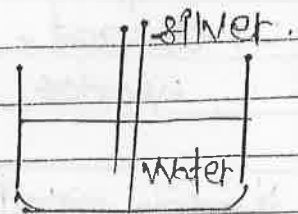
Capillarity or capillary Action :-



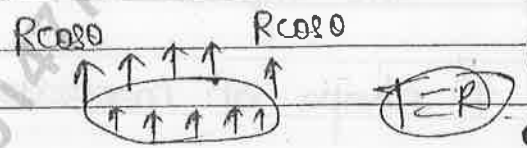
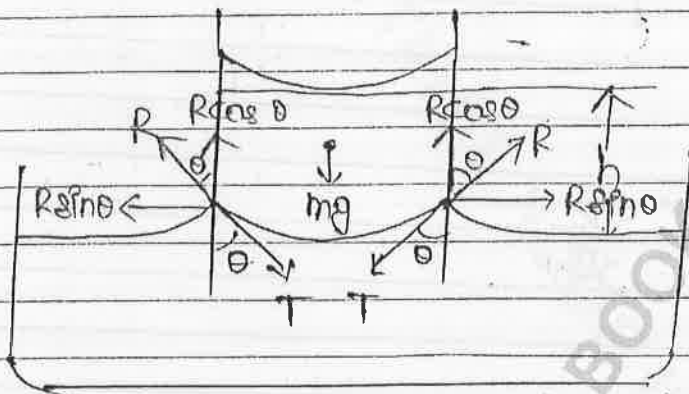
$\theta < 90^\circ$   
Liquid wet solid  
Rise



$\theta > 90^\circ$   
Liquid does not wet solid  
fall



$\theta = 90^\circ$   
neither rise nor fall.



$\downarrow = \uparrow, F = 2\pi R \cos \theta$   
 $\downarrow = 2\pi R T, F = 2\pi R T \cos \theta$

$2\pi R T \cos \theta = mg$   
 $2\pi R T \cos \theta = V \rho g$

$2\pi R T \cos \theta = \pi R^2 h \rho g$

$$h = \frac{2T \cos \theta}{\rho g}$$

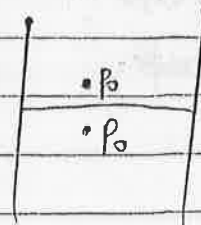
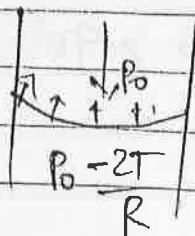
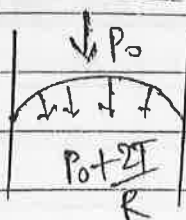
Ascent formula

For app mg

$\left(\frac{h + L}{3}\right) = \frac{2T \cos \theta}{\rho g}$

capillary correct<sup>n</sup> =  $\frac{1}{3}$

# pressure - Volume Balance.





BB-7 → 1st 5.

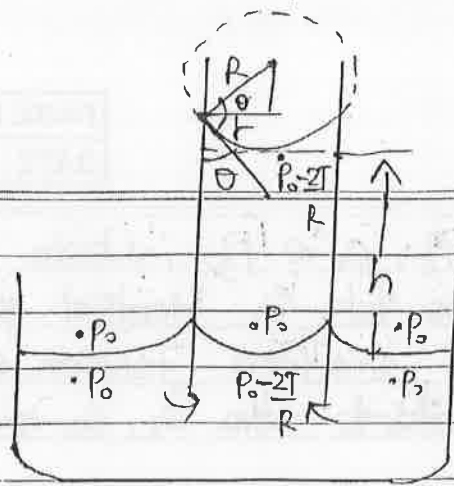
I → 74, 83, 84, 93,

II → 7, 32, 104, 1

III → 9, 10, 11, 12

PAGE NO.

DATE: / /



$$h \rho g = \frac{2T}{R}$$

$$\cos \theta = \frac{r}{R} \quad , \quad h \rho g = \frac{2T}{r/\cos \theta}$$

$$R = \frac{r}{\cos \theta}$$

$$* \quad h = \frac{2T \cos \theta}{r \rho g} \quad \rightarrow \text{ht given}$$

$$h \rho g = \frac{2T \cos \theta}{r}$$

$$* \quad 2T \cos \theta = mg \quad \rightarrow \text{wt of rise of mass given}$$

$$h = \frac{2T \cos \theta}{r \rho g}$$

12.09.19

Q. Wt. of water rise in a capillary is  $6.28 \times 10^{-4} \text{ N}$ , S.T. is  $2 \times 10^{-2} \text{ N/m}$ . If  $\theta = 0^\circ$ . Find —

- (i) radius of the capillary.
- (ii) inner circumference.

$$\rightarrow \text{(i) } r = \frac{mg}{2T \cos \theta} = \frac{6.28 \times 10^{-4}}{2 \times 3.14 \times 2 \times 10^{-2} \times 1} \Rightarrow 0.5 \times 10^{-2} \text{ m} = 0.5 \text{ cm} = 5 \text{ mm}$$

$$\text{(ii) } 2T = \frac{mg}{\cos \theta} = \frac{6.28 \times 10^{-4}}{2 \times 10^{-2} \times 1} = 3.14 \times 10^{-2} = 3.14 \text{ cm}$$

Q. A capillary tube of diameter  $1.46 \times 10^{-3} \text{ m}$ , is held vertically in water upto some length. If S.T. is  $0.073 \text{ N/m}$ . Find the ht upto which water rises.

$$\rightarrow \quad h = \frac{2T \cos \theta}{r \rho g} = \frac{2 \times 0.073 \times 1}{\frac{1.46 \times 10^{-3}}{2} \times 10^3 \times 10} = \frac{2}{100} \text{ m} = 2 \text{ cm}$$

Q. Three liquids of densities  $\rho_1, \rho_2$  &  $\rho_3$ , where  $\rho_1 > \rho_2 > \rho_3$ . All rises upto same ht in identical capillaries. All the three have same surface tension. If their corresponding  $\angle$  of contacts are  $\theta_1, \theta_2$  and  $\theta_3$ . then WOF is correct?

(i)  $\frac{\pi}{2} < \theta_1 < \theta_2 < \theta_3 < \pi$ .

(ii)  $\pi > \theta_1 > \theta_2 > \theta_3 > \frac{\pi}{2}$ .

(iii)  $\frac{\pi}{2} > \theta_1 > \theta_2 > \theta_3 \geq 0^\circ$ .

(iv)  $0^\circ \leq \theta_1 < \theta_2 < \theta_3 < \frac{\pi}{2}$ .

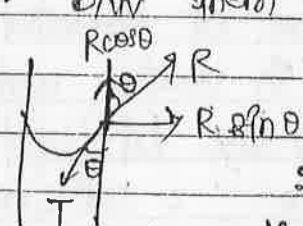
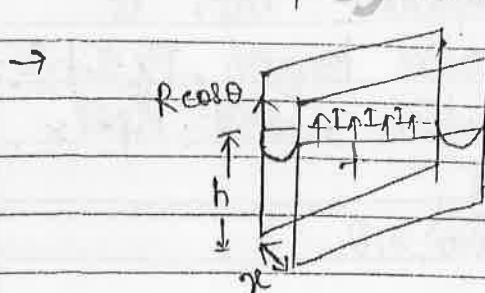
} obtuse  
} acute

(h)  $= \frac{2T \cos \theta}{\rho g} \Rightarrow h \propto [\cos \theta]$

Hises  $\rightarrow \theta$  acute

$0^\circ \leq \theta_1 < \theta_2 < \theta_3 < \frac{\pi}{2}$

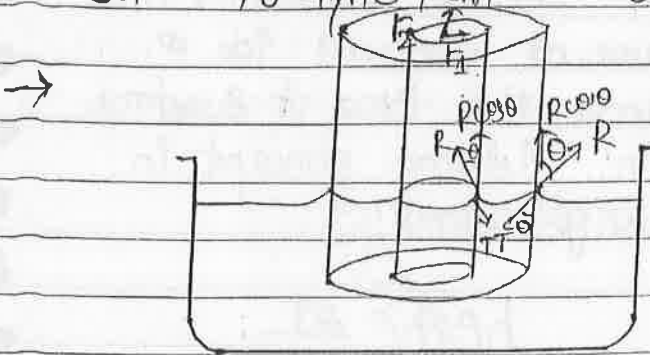
Q. Two identical rectangular glass plates are kept || to each other with very small separation  $x$ . Their lower edges touch water. upto what ht. b/w them, water will rise?



$l = 1, R \cos \theta$   
 $2x \cdot x R \cos \theta = mg$   
 $2x \cdot l T \cos \theta = l x x h \rho g$

$h = \frac{2T \cos \theta}{x \rho g}$

Q. A capillary tube of radius 2 mm is kept vertically in water upto some length. Now a cylindrical glass rod of radius 4 mm is inserted inside it coaxially. Find ht of water rise,  $\sigma = 70 \text{ dyne/cm}$  &  $\theta = 0^\circ$ .



$$2\pi r_2 \sigma \cos \theta + 2\pi r_1 \sigma \cos \theta = mg$$

$$= V\rho g$$

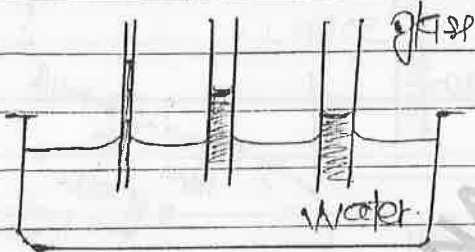
$$= (\pi r_2^2 - \pi r_1^2) h \rho g$$

$$2\pi \sigma \cos \theta (r_1 + r_2) = \pi (r_2^2 - r_1^2) h \rho g$$

$$h = \frac{2\sigma \cos \theta}{(r_2 - r_1) \rho g} = \frac{2 \times 70 \times 1}{(0.2 - 0.1) \times 1 \times 1000}$$

$$= 1.4 \text{ cm}$$

# case 01



$$h = \frac{2\sigma \cos \theta}{r \rho g}$$

$$h \propto \frac{1}{r}$$

∴  $\frac{h_1}{h_2} = \frac{r_2}{r_1}$

$$\Rightarrow \frac{h_1}{h_2} = \frac{r_2}{r_1}$$

Eg. If radius is doubled then ratio of rise of liquid in both case.

$$\rightarrow \frac{V_1}{V_2} = \frac{m_1 g h_1/2}{m_2 g h_2/2}$$

$$= \frac{m_1}{m_2} \times \frac{h_1}{h_2}$$

$$= \frac{m_1}{m_2} \times \frac{2}{1} = 4:1$$

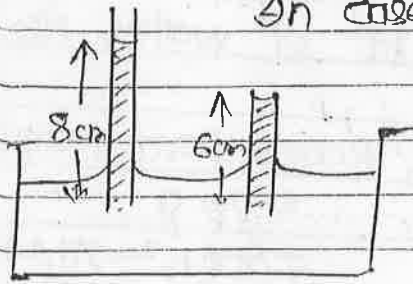
$$\frac{m_1}{m_2} \times \frac{2}{1} = 4:1$$

$$2\pi r \sigma \cos \theta = m g$$

$$\boxed{r \propto m}$$

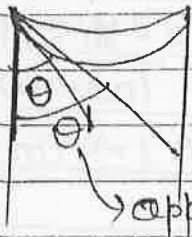
$$\frac{h_1}{h_2} = \frac{2r}{r} = \frac{2}{1}$$

2.



$$2\gamma \cos \theta$$

In case of insufficient length, liquid fills up to full length, but never spills out because the radius of curvature of meniscus  $r_c$  balances the force of surface tension. (Same concept in weightlessness)



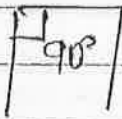
apparent angle of contact

$$2\gamma \cos \theta$$

$$h\rho g = \frac{2\gamma}{R}$$

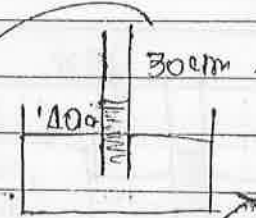
$$h \propto \frac{1}{R}$$

# weightlessness



$$2\gamma \cos 90^\circ$$

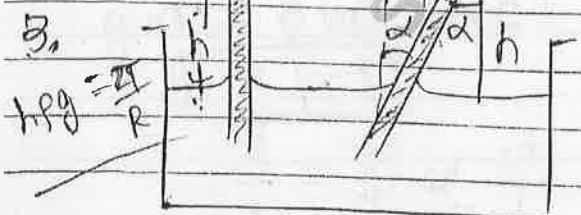
$$\cos 90^\circ = 0$$



Natural satellite  
moon etc,  
or escape  
mass greater  
than host

- I) free fall
- II) Artificial satellite
- III) center of earth
- IV) Well pt. on or in outer space

upto 30 cm full length ho. 100



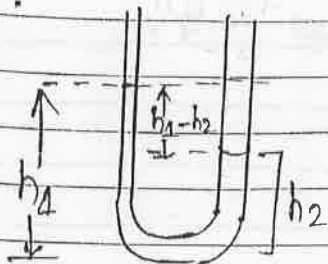
length of the liquid in the tube,

$$\cos \alpha = \frac{h}{L}$$

$$L = \frac{h}{\cos \alpha}$$

Ht of meniscus will remain same

4.



