SHM

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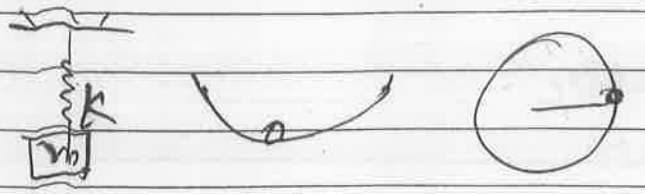
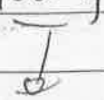
* Periodic (Harmonic) motion

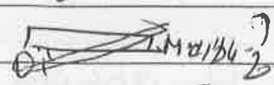


* oscillatory or vibratory motion

↓ Low 'f'



↓ High fr.



*Periodic functions :-

* SHM ⇒ Present in form of 'sin' or 'cos'. have const. amplitude and have only single frequency.

* Periodic but not SHM ⇒ Present in form of "sin" or 'cos' have const. amplitude and have integral multiple freq. of lowest freq.

* Non-periodic function → Present in form of tan, cot, sec. cosec e^t log t etc
or
have variable amplitude
or
have not integral multiple freq.

Q. Find nature of following functions :-

① $y = a\sin(\omega t) \longrightarrow$ SHM

$$\frac{2\pi}{T} = \omega \Rightarrow T = \frac{2\pi}{\omega}$$

② $y = a\sin(2\omega t) \longrightarrow$ SHM

$$\frac{2\pi}{T} = 2\omega \qquad T = \frac{\pi}{\omega}$$

③ $y = 3\sin\omega t + 4\cos\omega t$

$$\boxed{\sin^2\theta = \frac{1-\cos 2\theta}{2}} \qquad \boxed{\cos^2\theta = \frac{1+\cos 2\theta}{2}}$$

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(4) $y = \frac{5\sin\omega t}{6\cos\omega t} = \frac{5}{6}\tan\omega t \rightarrow$ Non-periodic

(5) $y = 1 + \omega t + \omega^2 t^2 \rightarrow$ Non-periodic

(6) $y = \underbrace{a\sin\omega t}_{T} + \underbrace{b\cos\omega t}_{T} \rightarrow$ SHM

$T = \frac{2\pi}{\omega}$

(7) $y = \underbrace{a\sin\omega t}_{\downarrow \, T} + b\underbrace{\cos 2\omega t}_{\downarrow \, T/2} \longrightarrow$ Periodic but not SHM

freq diff.

Time period $= T = \frac{2\pi}{\omega}$

(8) $y = \underbrace{a\sin\omega t}_{T} + \underbrace{b\cos 2\omega t}_{T/2} + \underbrace{c\sin 4\omega t}_{T/4}$ Periodic but not SHM.

$\Rightarrow$ Req: T (4 times) $\quad T/2$ 2 time $\quad T/4$ 1 time

$T = \frac{2\pi}{\omega}$

(9) $y = \log\omega t \rightarrow$ Non-periodic

(10) $y = e^{t} \rightarrow$ Non-periodic

Imp

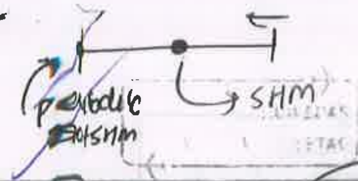
(11) $y = a\sin^2\omega t$

$y = a\left[\frac{1-\cos 2\omega t}{2}\right]$

$y = \frac{a}{2} - \underbrace{\frac{a}{2}\cos 2\omega t}_{\text{SHM}}$

It is SHM about $\frac{a}{2}$ mean position and periodic but not SHM about origin.

*



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$\frac{2\pi}{T} = 2\omega$ $\quad T = \frac{\pi}{\omega}$

(12) $y = a\sin^3\omega t$

$y = a[\sin^2\omega t]\cdot \sin\omega t$

$$y = a\left[\frac{1-\cos 2\omega t}{2}\right]\cdot \sin\omega t$$

$$y = \frac{a}{2}\sin\omega t - \frac{a}{2}\cos 2\omega t \sin\omega t$$

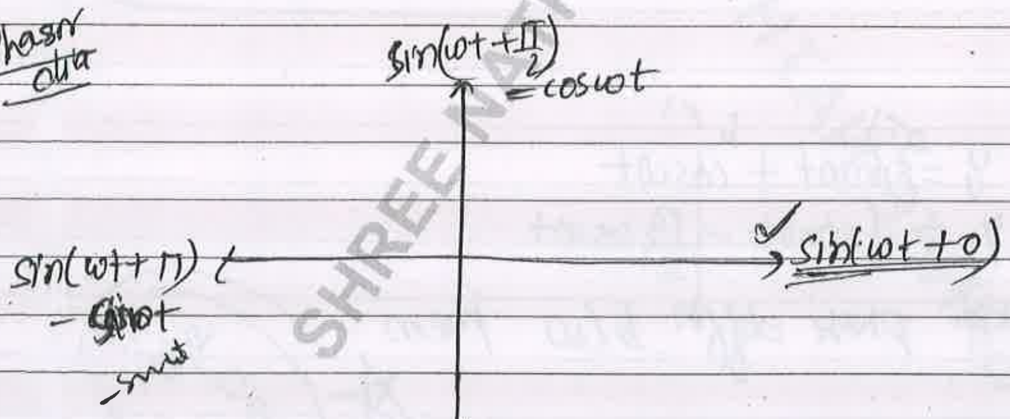
$\Rightarrow$ Periodic but not SHM $\quad T = \frac{2\pi}{\omega}$