Level of difference,

$$h_1 - h_2 = \frac{2T \cos \theta}{r_1 \rho g} - \frac{2T \cos \theta}{r_2 \rho g}$$

$$h_1 - h_2 = \frac{2T \cos \theta}{\rho g} \left[ \frac{1}{r_1} - \frac{1}{r_2} \right]$$

13.9.19

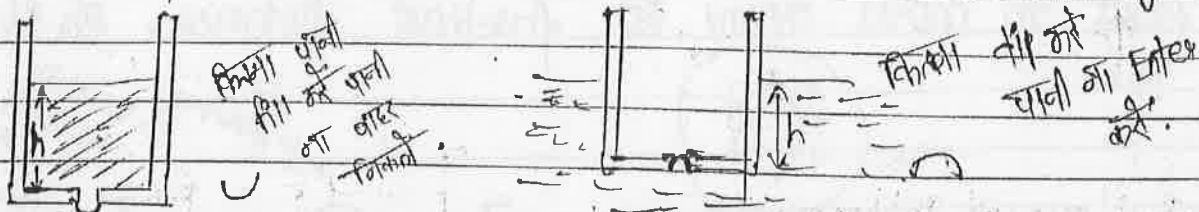
Q. Two capillaries of diameter 1 mm and 2 mm are connected and some water is poured in 1 atm. Find level diff. of water in both arms.  $T = 70 \text{ dyne/cm}$ ,  $\alpha \theta = 0$

$$\begin{aligned} \rightarrow h_1 - h_2 &= \frac{2T \cos \theta}{\rho g} \left[ \frac{1}{r_1} - \frac{1}{r_2} \right] \\ &= \frac{2 \times 70 \times 1 \times 2}{1 \times 1000} \left[ \frac{1}{0.1} - \frac{1}{0.2} \right] \quad \rightarrow \text{Answer} \\ &= \frac{2 \times 7 \times 2}{100} \times (0.2 - 0.1) = 4 \times 0.1 = 1.4 \text{ cm} \end{aligned}$$

$$* h = \frac{2T \cos \theta}{r \rho g} \quad \uparrow \downarrow h \propto \frac{1}{r} \quad \uparrow \quad h_{\text{moon}} = \frac{g_{\text{earth}}}{6}$$

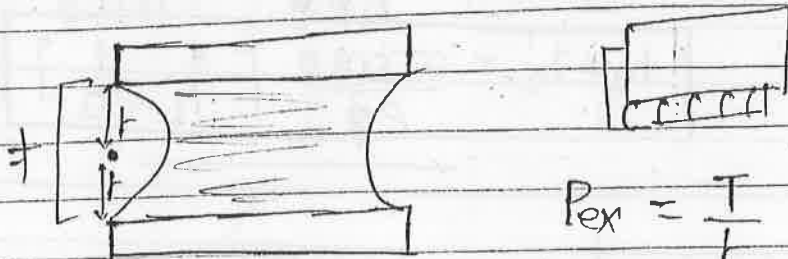
$$\rightarrow h_{\text{moon}} = 6 h_{\text{earth}}$$

\* Maxm ht upto which water can be filled without leakage.



$$\text{Defect} \rightarrow P_0 + \frac{2T}{R} = P_0 + h \rho g \quad \rightarrow \quad h = \frac{2T \cos \theta}{r \rho g}$$

\* Min force reqd to separate 2 glass plates which hold a water film b/w them of thickness  $\rightarrow t$ .



$t = 2t$   
 $t = t/2$

glass plate

$$P_{ex} = \frac{T}{r}$$

$$\frac{F}{A} = \frac{T}{t/2} \Rightarrow F = \frac{2TA}{t}$$

eg: Q. A water film of volm  $0.05 \text{ cm}^3$  is held b/w 2 glass plates, of thickness  $0.1 \text{ cm}$ . Find min force reqd to separate them.  $T = 70 \text{ dyne/cm}$

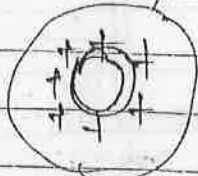
$\rightarrow V = A t$

$$A = \frac{V}{t} = \frac{0.05}{0.1} = 0.5 \text{ cm}^2$$

$$F = \frac{2TA}{t} = \frac{2 \times 70 \times 0.5}{0.1}$$

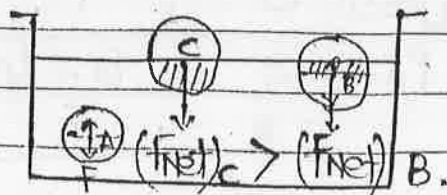
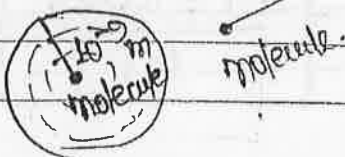
$$F = 700 \text{ dyne}$$

\* If a soap bubble is charged, then due to mutual repulsion, it tries to move away @ farthest distance. So its Volm  $\uparrow$ es.



Net force of molecule  $\rightarrow$  away from center

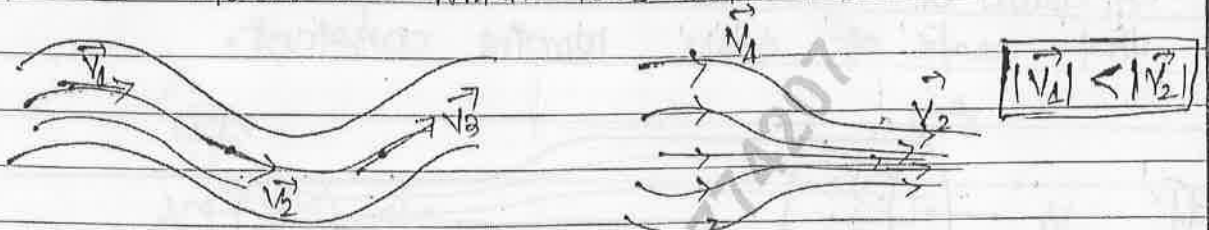
Sphere of influence  $\rightarrow 10^{-9} \text{ m}$



Nature of flow -

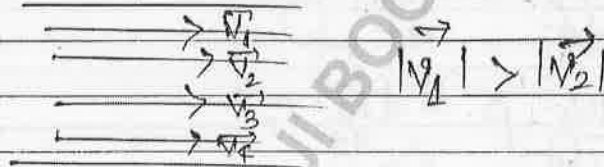
I. Stream line flow -

During the flow of a fluid at a time, each molecule passes with same velocity both in magnitude & dir<sup>n</sup>, their paths are stream line which does not intersect.



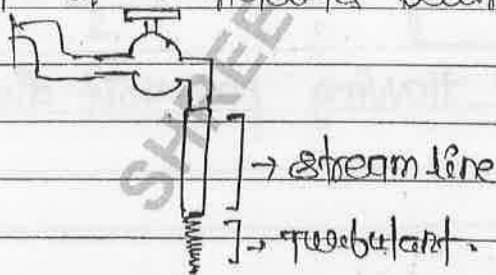
II. Laminar flow -

At different points in the flow, magnitude of velocities may be different but their dir<sup>n</sup> are same.



III. Turbulent flow -

If speed of flow exceeds critical speed, then mot<sup>n</sup> of molecules becomes irregular.



\* Ideal fluid -

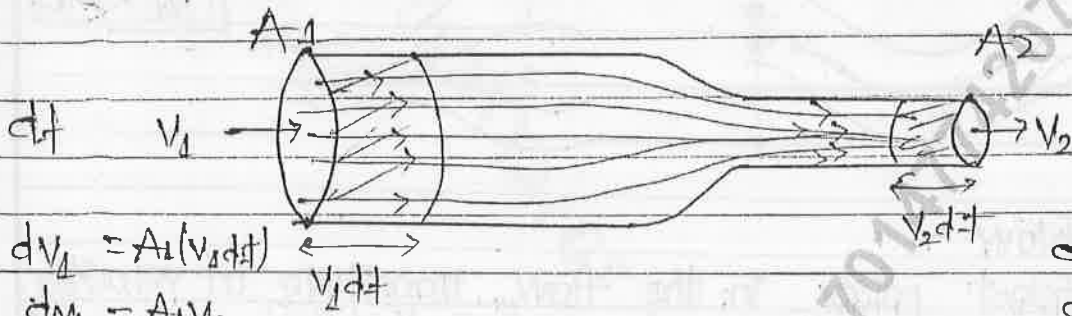
→ Incompressible.  $\Delta V = 0$ .  $\rho = \text{const.}$   
 $B = \infty$ .

→ Non-viscous fluid def<sup>n</sup> viscosity = 0.

→ streamline or steady.

Equation of continuity :-

During an ideal fluid flow at each and every point the vol<sup>m</sup> flowing per unit time or rate of flow remains constant. If area of cross-section ↓ then speed of flow ↑ in a way that rate of flow remains constant.



$$dV_1 = A_1(v_1 dt)$$

$$\frac{dm_1}{dt} = A_1 v_1$$

$$\frac{dm_1}{dt} = \frac{dm_2}{dt}$$

$$\frac{dV_1}{dt} \rho = \frac{dV_2}{dt} \rho$$

$$A_1 v_1 = A_2 v_2$$

$$dV_2 = A_2(v_2 dt)$$

$$\frac{dm_2}{dt} = A_2 v_2$$

$$\left[ \frac{dV}{dt} = AV = \text{constant} \right] \text{ m}^3/\text{s} \text{ or Litre/s}$$

Rate of flow or vol<sup>m</sup> flowing per unit time,

$$v \propto \frac{1}{A}$$

$$\frac{v_1}{v_2} = \frac{A_2}{A_1} = \left( \frac{r_2}{r_1} \right)^2$$

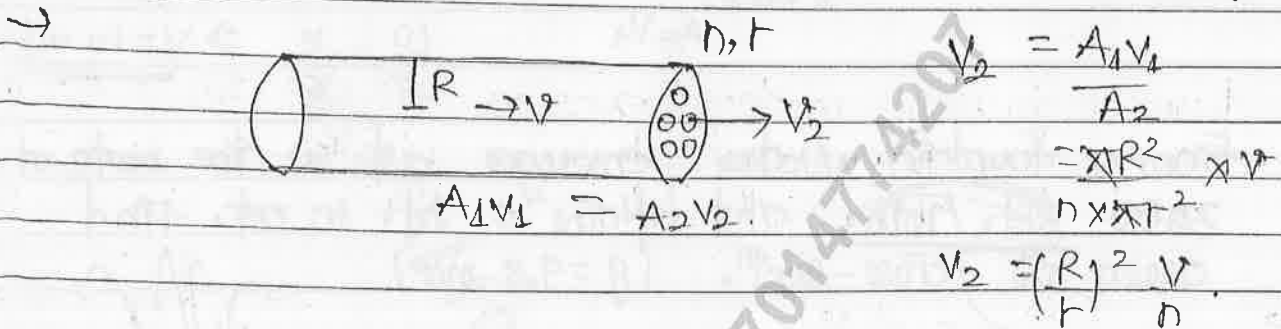
$$r_1^2 v_1 = r_2^2 v_2$$

$$1 \text{ Litre} = 10^{-3} \text{ m}^3$$



Q. One end of a horizontal tube of radius 8 cm is closed with a plate having 40 narrow holes each of radius 1 mm. If speed of water flow in the tube is 10 cm/s. Find the speed of water emerges out from each hole.

(No. pores  $\rightarrow$  Eq<sup>n</sup> of continuity)  
 Only speed, area.



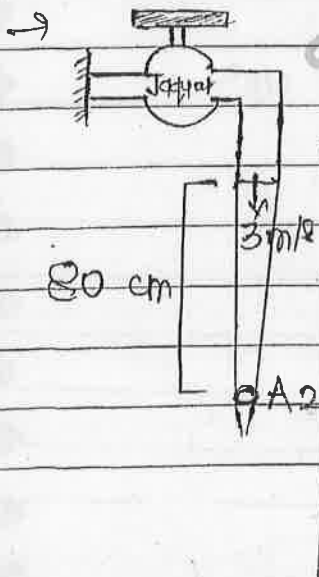
$$v_2 = \left( \frac{8}{0.01} \right)^2 \times \frac{10}{40} \text{ cm/s.}$$

$$= \frac{1600}{40} \times 10 \text{ cm/s.}$$

$$= 1600 \text{ cm/s.}$$

$$= 16 \text{ m/s.}$$

Q. Water stream emerges out from a tap of area of cross-section 2 cm<sup>2</sup> with speed 3 m/s. Find area of cross-section of stream 80 cm below the tap.



$$A_1 v_1 = A_2 v_2$$

$$A_1 = 2 \text{ cm}^2. \quad A_2 = \frac{v_1}{v_2} A_1$$

$$= \frac{3}{5} \times 2 \text{ cm}^2.$$

$$\uparrow v \propto \frac{1}{A} \downarrow$$

So necessary to maintain rate of flow

$$v_2^2 = v_1^2 + 2gh$$

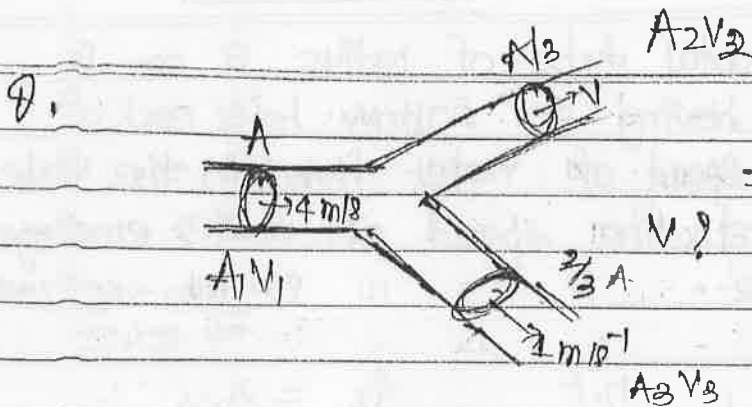
$$= 9 + 2 \times 10 \times 0.8$$

$$= 9 + 16$$

$$v_2^2 = 25$$

$$v_2 = 5 \text{ m/s.}$$

$$= \frac{6}{5} \text{ cm}^2 = 1.2 \text{ cm}^2.$$



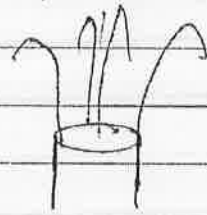
$$A_1 V_1 = A_2 V_2 + A_3 V_3$$

$$\Rightarrow A \times 4 = \frac{A}{3} \times V + \frac{2}{3} \times 2$$

$$\Rightarrow 4 - \frac{2}{3} = \frac{V}{3}$$

$$\frac{10}{3} = \frac{V}{3} \Rightarrow V = 10 \text{ m/s}$$

Q. From a fountain water emerges at the rate of  $3000 \text{ lit. / min.}$  and attains a ht. of  $10 \text{ m}$ . Find area of cross-section. ( $g = 9.8 \text{ m/s}^2$ )



$\rightarrow \frac{dV}{dt} = AV$

$$\frac{3000 \times 10^{-3} \text{ m}^3}{60} = A \times \sqrt{2gH}$$

$$5 \times 10^{-2} = A \sqrt{2 \times 9.8 \times 10}$$

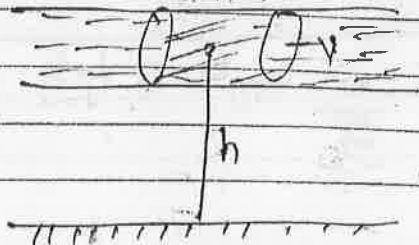
$$= A \sqrt{196}$$

$$A = \frac{5 \times 10^{-2}}{14} \text{ m}^2$$

$$\Rightarrow A = \frac{5 \times 10^{-2} \times 10^4}{14} \text{ cm}^2$$

$$= \frac{5000}{14} = 357.14 \text{ cm}^2$$

\* Energy possessed by flowing fluid -



Page-2  
12/10/16

BB-4 -4

Ex I-46-48

PAGE NO.   
 DATE: / /

53, 54, 57, 59.

II-3, 8, 13, 17

1. per unit vol<sup>m</sup> K.E.

$$\frac{K.E.}{dV} = \frac{1}{2} \frac{dm}{dV} v^2$$

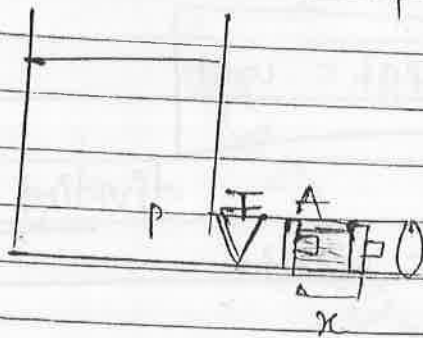
$$= \frac{1}{2} \rho v^2$$

2. Per unit vol<sup>m</sup> P.E.

$$\frac{P.E.}{dV} = \frac{dm}{dV} gh$$

$$= \rho gh$$

3. per unit volume pressure energy -



$$W = Fx$$

$$= PAx$$

$$W = PdV$$

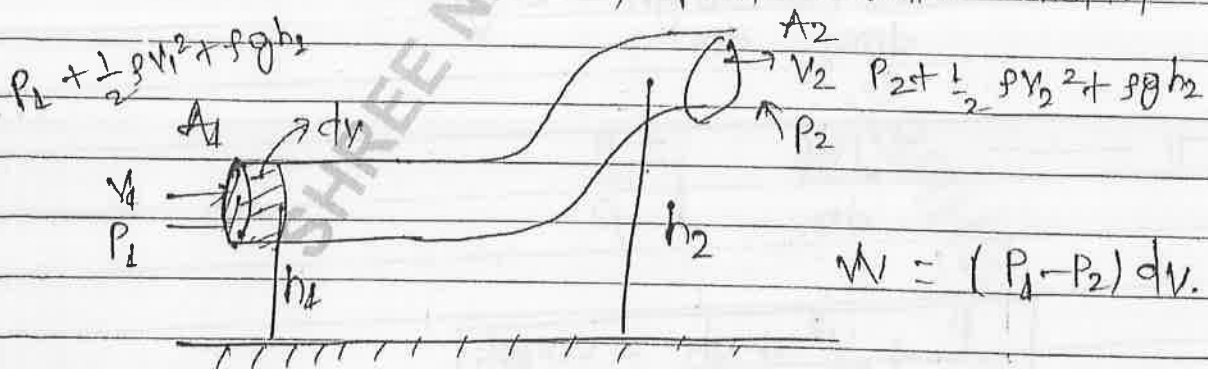
$$W = P \cdot dV$$

$$P = \frac{F}{A}$$

$$F = PA \quad (\text{ma})$$

### # Bernoulli's Theorem :-

During an ideal fluid flow, the sum of per unit vol<sup>m</sup> pressure energy, per unit vol<sup>m</sup> K.E., & per unit vol<sup>m</sup> P.E. @ each & every point remains constant.



$$P_1 + \frac{1}{2} \rho v_1^2 + \rho g h_1$$

$$W = (P_1 - P_2) dV$$

$$\Delta K = \frac{1}{2} dm (v_2^2 - v_1^2)$$

$$W = \Delta K + \Delta U$$

$$\Delta U = dm g (h_2 - h_1)$$

$$(P_1 - P_2) \, dV = \frac{1}{2} \, dm \, (V_2^2 - V_1^2) + dm \, g \, (h_2 - h_1)$$

$$P_1 - P_2 = \frac{1}{2} \rho (V_2^2 - V_1^2) + \rho g (h_2 - h_1)$$

$$P_1 + \frac{1}{2} \rho V_1^2 + \rho g h_1 = P_2 + \frac{1}{2} \rho V_2^2 + \rho g h_2$$

$$\boxed{P + \frac{1}{2} \rho V^2 + \rho g h = \text{const}}$$

16.9.19

dividing by  $\rho g$

$$\frac{P}{\rho g} + \frac{V^2}{2g} + h = \text{const}$$

← Pressure head
↓ Velocity head
→ gravitational head

per unit mass,  $\frac{KE}{dm} = \frac{1}{2} \frac{dm V^2}{dm} = \frac{V^2}{2}$

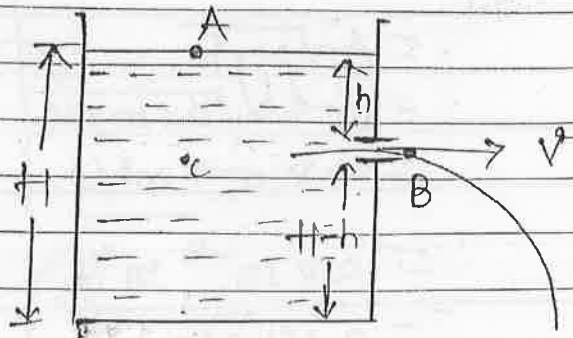
||  $\frac{PE}{dm} = \frac{dm g h}{dm} = gh$

||  $\frac{P \, dV}{dm} = \frac{P}{\rho}$

$$\boxed{\frac{P}{\rho} + \frac{V^2}{2} + gh = \text{const}}$$

Applications →

I. Speed of Efflux :-



$$P + \frac{1}{2} \rho v^2 + \rho gh = \text{const}$$

$v_1 \ll v_2$

$$P_1 + \left( \frac{1}{2} \rho v_1^2 \right) + \rho gh_1 = P_2 + \frac{1}{2} \rho v_2^2 + \rho gh_2$$

$$P_0 + 0 + \rho gH = P_0 + \frac{1}{2} \rho v^2 + \rho g(H-h)$$

$$\rho gH = \frac{\rho}{2} v^2 + \rho gH - \rho gh$$

$$\frac{v^2}{2} = gh$$

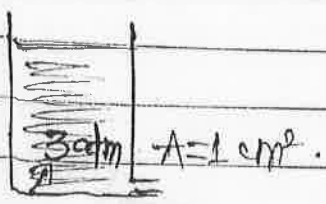
$$v = \sqrt{2gh}$$

# Torricelli's theorem →

Speed of efflux = speed acquired by the particle falling freely in air from level of water to the orifice.

Q. Pressure at the bottom of a water filled open container is 3 atm. There is an orifice of area  $1 \text{ cm}^2$  at its base. Find the vol<sup>n</sup> of water emerges out in 1 sec. initially.

$\frac{dv}{dt}$



$$\frac{dV}{dt} = AV$$

$$= A\sqrt{2gh}$$

$$= 10^{-4} \sqrt{2 \times 10 \times 20}$$

$$= 20 \times 10^{-4} \text{ m}^3/\text{s}$$

$$= 2 \times 10^{-3} \text{ m}^3/\text{s}$$

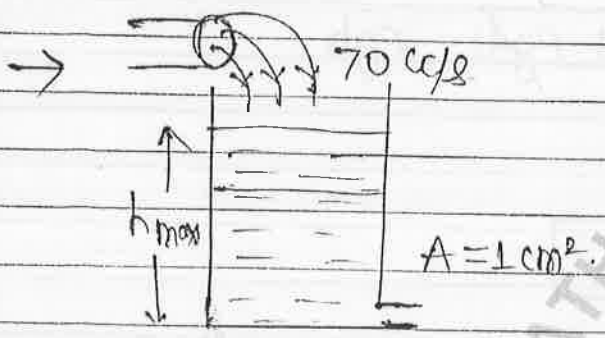
$$= 2 \text{ Litres}/\text{s}$$

$$P = P_0 + h\rho g$$

$$\exists P_0 = P_0 + h\rho g$$

$$h = \frac{2 \times 10^5}{10^3 \times 10} = 20$$

Q. Water is poured in an open container at the rate 70 cc/s. There is an orifice at its base. Find the max height upto which water can be filled in the container. (Area of orifice = 1 cm<sup>2</sup>).



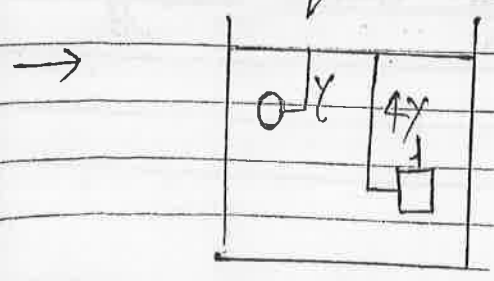
$$\frac{dV}{dt} = AV$$

$$\left(\frac{dV}{dt}\right)_{in} = \left(A\sqrt{2gh_{max}}\right)_{out}$$

$$70 = 1 \sqrt{2 \times 980 \times h_{max}}$$

$$h_{max} = \frac{4900}{2 \times 980} = \frac{10}{4} = 2.5 \text{ cm}$$

Q. If vol<sup>m</sup> emerging out per sec from both holes are equal find radius of circular hole in terms of side 'l' of square hole.



$$\left(\frac{dV}{dt}\right)_1 = \left(\frac{dV}{dt}\right)_2$$

$$A_1 V_1 = A_2 V_2$$

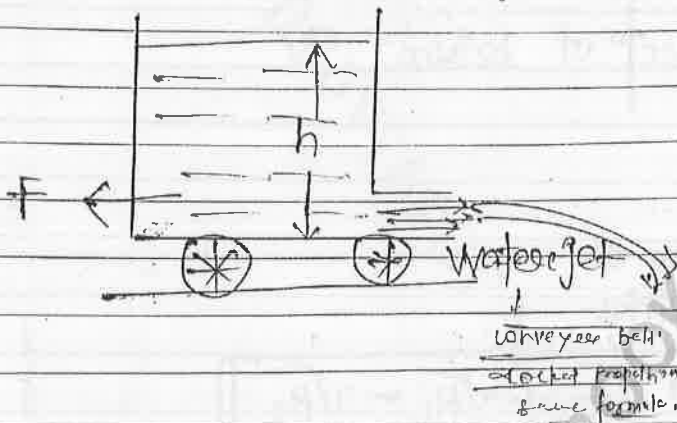
$$A_1 v_1 = A_2 v_2$$

$$\pi R^2 (\sqrt{2gh}) = 1^2 \sqrt{2g \times 4g}$$

$$\pi R^2 = 1^2 \times 2$$

$$R = \sqrt{\frac{2}{\pi}}$$

Q. When water emerges out, find the Hz force on container.



$$F = v \frac{dm}{dt}$$

$$= v \left( \frac{dV}{dt} \right) \rho$$

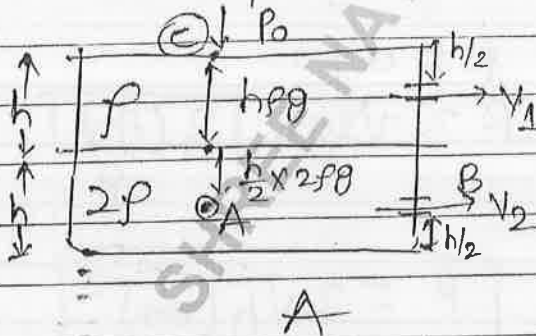
$$= v (AV) \rho$$

$$F = AV^2 \rho$$

$$= A (2gh) \rho$$

$$F = 2gAh\rho$$

Q. Find the speed of fluid emerging out of fluids.



$$v_1 = \sqrt{2gh/2} = \sqrt{gh} \quad \text{--- (1)}$$

$$P_1 + \frac{1}{2} \times 2\rho v_2^2 = P_2 + \frac{1}{2} \times 2\rho v_1^2$$

$$P_0 + h\rho g + \frac{h}{2} \times 2\rho g = P_0 + \rho v_2^2$$

$$2gh = v_2^2$$

$$v_2 = \sqrt{2gh} \quad \text{--- (2)}$$

$$\frac{v_1}{v_2} = \frac{1}{\sqrt{2}}$$

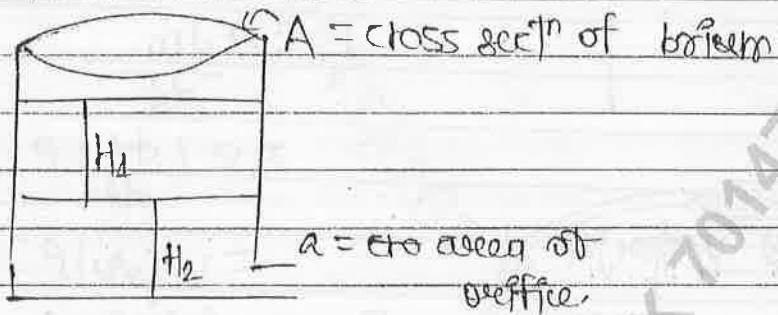
(C)

(B)

$$\rho_0 + 0 + \rho_0 h + 2 \rho_0 h = \rho_0 + \frac{1}{2} \times \rho_0 \times v^2 + \frac{h}{2} \times \rho_0 g$$

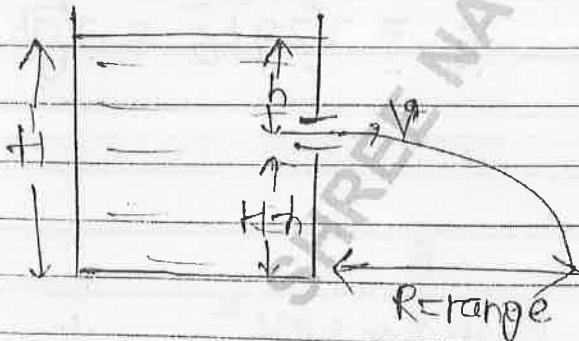
$$3 h \rho_0 g - h \rho_0 g = \rho_0 v^2$$

# Time taken by the level to fall from  $H_1$  to  $H_2$  above orifice.



$$T = \frac{A \sqrt{2}}{a \sqrt{g}} [\sqrt{H_1} - \sqrt{H_2}]$$

\* Range of the jet  $\rightarrow$

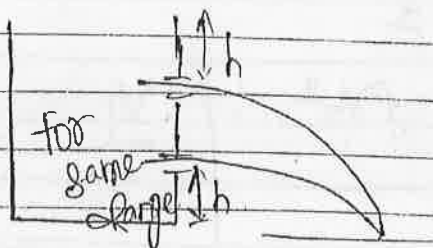


$$R = uT$$

$$R = \sqrt{2gh} \sqrt{\frac{2(H-h)}{g}}$$

$$R = 2\sqrt{h(H-h)}$$

For max<sup>m</sup> Range,  $h = \frac{H}{2}$



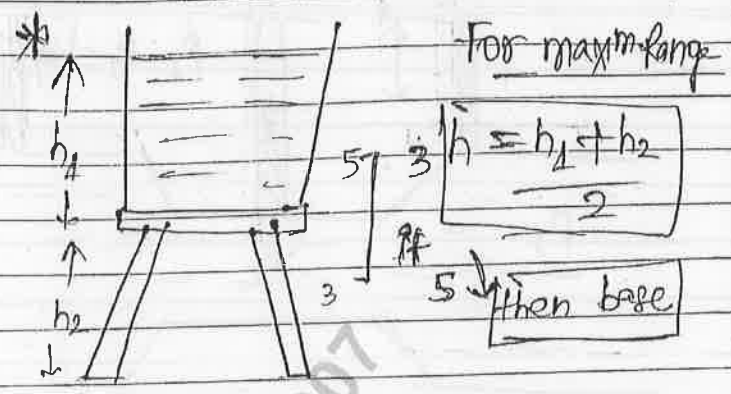
$$R_{\max} = H$$



$H_2O \rightarrow BB \rightarrow 4 \rightarrow 2$   
 Ex-I  $\rightarrow 50, 53, 54, 59$   
 II  $\rightarrow 4, 2, 8, 33$

PAGE NO. III - 18, 19.  
 DATE: 1 1 P-2  $\rightarrow 2, 3, 4, 5, 16$

10 m		
9 m	1 x 9	9
8 m	2 x 8	16
7 m	3 x 7	21
6 m	4 x 6	24
5 m	5 x 5	25
4 m	6 x 4	24
3 m	7 x 3	21



Result approx. start

18.9.19.

2. Venturimeter -

Eq<sup>n</sup> of continuity

$A_1 > A_2$

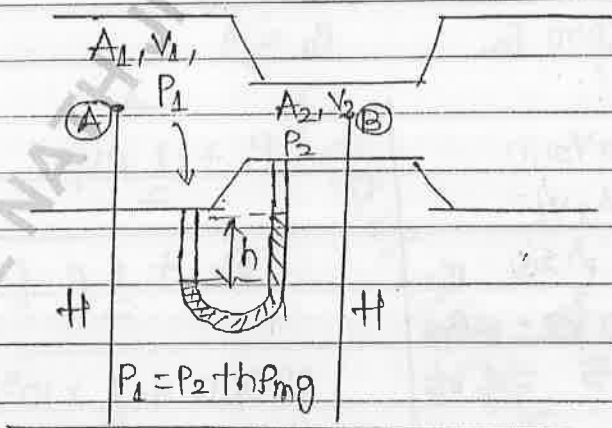
$v_1 < v_2$

Bernoulli's

$\uparrow P + \frac{1}{2} \rho v^2 \downarrow = \text{const.}$

$v_1 < v_2$

$P_1 > P_2$



$P_1 = P_2 + h \rho_m g$

$P_1 - P_2 = h \rho_m g$

$P_1 + \frac{1}{2} \rho v_1^2 = P_2 + \frac{1}{2} \rho v_2^2$

$P_1 - P_2 = \frac{1}{2} \rho [v_2^2 - v_1^2]$

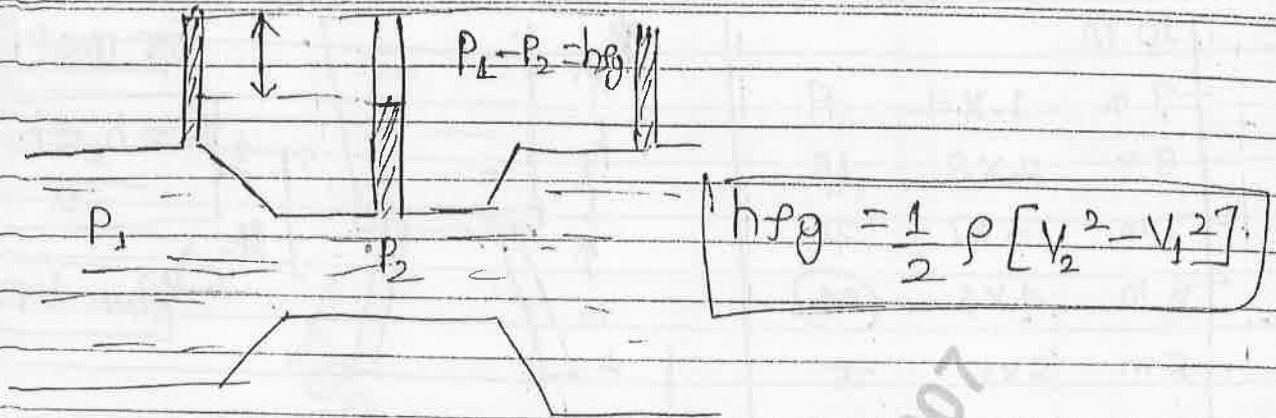
$h \rho_m g = \frac{1}{2} \rho [v_2^2 - v_1^2]$

$\frac{dV}{dt} = A_1 v_1 = A_2 v_2 = Q$

$v_1 = \frac{Q}{A_1}, v_2 = \frac{Q}{A_2}$

$h \rho_m g = \frac{1}{2} \rho \left[ \frac{Q^2}{A_2^2} - \frac{Q^2}{A_1^2} \right]$

$h \rho_m g = \frac{1}{2} \rho Q^2 \left[ \frac{A_1^2 - A_2^2}{A_1^2 A_2^2} \right]$



Q. Water is flowing through a horizontal tube of cross-section  $10 \text{ cm}^2$  with speed  $2 \text{ m/s}$ . Pressure is  $40 \text{ k pascal}$ . At a place where area of cross-section is  $5 \text{ cm}^2$ . find pressure.

→  $A_1 = 10 \text{ cm}^2$        $A_2 = 5 \text{ cm}^2$

$V_1 = 2 \text{ m/s}$        $V_2 = ?$

$P_1 = 40,000 \text{ Pa}$        $P_2 = ?$

$$A_1 V_1 = A_2 V_2$$

$$V_2 = \frac{A_1 V_1}{A_2}$$

$$= \frac{10 \times 2}{5} = 4 \text{ m/s}$$

A      B

$$P_1 + \frac{1}{2} \rho V_1^2 = P_2 + \frac{1}{2} \rho V_2^2$$

$$P_1 + \frac{1}{2} \rho [V_1^2 - V_2^2] = P_2$$

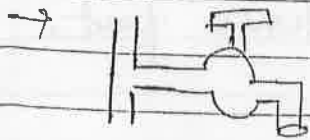
$$40,000 + \frac{1}{2} \times 10^3 [4 - 16] = P_2$$

$$40,000 - 500 \times 12 = P_2$$

$$P_2 = 10,000 - 6000$$

$$= 4000 \text{ Pascal}$$

Q. When tap is closed pressure in the tube is  $2.5 \times 10^5 \text{ Pascal}$ . When tap is opened, pressure in the tube is  $2 \times 10^5 \text{ Pascal}$ . Find speed of flow of water.



$$P_1 + \frac{1}{2} \rho V_1^2 = P_2 + \frac{1}{2} \rho V_2^2$$

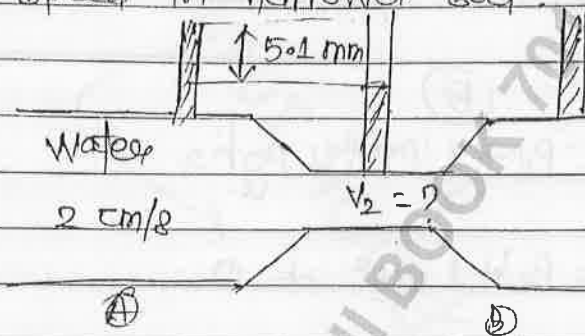
$$2.5 \times 10^5 + 0 = 2 \times 10^5 + \frac{1}{2} \times 10^3 \times V_2^2$$

$$0.5 \times 10^5 = 500 V_2^2$$

$$V_2^2 = \frac{0.5 \times 10^5}{500} = 100$$

$$V_2 = 10 \text{ m/s}$$

Q. Find speed in narrower section.  $V_2 = ?$



$$\rightarrow P_1 + \frac{1}{2} \rho V_1^2 = P_2 + \frac{1}{2} \rho V_2^2$$

$$P_1 - P_2 + \frac{1}{2} \rho V_1^2 = \frac{1}{2} \rho V_2^2$$

$$h \rho g = \frac{1}{2} \rho V_1^2 = \frac{1}{2} \rho V_2^2$$

$$V_2^2 = 2gh + V_1^2$$

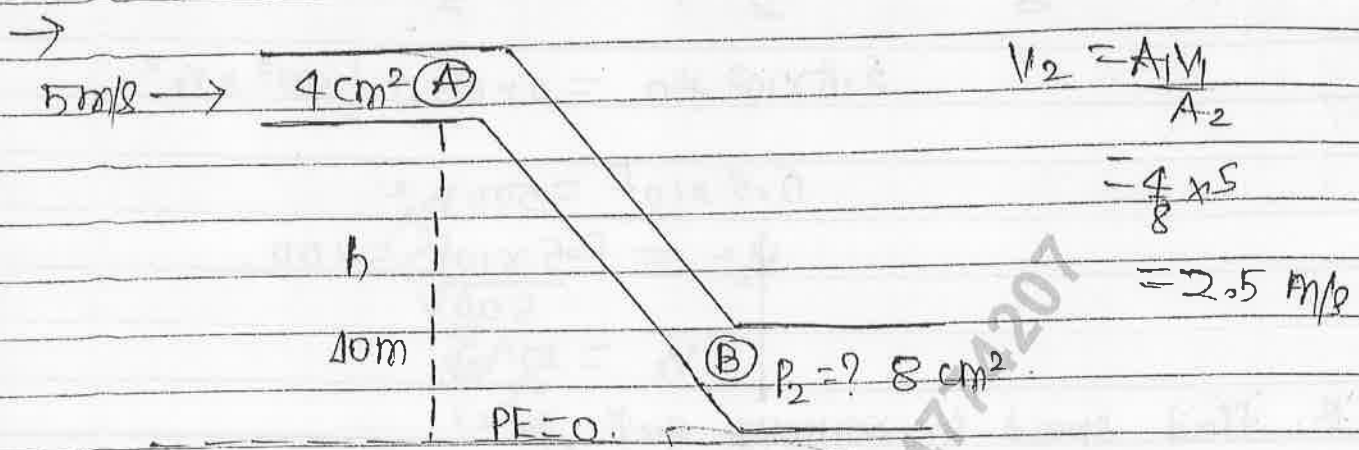
$$= 2 \times 1000 \times 0.51 + 4$$

$$= 1020 + 4$$

$$V_2^2 = 1024 \quad \Rightarrow V_2 = 32 \text{ cm/s}$$

Q. Water is flowing through a tube of cross-section  $4 \text{ cm}^2$  with speed  $5 \text{ m/s}$ . Water gradually descends by  $10 \text{ m}$ . Wherever cross-section becomes  $8 \text{ cm}^2$  of pt. in the upper

Ex-11  
 part is  $1.5 \times 10^5$  Pa. find pressure at the lower part.



(A)  $P_1 + \frac{1}{2} \rho V_1^2 + \rho g h_1 = P_2 + \frac{1}{2} \rho V_2^2 + \rho g h_2$

$P_1 + \frac{1}{2} \rho V_1^2 + \rho g h_1 = P_2 + \frac{1}{2} \rho V_2^2 + 0$

$P_1 + \frac{1}{2} \rho [V_1^2 - V_2^2] + \rho g h_1 = P_2$

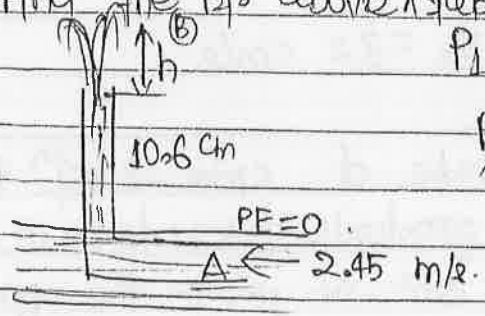
$1.5 \times 10^5 + \frac{1}{2} \times 10^3 [25 - 6.25] + 10^3 \times 10 \times 10 = P_2$

$1.5 \times 10^5 + 500 [18.75] + 10^5 = P_2$

$P_2 = 2.5 \times 10^5 + 0.09375 \times 10^5$

$P_2 = 2.59375 \times 10^5 \text{ Pascal}$

Q. find the ht. above <sup>end of the</sup> tube where water spills out.



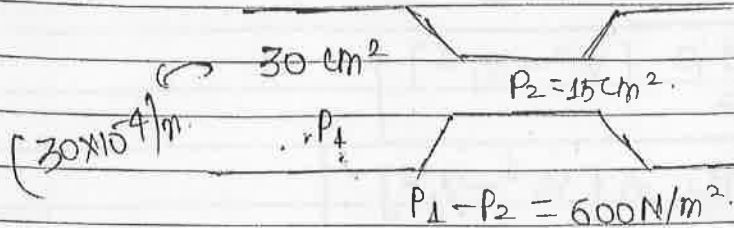
$P_1 + \frac{1}{2} \rho V_1^2 + \rho g h_1 = P_2 + \frac{1}{2} \rho V_2^2 + \rho g h_2$

$\rho/2 + \frac{1}{2} \rho V_1^2 + 0 = \rho/2 + 0 + \rho g (10.6 \times 10^{-2} + h)$

$10.6 \times 10^{-2} + h = \frac{V_1^2}{2g}$

$h = \frac{2.45 \times 2.45}{2 \times 9.8} - 10.6 \times 10^{-2} = 30.6 - 10.6 = 20 \text{ cm}$

Q. Find vol<sup>m</sup>-flowing per unit time through the tube.



$$P_1 + \frac{1}{2} \rho V_1^2 = P_2 + \frac{1}{2} \rho V_2^2$$

$$P_1 - P_2 = \frac{1}{2} \rho Q^2 \left[ \frac{A_1^2 - A_2^2}{A_1^2 A_2^2} \right]$$

$$600 = \frac{1}{2} \times 10^3 \times Q^2 \left[ \frac{900 \times 10^{-8} - 225 \times 10^{-8}}{900 \times 10^8 \times 225 \times 10^{-8}} \right]$$

$$1.2 = Q^2 \left[ \frac{575 \times 10^{-8}}{900 \times 225 \times 10^{-16}} \right]$$

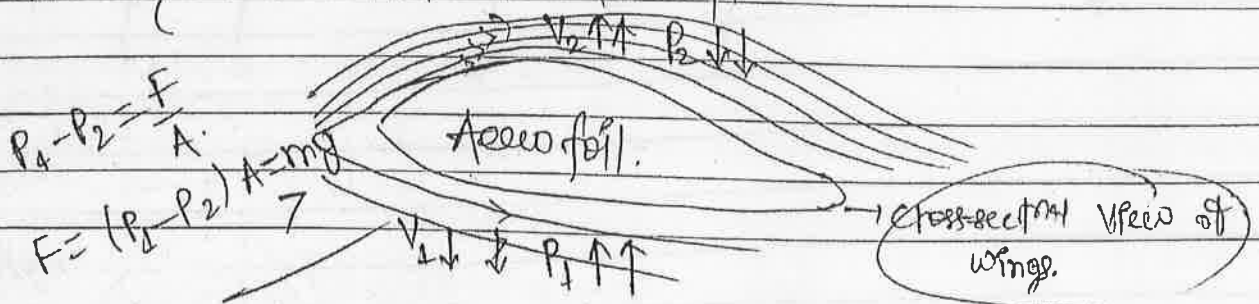
$$A_{22} = Q^2 \times 300 \times 10^{-8}$$

$$Q^2 = \frac{360 \times 10^{-8}}{36 \times 10^{-7}}$$

$$Q^2 = \frac{36 \times 10^{-8}}{10}$$

$$Q = \frac{6 \times 10^{-3} \text{ m}^3/\text{s}}{\sqrt{10}}$$

### III. Dynamic lift of Aeroplane -



\*-27  
 PG-16, 15, 17  
 III-18, 19

BB-4 comp

EX-1 →

16-59  
 181

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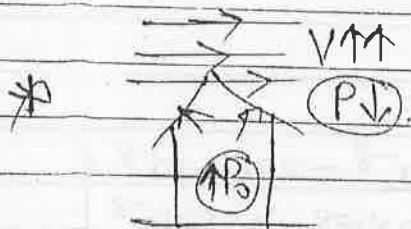
8/12/17  
 181

$$P_1 + \frac{1}{2} \rho V_1^2 = P_2 + \frac{1}{2} \rho V_2^2$$

$$P_1 = P_2 = \frac{1}{2} \rho [V_2^2 - V_1^2]$$

$$F = \frac{1}{2} \rho_{air} A [V_2^2 - V_1^2]$$

$$\rho_{air} = 1.29 \text{ kg/m}^3$$



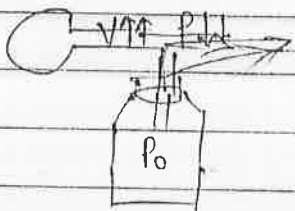
$$F = \frac{1}{2} \rho_{air} A [V_2^2 - 0]$$

Inside room.

\* Atomizer

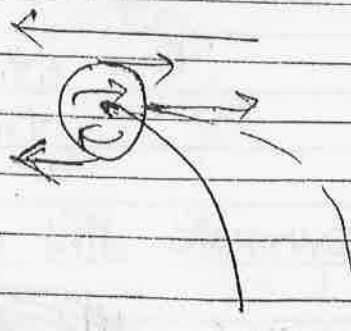
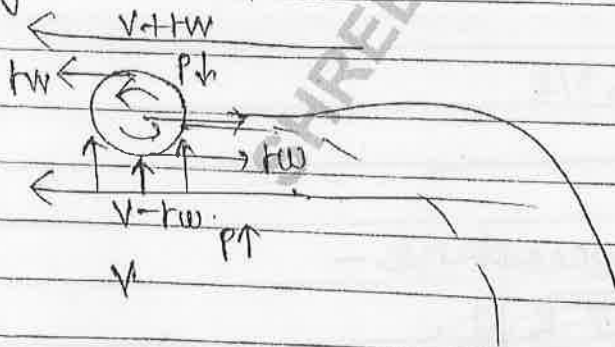
Paint gun

pesticide spray



19.9.19

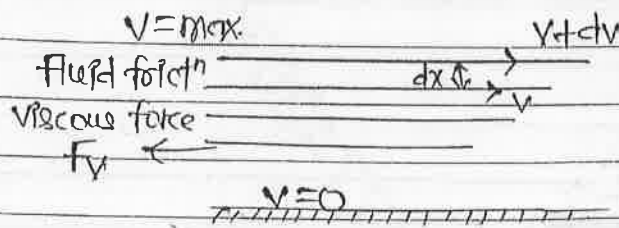
Magnus Effect →



# VISCOSITY

Due to relative motion b/w different layers of a fluid a force comes into play which opposes their relative motion. This force is called viscous force & the phenomena viscosity.

Reason - IMF of attract<sup>n</sup> or cohesive force. (electromagnetic).



$$F_v \propto A$$

$$F_v \propto \frac{dv}{dx}$$

$$F_v \propto A \frac{dv}{dx}$$

Velocity gradient =  $\frac{dv}{dx}$

$$F_v = \eta A \frac{dv}{dx}$$

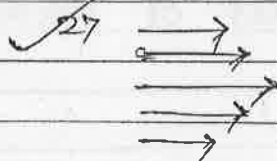
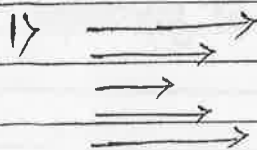
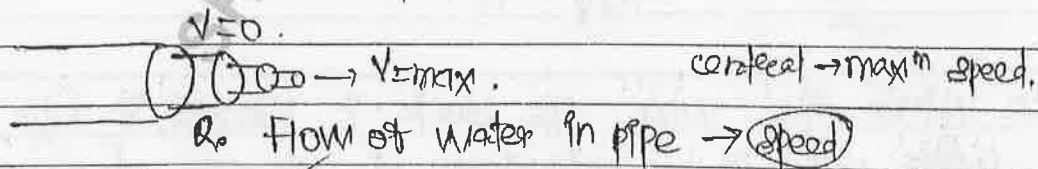
$$\eta = \frac{F_v dx}{A dv} \quad \frac{N \cdot m \cdot s}{m^2 \cdot m}$$

$$\frac{M^1 L^1 T^{-2} T^1}{L^2}$$

$$[M^1 L^{-1} T^{-1}]$$

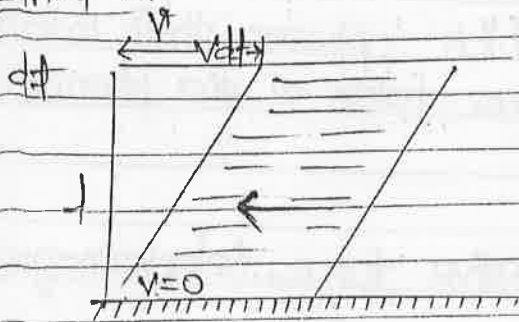
Coefficient of viscosity  
↓  
Nature of fluid

	M.K.S.	C.G.S.
→	$\frac{N \cdot s}{m^2}$ or Poiseals	→ $\frac{dyne \cdot s}{cm^2}$
→	Poiseuille decapoise	→ Poise



4) can't predict

Method-2



$$\text{shear stress} = \frac{F}{A}$$

$$\text{shear strain} = \frac{vd}{d}$$

$$\text{shear strain rate} = \frac{v}{d}$$

coefficient viscosity,

$$\eta = \frac{\text{shear stress}}{\text{shear strain rate}}$$

$$\eta = \frac{Fv/A}{v/d} \Rightarrow$$

$$\boxed{\eta = \frac{Fd}{AV}}$$

Q. A glass slab of length 3 m & breadth 2 m is placed on a layer of honey of thickness 10 cm. Find force reqd to drag it with velocity 20 cm/s.  $\eta$  of honey =  $2.5 \times 10^{-4}$  Pa-s.

$$\begin{aligned} \rightarrow F_v &= \frac{\eta AV}{d} = \frac{2.5 \times 10^{-4} \times 6 \times 0.2}{0.1} \\ &= 30 \times 10^{-4} \\ &= 3 \times 10^{-3} \text{ N} \end{aligned}$$

NCERT

Q. On the given fig. when the block is released, it falls with speed 0.085 m/s. Find coefficient of viscosity of glycerine.

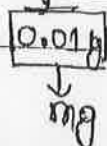


$$F_v = mg$$

$$F_v = 0.01 \times 10$$

$$F_v = 0.1 \text{ N}$$

$$\downarrow 0.085 \text{ m/s}$$



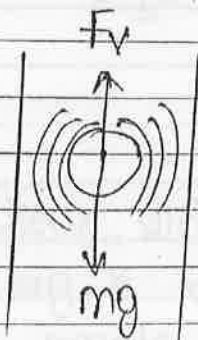


$$\eta = \frac{F_v \cdot l}{A \cdot v}$$

$$= \frac{0.4}{0.1} \times \frac{0.3 \times 10^{-3}}{0.085} = \frac{0.3 \times 10^{-3} \times 10^{-3}}{85} = \frac{3.5}{85} \times 10^{-3} \text{ Pascal}$$

## II. STOKES' LAW :-

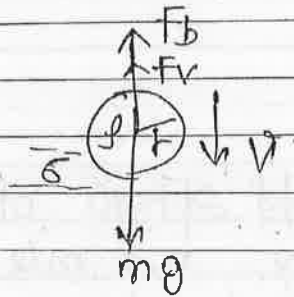
When a spherical body is released in viscous med<sup>m</sup>, due to relative mot<sup>n</sup>, viscous force acts on it which counter-balance the effective wt. & body together falls with a const. velocity which is called terminal velocity.



$$F_v = 6\pi\eta r v$$

$$F_v = -6\pi\eta r \vec{v}$$

$$F_v = -k v$$



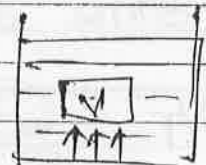
$$F_v + F_b = mg$$

$$F_v = mg - F_b$$

$$= \frac{4}{3} \pi r^3 \rho g - \frac{4}{3} \pi r^3 \sigma g$$

$$6\pi\eta r v_T = \frac{4}{3} \pi r^3 (\rho - \sigma) g$$

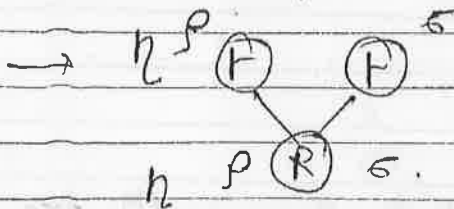
$$v_T = \frac{2r^2 (\rho - \sigma) g}{9\eta}$$



$V \sigma g = \text{Buoyancy}$   
 Body  $\rho$  fluid  $\sigma$

Q. Two identical rain droplets were falling in air with speed 5 m/s. Both the droplets coalesce. Find constant speed of new drop.

$$V_T = \frac{2r^2}{9\eta} (\rho - \sigma)g$$



$$V \propto r^2$$

$$\frac{V_1}{V_2} = \left(\frac{r}{R}\right)^2$$

$$= \left[\frac{r}{2^{1/3}r}\right]^2 = \frac{1}{2^{2/3}}$$

$$V_2 = 5 \times 2^{2/3}$$

$$= 5 \times 4^{1/3} \text{ cm/s}$$

20.09.19

Q. A gold sphere of density 19 gm/cc falls with constant velocity 0.4 m/s in a liquid of density 3 gm/cc. Find terminal velocity acquired by Cu sphere of same size in the same liquid, density of Cu = 8 gm/cc.

$$\rightarrow V_T = \frac{2r^2}{9\eta} (\rho - \sigma)g$$

(9)

$$\frac{V_1}{V_2} = \frac{\rho_1 - \sigma}{\rho_2 - \sigma} \Rightarrow \frac{0.4}{V_2} = \frac{19 - 3}{8 - 3} = \frac{16}{5}$$

$$\Rightarrow V_2 = 0.2 \text{ m/s}$$

Q. Two spheres of mass  $m_1$  &  $m_2$  of same material are falling in same liquid. Find ratio of their terminal velocity.

$$\rightarrow V_T \propto r^2$$

$$\frac{V_1}{V_2} = \left(\frac{r_1}{r_2}\right)^2$$

$$m = V\rho$$

$$m = \frac{4}{3} \pi r^3 \rho$$

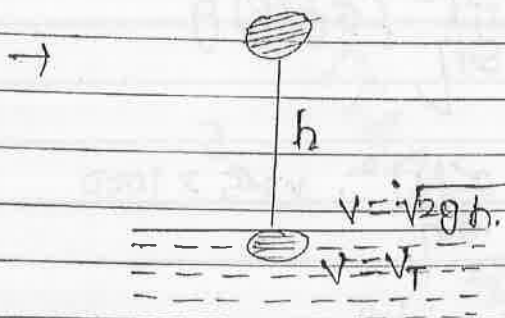
$$\frac{m_1}{m_2} = \left(\frac{r_1}{r_2}\right)^3$$

III.  $v \propto (p-6)$   
 $\frac{v_1}{v_2} = \frac{\eta_2}{\eta_1} \left( \frac{p_1-6}{p_2-6} \right)$   $\frac{10}{v_2} = \frac{13.2}{8.5 \times 10^4} \left( \frac{7.8-1.2}{7.8-1} \right)$

$\frac{m}{8m} = \left( \frac{\eta_1}{\eta_2} \right)^3$

$\Rightarrow \frac{v_1}{v_2} = \left( \frac{1}{2} \right)^2 = \frac{1}{4}$   $\Rightarrow \left( \frac{\eta_1}{\eta_2} = \frac{1}{4} \right)$

Q. A ball of radius  $10^{-4}$  m & density  $40^4$  kg/m<sup>3</sup>. Is dropped from ht. 'h' above water surface. Just after entering in water it further falls with a const. vel. find 'h'.  $\eta$  of water =  $10^{-5}$  Pa-s.



$\sqrt{2gh} = \frac{2r^2 (p-6) g}{9\eta}$

$= \frac{2 \times (10^{-4})^2 \times [40 \times 10^3 - 10^3] \times 10}{9 \times 10^{-5}}$

$= \frac{2 \times 10^{-8} \times 39 \times 10^3 \times 10}{9 \times 10^{-5}}$

$\sqrt{2gh} = 20$

$h = \frac{20 \times 20}{2 \times 10} \text{ m}$

Q. A sphere of radius 'r' is dropped in a viscous fluid. Heat is produced due to viscous force. When the sphere acquires constant velocity. then rate of heat production depends on radius of sphere a/c to the relation -

- 1) r
- 2) r<sup>2</sup>
- 3) r<sup>5</sup>
- 4) r<sup>4</sup>

$\frac{dQ}{dt} = \text{Power} = F_v v$   
 $= 6\pi\eta r v v$

Power  $\propto r v^2$

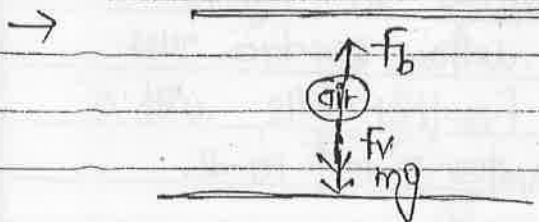
Power  $\propto r \times r^4$

Power  $\propto r^5$

$v \propto r^2$

$v^2 \propto r^4$

Q. An air bubble of radius 1 cm is rising in a liquid of density 1.5 gm/cc with speed 2 mm/sec. Find  $\eta$ .  
Ignore air density.



$$f_b = f_v + mg$$

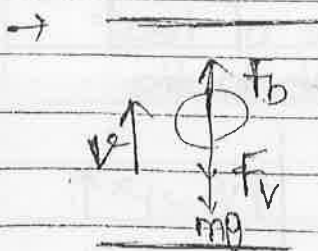
$$f_v = f_b - mg$$

$$V_T = \frac{2r^2 (\sigma - \rho) g}{9\eta}$$

$$\frac{0.2}{0.2} = \frac{2(1)^2}{9\eta} \times 1.5 \times 1000$$

$$\eta = \frac{5}{3} \times 10^3 = 1.6 \times 10^3 \text{ Poise}$$

Q. A ball is rising with constant speed in water. Density of water is 5 times that of ball. Find ratio of force of friction to the wt of ball.



$$f_b = f_v + mg$$

$$f_v = f_b - mg$$

$$= V \cdot 5g \frac{\rho}{\rho} - V \rho g$$

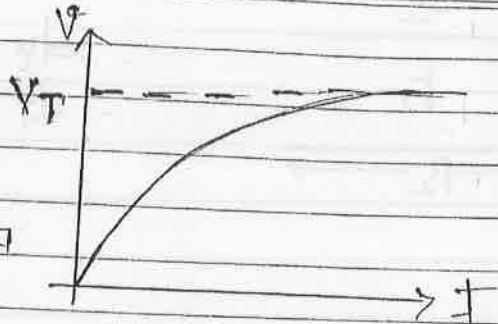
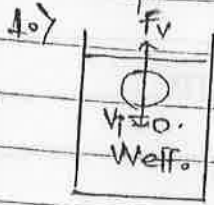
$$= V \rho g \left( \frac{5}{\rho} - 1 \right)$$

$$f_v = W \left[ \frac{5}{\rho} - 1 \right]$$

$$\frac{f_v}{W} = \left[ \frac{5}{\rho} - 1 \right]$$

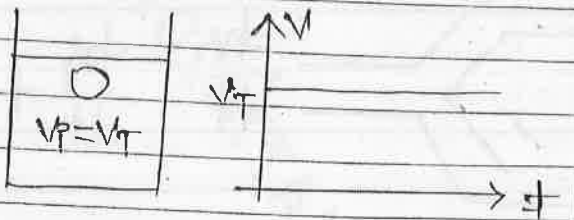
$$\frac{f_v}{W} = \frac{5}{4}$$

Graph →

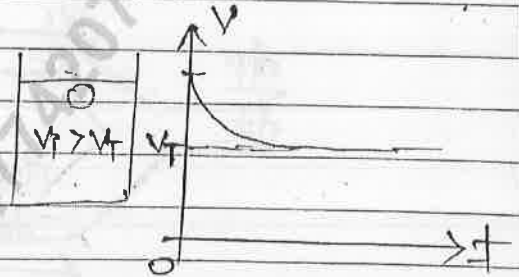


$$W_{eff.} = \frac{(6\pi\eta d v T)}{m} = g$$

2. >

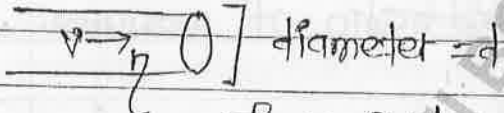


3. >



# Reynolds' No. →

Estimates nature of flow.



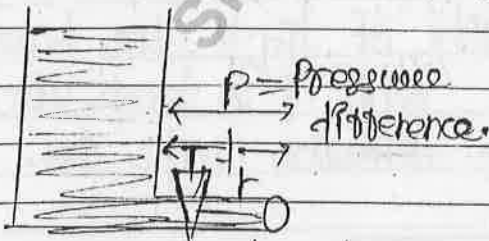
$$R_n = \frac{\rho v d}{\eta}$$

change

- \*  $R_n < 1000$  Flow is streamline
- \*  $1000 < R_n < 2000$  Streamline converting into turbulent.
- \*  $R_n > 2000$  Turbulent.

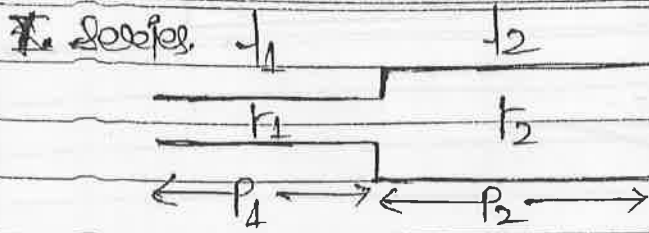
$$R_n = \frac{\rho v d}{\eta} \times \frac{A v}{A v} = \frac{A v^2 \rho}{\eta A v} = \frac{\text{Inertial force}}{\text{Viscous force}}$$

# Poiseuille's formula → estimates rate of flow of vol<sup>m</sup> flowing per unit time.



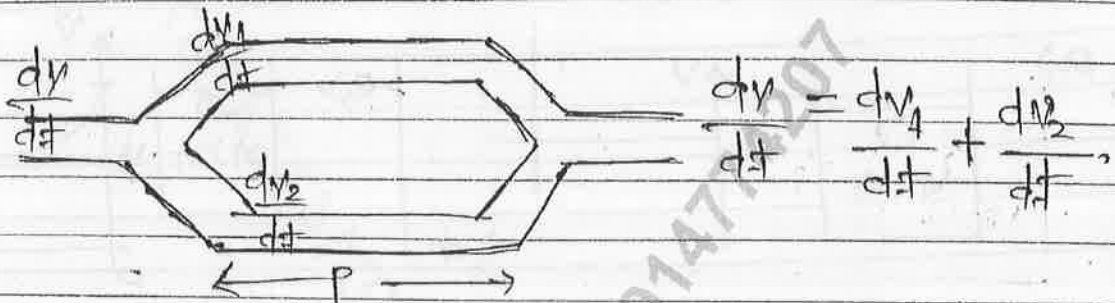
$$\frac{dV}{dt} = \frac{\pi P r^4}{8 \eta l} \text{ m}^3/\text{s} \text{ or } \text{lit}/\text{sec.}$$

$$\frac{dV}{dt}$$



$$\frac{dy}{dt} = \text{volume}$$

II. Parallel.



Q. Two tubes of length 'l' & '2l' are connected in series, ratio of their radii is 1:2. Find ratio of pressure difference across them.

$$\rightarrow \left(\frac{dy}{dt}\right) = \frac{\Delta P l}{8 \eta r^4} \cdot P \propto \frac{l}{r^4}$$

$$\frac{P_1}{P_2} = \frac{l_1}{l_2} \times \left(\frac{r_2}{r_1}\right)^4$$

$$= \frac{l}{2l} \times \left(\frac{2}{1}\right)^4 = \frac{8}{4}$$

\* With ↑ in temperature viscosity of liquids ↓ coz, cohesive force ↓ while with ↑ in temp. viscosity of gases ↑ because their diffusion rate ↑.

— X —

# Fluid Statics

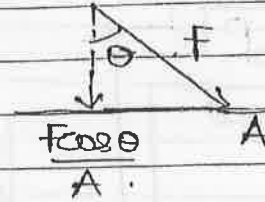
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Pressure =  $\frac{F}{A}$  N/m<sup>2</sup> or Pascal. (booklet) [M<sup>1</sup>L<sup>-1</sup>T<sup>-2</sup>]

Here force is normal to the area.

$F = \text{const.}$

$\downarrow \downarrow P \propto \frac{1}{A} \uparrow$



Types of Pressure :-

1. Atmospheric Pressure -

Pressure exerted by atmosphere or air column pressure.

1 atm = 760 mm of Hg = 76 cm of Hg =  $h \rho_m g = 1.013 \times 10^5$  Pascal.

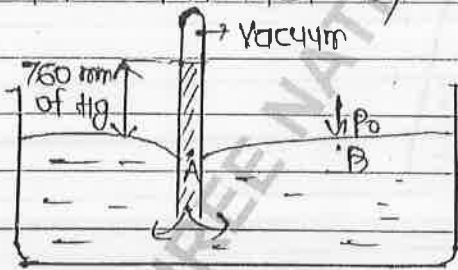
On case of water,  $h \rho_w g = 1.013 \times 10^5$   
 $h_{\text{water}} = 10.33 \text{ m}$

1 bar =  $10^5$  Pascal.

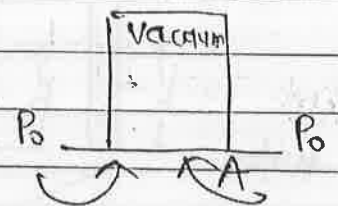
1 torr = 1 mm of Hg = 133 Pascal.

1 atm = 760 torr.

Barometer → Invented by Torricelli.



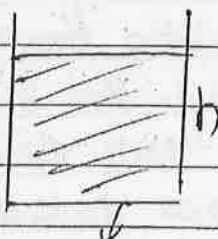
$P_A = P_B$   
 $h \rho_m g = P_0$



$F = (P_0 - P) A > mg$

2. Gauge Pressure -

Excess pressure over 1 atmospheric is gauge pressure

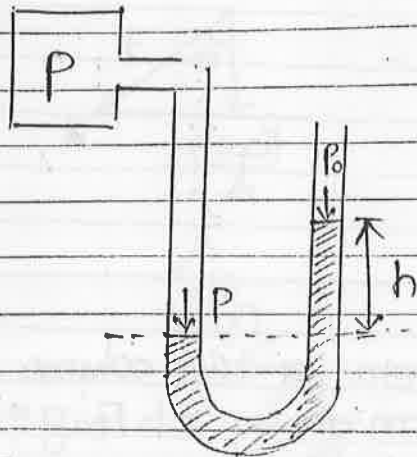


$P = P_0 + h \rho g$

$P = P_0 + \frac{mg}{A}$

$= P_0 + \frac{V \rho g}{A} = P_0 + \frac{Ah \rho g}{A}$

Manometer

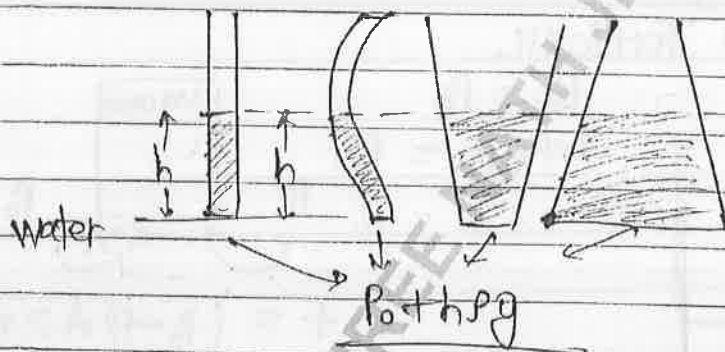


$$P = P_0 + h\rho g$$

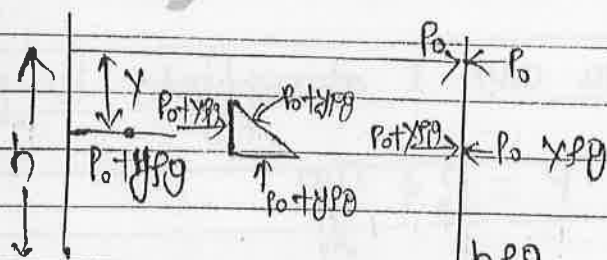
$$P - P_0 = h\rho g$$

3. Absolute Pressure - / Pressure / Total Pressure -  
 $P_{total} = P_0 + h\rho g$

\* Liquid column pressure depends on height. It is independent of shape & size of container & amount of liquid.



\* Avg. Pressure -



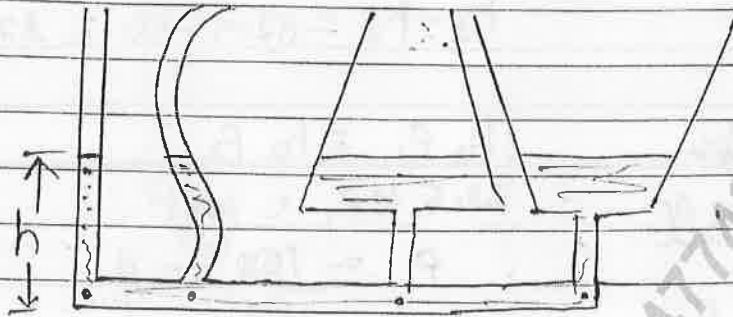
Fluid exerts equal pressure in all dir<sup>n</sup> @ a given depth. Whatever be the orientation of surface. That is why container wall also experiences pressure.

Avg. Pressure on container wall =  $\frac{0 + h\rho g}{2}$

Force on container wall,  $F = \rho_{avg} A = \frac{h\rho g}{2} \times A$

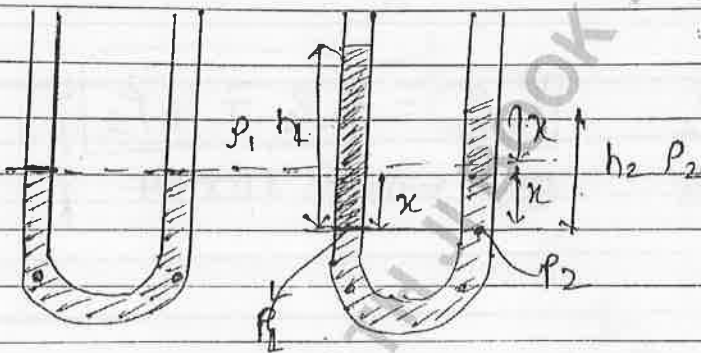


\* The height of liquid in interconnected open containers is same becoz liquid maintains equal pressure at same h<sub>l</sub> level. It is called hydrostatic paradox.



$P_0 + h\rho g$  Hydrostatic Paradox.

\* U-Tube



$$P_1 = P_2$$

$$P_0 + h_1 \rho_1 g = P_0 + h_2 \rho_2 g$$

$$\boxed{h_1 \rho_1 = h_2 \rho_2}$$

Q. Hg is partially filled in a U-shaped tube. Now some H<sub>2</sub>O is poured in one arm upto length 27.2 cm. By what amount level of Hg rises in other arm.

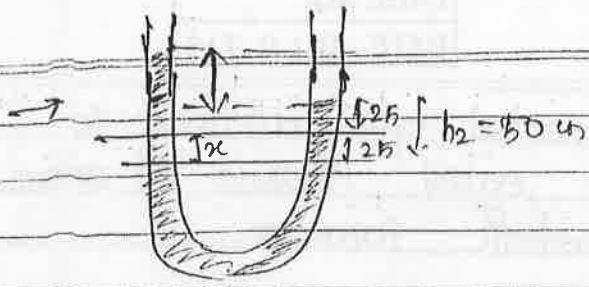
$$\rightarrow h_1 \rho_1 = h_2 \rho_2 \quad \text{for } h^2 = 2x.$$

$$27.2 \times 1 = 2x \times 13.6.$$

$$x = 1 \text{ cm.}$$

But value of x asked.

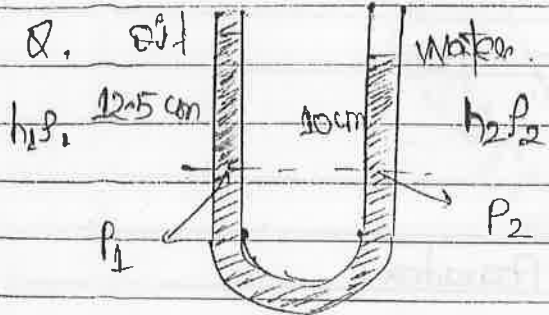
Q. A U-tube is partially filled with water. Now, an immiscible oil of density 0.8 gm/cc is poured in one arm. Until level of water rises by 25 cm in other arm. How much h<sub>2</sub>o of oil level above water level?



$$h_1 \times 0.8 = 50 \times 1$$

$$h_1 = \frac{500}{0.8} = 62.5 \text{ cm}$$

$$h_1 - h_2 = 62.5 - 50 = 12.5 \text{ cm}$$

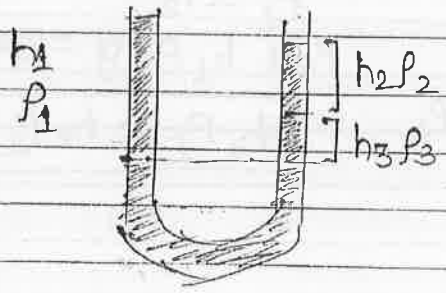


$$h_1 \rho_1 = h_2 \rho_2$$

$$12.5 \times \rho_1 = 10 \times 1$$

$$\rho_1 = \frac{100}{12.5} = 8$$

$$\rho_1 = 0.8 \text{ gm/cc}$$



$$h_1 \rho_1 = h_2 \rho_2 + h_3 \rho_3$$

$$62.5 \times 0.8 = 10 \times 1 +$$

Q. A bubble rises from bottom of lake on reaching surface, its radius becomes double. Find depth of the lake. If 1 atm  $P_0$  is eq. to 'H' ht of water column.

IT  
gradual  
process

$$\rho_1 V_1 = \rho_2 V_2$$

$$P_1 V_1 = P_2 V_2$$

$$(P_0 + h\rho g) \frac{4}{3} \pi R^3 = P_0 \frac{4}{3} \pi (2R)^3$$

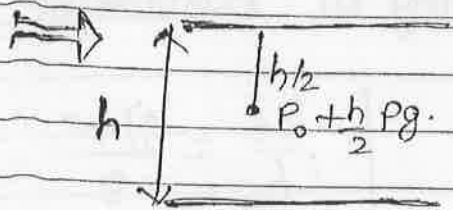
$$h\rho g = 7P_0$$

$$P_0 = H\rho g$$

$$h = \frac{7P_0}{\rho g} = 7 \frac{H\rho g}{\rho g}$$

$$h = 7H$$

Q. If pr. at half depth of a lake is  $\frac{2}{3}$  times that of at bottom. find depth of the lake ( $1 \text{ atm} = 10^5 \text{ Pa}$ )



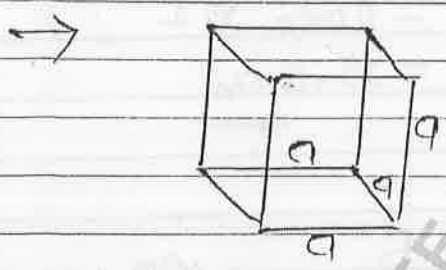
$$P_0 + \frac{h}{2} \rho g = \frac{2}{3} (P_0 + h \rho g)$$

$$\Rightarrow 3P_0 + \frac{3}{2} h \rho g = 2P_0 + 2h \rho g$$

$$\Rightarrow P_0 = \frac{1}{2} h \rho g \Rightarrow h = \frac{2 \times 10^5}{10^3 \times 10} = 20 \text{ m}$$

Q. A cube of side 'a' is completely filled with liquid of density 'ρ' find —

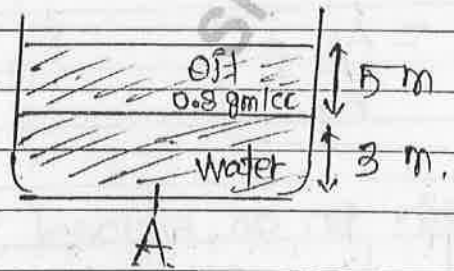
- (i) force on 1 wall due to liquid.
- (ii) force on base due to liquid.



$$(i) F_{\text{wall}} = \frac{\rho g a^3}{2} \times a^2 = \frac{\rho^2 g a^5}{2}$$

$$(ii) F_{\text{base}} = \rho g a^3 \times a^2 = \rho^2 g a^5$$

Q. Find pressure at Pt. 'A' due to liquid.  $1 \text{ gm/cc} = 10^3 \text{ kg/m}^3$

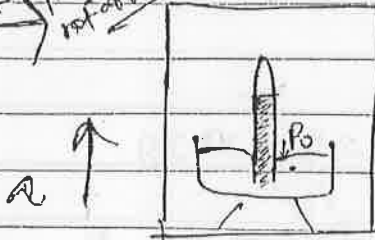


$$P_A = h_1 \rho_1 g + h_2 \rho_2 g$$

$$\begin{aligned} \text{due to oil} &= 5 \times 0.8 \times 10^3 \times 10 + 3 \times 10^3 \times 10 \\ \text{liquid} &= (4+3) 10^4 \\ &= 7 \times 10^4 \text{ Pascal} \end{aligned}$$

Q. A barometer is kept inside a stationary lift & its reading is 76 cm. Now the lift starts ascending with acc to  $9 \text{ m/s}^2$ . Find reading of Barometer.

atmospheric pressure not affected.



$$g_{\text{eff}} = g + a$$

$$P_0 = h_1 \rho g$$

$$h \propto \frac{1}{g}$$

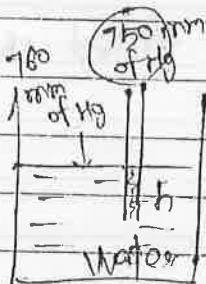
$$\frac{h_1}{h_2} = \frac{g_2}{g_1}$$

$$\frac{76}{h_2} = \frac{g+a}{g}$$

$$h_2 = 76 \times \frac{g}{g+a}$$

$$h_2 = 40 \text{ cm}$$

Ex 2. Q. 10



10 mm of Hg  $\geq h \rho g$ .

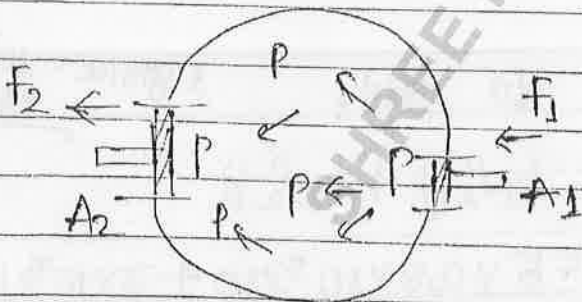
1 cm of Hg  $= h_{\text{max}} \cdot \rho g$ .

$$h_1 \rho_1 g = h_{\text{max}} \cdot \rho_2 g$$

$$1 \times 13.6 = h_{\text{max}} \times 1.$$

$$h_{\text{max}} = 13.6 \text{ cm}$$

# Pascal's Law :-



$$P = \frac{F_1}{A_1} = \frac{F_2}{A_2}$$

eg.

$$A_2 = 3A_1$$

$$F_2 = 3F_1$$

$$F_2 = \frac{A_2}{A_1} F_1$$

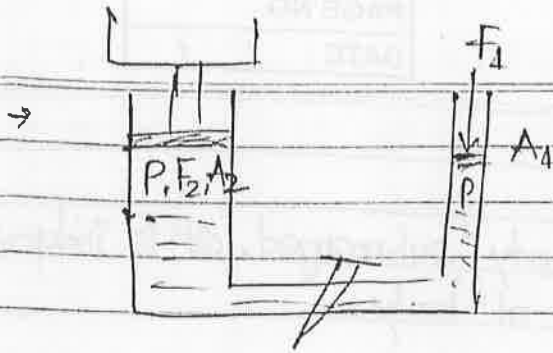
If a pressure is applied at a pt. on an enclosed fluid, then this pressure is equally transmitted to each & every pt. undiminished.

Q. → 3  
L15, 10, 11, 15,  
17,

P-238 - BB-73

8-2, 3A,  
I-29, 30, 33,  
40, 42, 44  
II-1, 26.  
III-3, 15, 16, 17  
13

PAGE NO. \_\_\_\_\_  
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$$F_2 = \frac{A_2}{A_1} F_1$$

$$F_2 = \left(\frac{h_2}{h_1}\right)^2 F_1$$

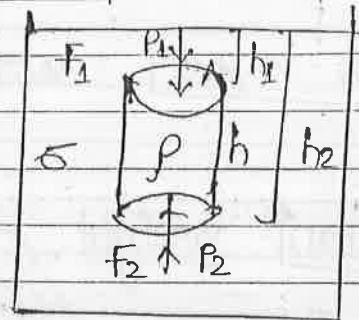
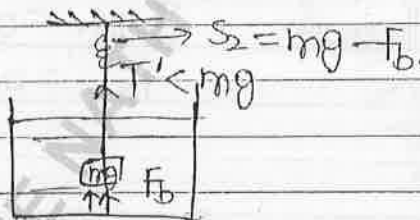
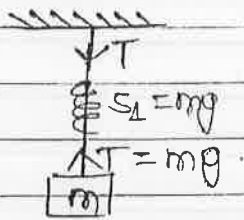
- 1) Hydraulic lift;
- 2) Hydraulic brakes.
- 3) Hydraulic Press.

24.9.19

### # Buoyancy / Archimedes Principle -

When a body completely or partially submerged in a fluid then it experiences upward force. It is called buoyant force & phenomena buoyancy.

A/c to Archimedes, Buoyant force = wt. of fluid displaced



$$F_1 = (P_0 + h_1 \sigma g) A$$

$$F_2 = (P_0 + h_2 \sigma g) A$$

$$\begin{aligned} F_{net} &= F_2 - F_1 = P_0 A + A h_2 \sigma g - P_0 A - A h_1 \sigma g \\ &= A (h_2 - h_1) \sigma g \\ &= A h \sigma g \end{aligned}$$

not depend on  
ht.  
when completely  
submerged.

$$F_b = V \sigma g$$

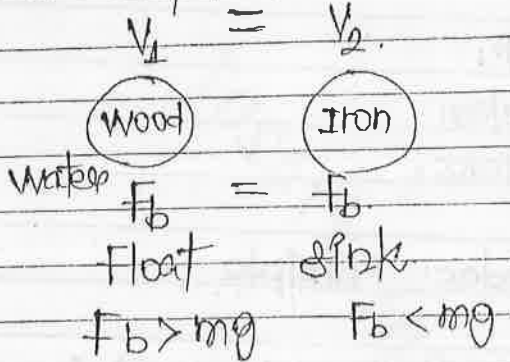
depends on mass & vol. with wt.

$$V \sigma g$$

\*  $F_b = V_{in} \rho g$

1.  $F_b \propto V_{in}$

\*  $F_b$  depends only on the vol<sup>m</sup> of body submerged. It is independent of shape, size & density of body.



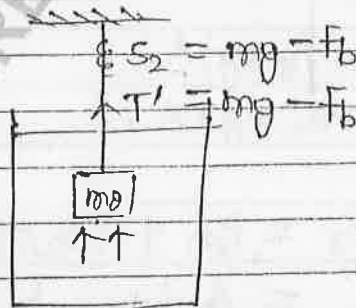
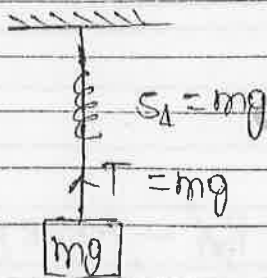
2.  $F_b \propto \rho \uparrow$   $\rho_{sea} > \rho_{fresh}$

→ It is easier to swim in sea than river.

→ When ship comes from sea to river, it further slightly sinks.

3.  $F_b \propto g \uparrow$  Weightlessness  $g_{net} = 0$   
 $F_b = 0$

Apparent weight →



$$W_{app} = mg - F_b$$

$$= V \rho g - V \rho_0 g$$

$$= V \rho g \left[ 1 - \frac{\rho_0}{\rho} \right]$$

$$W_{app}' = mg \left[ 1 - \frac{\rho_0}{\rho} \right]$$

$$W_{net} = mg \left[ 1 - \frac{\rho_0}{\rho} \right]$$

$$W_{app} = W \left[ 1 - \frac{\rho_0}{\rho} \right]$$

$$T = 2\pi \sqrt{\frac{l}{g}}$$

Q. A block of mass 3 kg is suspended by thread. Now the block is submerged in a liquid of relative density 0.5. Relative density of block is 10. What will be the tension in the string.

$$R.D. = \frac{\rho}{\rho_{w}}$$

$$\rightarrow W_{app} = mg \left[ 1 - \frac{\rho}{\rho} \right]$$

$$\rho = R.D. \times \rho_w$$

$$= 3 \times 10 \left[ 1 - \frac{0.5 \times 10^3}{10 \times 10^3} \right]$$

$$= 30 [1 - 0.05]$$

$$= 30 \times 0.95$$

$$= 28.5 \text{ N.}$$

$$= 28.5 \times \frac{1}{10} \text{ kg wt} = 2.85 \text{ kg wt.}$$

$$1 \text{ kg wt.} = 10 \text{ N.}$$

$$1 \text{ gm wt.} = 1000 \text{ dyne}$$

$$1 \text{ kg F} = 10 \text{ N.}$$

$$1 \text{ gm f} = 1000 \text{ dyne}$$

$$1 \text{ N} = \frac{1}{10} \text{ kg wt.}$$

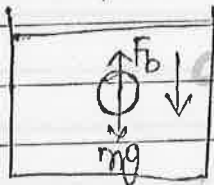
### # LAWS of floatation :-

1.  $W > F_b$

$$V \rho g > V \rho_w g$$

$$\rho > \rho_w$$

sink upto bottom.

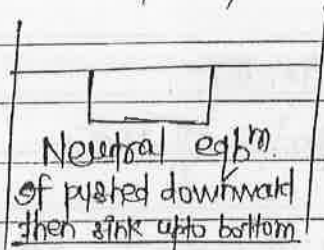


$$W_{app} = mg - F_b$$

2.  $W = F_b$

$$\rho = \rho_w$$

float completely submerged.

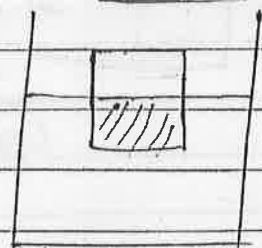


$$W_{app} = 0$$

3.  $W < F_b$

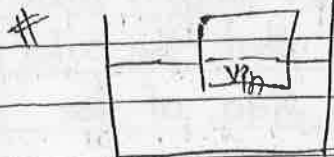
$$\rho < \rho_w$$

float partially submerged.



Stable eqbm.

$$W_{app} = 0$$



$$mg = V_m \rho_w g$$

$$V \rho = V_m \rho_w$$

Q. A cube of side 10 cm and mass 700 gm is floating in water. Find the vol<sup>m</sup> outside water.

→

$$mg = V_{in} \rho$$

$$700 = V_{in} \times 1.$$

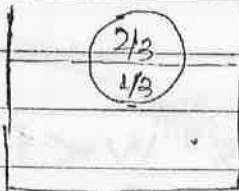
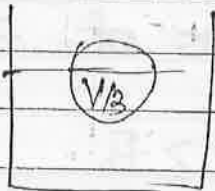
$$V_{in} = 700 \text{ cc.}$$

$$V = (10)^3 = 1000 \text{ cc.}$$

$$V_{out} = 300 \text{ cc.}$$

Q. A sphere floats in water by  $\frac{2}{3}$  part vol<sup>m</sup> outside.  
Q. floats in a liq<sup>d</sup> by 80% vol<sup>m</sup> outside. Find density of sphere & liq<sup>d</sup>.

→

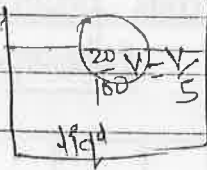



$$mg = V_{in} \rho$$

$$V \rho = V_{in} \rho$$

$$V \rho = \frac{1}{3} \rho$$

$$\rho = \frac{1}{3} \text{ gm/cc.} = \frac{1}{3} \times 10^3 \text{ kg/m}^3.$$



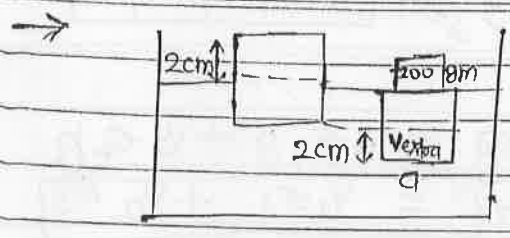
$$V \rho = V_{in} \rho_1$$

$$V \times \frac{1}{3} \times 10^3 = \frac{V}{5} \times \rho_1.$$

$$\rho_1 = \frac{5 \times 10^3}{3} \text{ kg/m}^3.$$

Q. A cube is floating in water such that, its one phase is 2 cm above water surface. Now a block of mass 200 gm is placed over it. So that the cube floats just completely submerged. Find side of the cube.





$(m g)_{extra} = (F_b)_{extra}$

$m g = V_{extra} \rho g$

$200 = 4 \times 4 \times 2 \times 1$

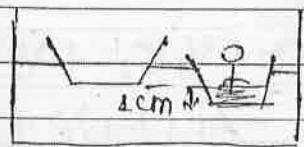
$a^2 = 100$

$a = 10 \text{ cm}$

Q. A boat of length 3 m & breadth 2 m is floating on water. Now a man gets inside it & both boat & man sink by 1 cm. Find mass of the man. (50 kg)

$m g = V_{in} \rho g$   
 $m = 3 \times 2 \times 10^{-2} \times 10^3 = 60 \text{ kg}$

25.9.19



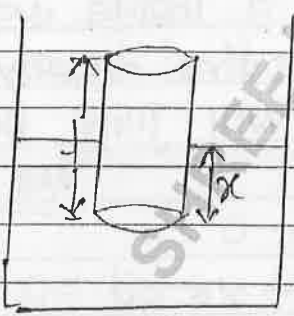
$(m g)_{extra} = F_{b, extra}$

$m g = V_{extra} \rho g$

$m = 3 \times 2 \times 10^{-2} \times 10^3$

$m = 60 \text{ kg}$

#



$V \rho = V_{in} \sigma$

$A \rho = A \times \sigma$

$\frac{L}{x} = \frac{\sigma}{\rho} = \text{const.}$

Candle burns @ 2 cm/hr.

after 1 hr,

$\frac{L_1}{x_1} = \frac{L_2}{x_2}$

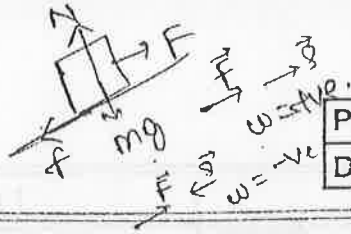
$\frac{2L}{L} = \frac{2L-2}{x_2}$

$x_2 = L-1$

① Length out side falls @ 1 cm/hr.

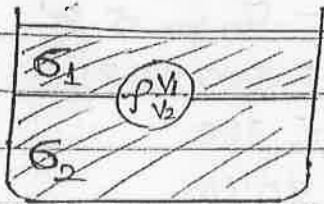
② Length inside also falls @ 1 cm/hr.

WE Theory



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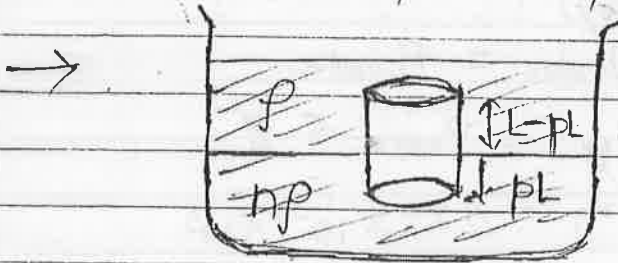


$$mg = V_1 \sigma_1 g + V_2 \sigma_2 g$$

$$V \rho = V_1 \sigma_1 + V_2 \sigma_2$$

$$\sigma_2 > \sigma_1 \quad \sigma_2 > \rho > \sigma_1$$

Q. Two immiscible liquids of density ' $\rho$ ' and ' $n\rho$ ' ( $n > 1$ ), are placed in a container. Both have same ht. ' $h$ '. A cylinder of length ' $L$ ' & density ' $d$ ' floats in them with axis across vertical. If  $pL$  length is in denser med<sup>n</sup> ( $p < 1$ ) find density of cylinder.



$$V \rho = V_1 \sigma_1 + V_2 \sigma_2$$

$$ALd = AL(1-p)\rho + A p L n\rho$$

$$d = (1-p + np)\rho$$

$$d = [1 + p(n-1)]\rho$$

Q. A wooden ball is taken to a depth ' $d$ ' inside water & released. If density of ball ' $\rho$ ' is less than density of water ' $\sigma$ '. upto what ht. ball will jump out.



$$W_g + W_b = 0 - 0$$

$$W_{air} = k_f - k_b$$

$$a_{net} = g \left( \frac{\sigma}{\rho} - 1 \right)$$

$$-V \rho g (h+d) + V \sigma g d = 0$$

$$\sigma d = \rho h + \rho d$$

$$\rho < \sigma \quad v_i = 0$$

$$(\sigma - \rho) d = \rho h$$

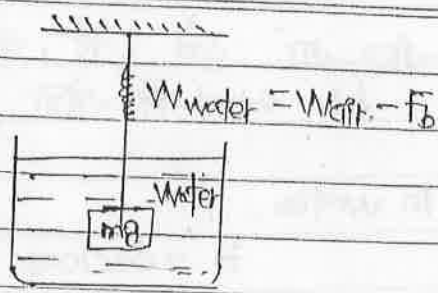
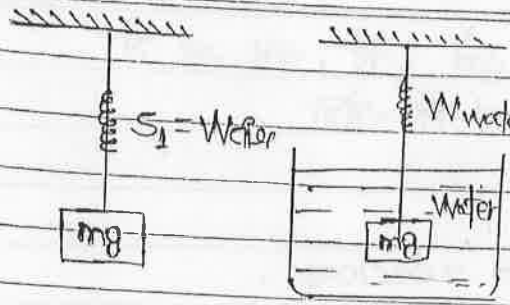
$$v = \sqrt{2gd \left( \frac{\sigma}{\rho} - 1 \right)}$$

$$h = \left( \frac{\sigma - \rho}{\rho} \right) d = d \left( \frac{\sigma}{\rho} - 1 \right)$$

not  $\rightarrow P \rightarrow 2 \text{ km}$   
 Note  $\rightarrow$  all o.  $\text{g.c.}$   
 NCEET - solved  
 Examp.

PAGE NO. iron  
 DATE: / /

Density -



$F_b = W_{air} - W_{water}$

$R.D. = \frac{V \rho g}{V \rho_w g}$

$R.D. = \frac{W_{air}}{F_b \text{ in water}}$  (solid)

$R.D. = \frac{W_{air}}{W_{air} - W_{water}}$

$\rho = R.D. \times \rho_w = R.D. \text{ gm/cc}$   
 $= R.D. \times 10^3 \text{ kg/m}^3$

$R.D. = \frac{V \rho_l g}{V \rho_w g} = \frac{F_b \text{ in liquid}}{F_b \text{ in water}}$

(liquid)

$R.D. = \frac{W_{air} - W_{liquid}}{W_{air} - W_{water}}$

Q. Wt of body in air = 10 N & in water = 8 N & in liquid = 4 N.  
 Find density of body & liquid.

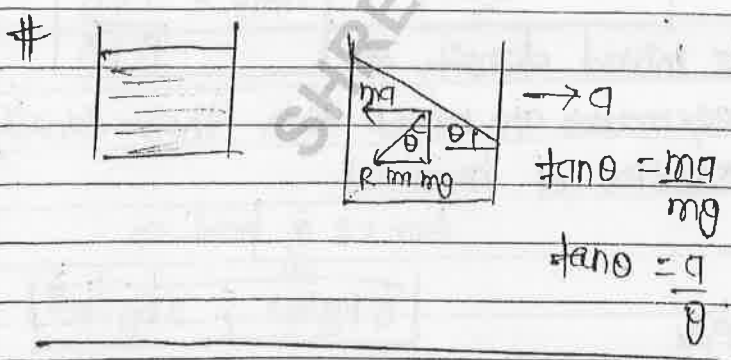
$\rightarrow$  (solid)  $R_{body} = R.D. = \frac{10}{10-8} = \frac{10}{2} = 5$

$\rho = 5 \text{ gm/cc} = 5 \times 10^3 \text{ kg/m}^3$

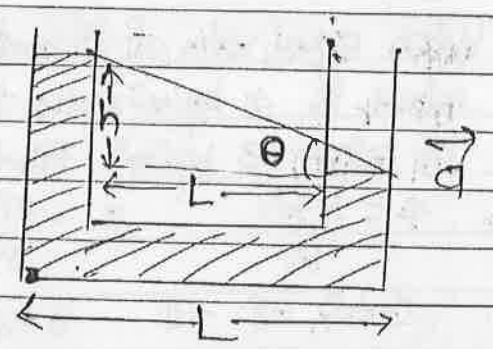
(liquid)

$R.D._{liquid} = \frac{10-4}{10-8} = \frac{6}{2} = 3$

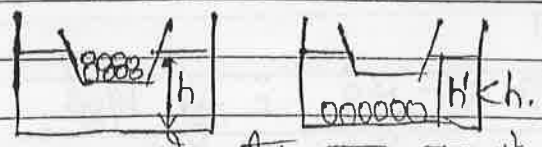
$\rho_{liquid} = 3 \text{ gm/cc} = 3 \times 10^3 \text{ kg/m}^3$



$\tan \theta = \frac{mg \sin \theta}{mg \cos \theta}$   
 $\tan \theta = \frac{g}{\theta}$

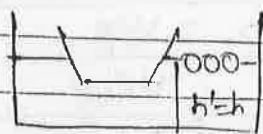


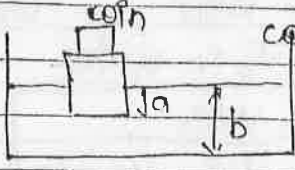
$\tan \theta = \frac{h}{L} = \frac{g}{\theta}$



आ - चीज दूष बार के  
 उरों level कम ही जाता है.

$h = \frac{gL}{\theta}$

\*  जी-चीज जो डूबे :  $\rho_{obj} < \rho_{liq}$ , इस case में  $h$  equal ही रहेगा.

\*  coin slips in water.  
 $b \rightarrow$  decrease.  
 $a \rightarrow$  decrease.

26.9.19

\* 

ICE	$mg = V_i \rho_i g$	$\frac{V_i}{V_f} = \frac{\rho_w}{\rho_i}$	(1) $\rho_w > \rho_i$	(2) $\rho_w < \rho_i$
Liquid	$V_i = \frac{m}{\rho_i}$ — (1)	$\frac{\rho_w}{\rho_i} > 1$	$\frac{\rho_w}{\rho_i} < 1$	
	melt water,	$\frac{V_i}{V_f} > 1$	$V_i < V_f$	Level Hse.
	$V_f = \frac{m}{\rho_w}$ — (2)	$V_i > V_f$	(3) $\rho_w = \rho_i$	$V_i = V_f$

\* Density -  $\rho = \frac{m}{V}$   $\text{kg/m}^3$ .  
 $\text{gm/cc}$ . ( $1 \text{ gm/cc} = 10^3 \text{ kg/m}^3$ )

Level fall. Level remain same.

\* Density of mixture -

$m_1$	$m_2$	$\rho_{mix} = \frac{m_1 + m_2}{V_1 + V_2}$
$V_1$	$V_2$	
$\rho_1$	$\rho_2$	

(1) Equal vol<sup>m</sup> mixed  $V_1 = V_2 = V$   
 $\rho_{mix} = V\rho_1 + V\rho_2$   
 $\rho_{mix} = \frac{\rho_1 + \rho_2}{2}$

(2) Equal masses  $m_1 = m_2 = m$   
 $\rho_{mix} = \frac{m + m}{\frac{m}{\rho_1} + \frac{m}{\rho_2}}$   
 $\rho_{mix} = \frac{2\rho_1\rho_2}{\rho_1 + \rho_2}$

Q. When equal vol<sup>m</sup> of 2 mat. are mixed density of mixer is  $4 \text{ kg/m}^3$ . when their eq. masses are mixed then their density of mixer  $3 \text{ kg/m}^3$ . Find densities of material.

$\rightarrow 4 = \frac{\rho_1 + \rho_2}{2}$        $3 = \frac{2\rho_1\rho_2}{\rho_1 + \rho_2}$        $\rho_1 + \rho_2 = 8$  — (1)       $3 \times 8 = 2\rho_1\rho_2$        $\rho_1\rho_2 = 12$  — (2)

$\rho_1 + \rho_2 = 8$        $\rho_1\rho_2 = 12$

$\rho_1 = 4 + \sqrt{4 - 12}$        $\rho_2 = 4 - \sqrt{4 - 12}$

$\rho_1 = 4 + 2i$        $\rho_2 = 4 - 2i$

$\rho_1 = 6 \text{ kg/m}^3$        $\rho_2 = 2 \text{ kg/m}^3$

\* Relative density -  $R.D. = \frac{\rho_{sub}}{\rho_w}$  unitless, dimensionless.

Specific Wt =  $\frac{mg}{V}$  =  $\rho g \text{ N/m}^3$

Specific Gravity =  $\frac{\rho_{sub}}{\rho_w}$  sp. wt. of water @  $4^\circ\text{C}$  contd.

$\rho_{s.g.} = \frac{\rho}{\rho_w}$