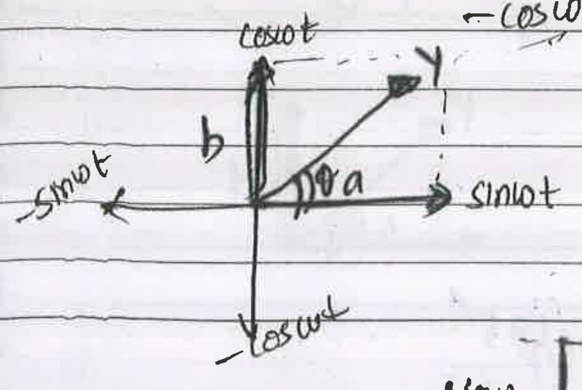
Ncert Q Theory. A function $y = a\sin\omega t + b\cos\omega t$ is given, find it's nature & result form.

Soln $y = a\sin\omega t + b\cos\omega t$

cau $\boxed{y = y_1 + y_2}$

Phasor dia





$\Delta\phi = \frac{\pi}{2}$

$A = \sqrt{a^2+b^2}$

$\tan\theta = \frac{b}{a} \Rightarrow \theta = \tan^{-1}\left(\frac{b}{a}\right)$

Now. $\boxed{y = \sqrt{a^2+b^2}\,\sin\left(\omega t + \tan^{-1}\frac{b}{a}\right)}$

$\sin \omega t \rightarrow \cos(\omega t + \frac{\pi}{2})$

$-\sin \omega t = \sin(\omega t + \pi)$

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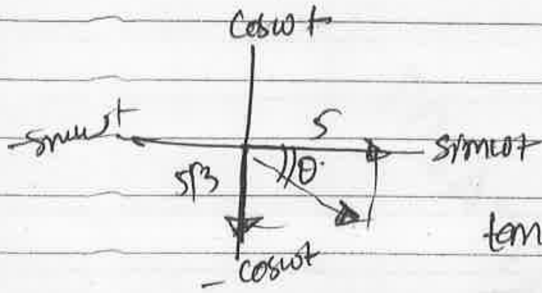
Q. A function $y = 5[\sin \omega t - \sqrt{3}\cos \omega t]$ is given find it's nature and result form.

$$y = 5\sin \omega t + (-5\sqrt{3}\cos \omega t)$$



$\tan\theta = \frac{5\sqrt{3}}{5}$

$\tan\theta = \sqrt{3}$

$\theta = 60 = \frac{\pi}{3}$

$$\boxed{y = 10\sin(\omega t - \frac{\pi}{3})}$$

Q. Two SHM → $y_1 = \overset{a=1}{\sin \omega t} + \overset{b=1}{\cos \omega t}$

$y_2 = \frac{1}{2}\sin \omega t - \frac{\sqrt{3}}{2}\cos \omega t$

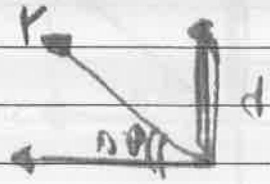
are given, find phase diff^n b/w them.

$\tan\theta = b/a$

Sol^n

$y_1 = \sqrt{1^2 + 1^2}\,\sin(\omega t + \frac{\pi}{4})$

$y_2 = \sqrt{(\frac{1}{2})^2 + (\frac{\sqrt{3}}{2})^2}\,\sin(\omega t - \pi/3)$



$\Delta\phi = \omega t + \frac{\pi}{4} - \omega t + \frac{\pi}{3}$

$= \frac{\pi}{4} + \frac{\pi}{3} = \frac{7\pi}{12}$ rad or 105^0

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SHM

Lineal SHM

$\Rightarrow F \propto -x$

or

$a \propto -x$

$\Rightarrow F = -kx$

Restoring force constant

$K = \frac{F}{x} = N/m$

$\Rightarrow$ ~~Fc~~ $ma = -kx$

$a = -\frac{k}{m}x$

$$\boxed{\frac{d^2x}{dt^2} + \frac{k}{m}x = 0}$$

$\Rightarrow x = A\sin\omega t$

$\frac{dx}{dt} = A\omega\cos\omega t$

$\frac{d^2x}{dt} = -\omega^2 A\sin\omega t$

$$\boxed{\frac{d^2x}{dt^2} + \omega^2 x = 0}$$

Stand. diff^n equ^n of ~~SHM~~ lineal SHM

Comparing.

$\rightarrow \omega^2 = \frac{k}{m}$ $\Rightarrow \boxed{k = m\omega^2}$

$$\boxed{T = 2\pi\sqrt{\frac{m}{k}}}$$

Angular SHM

$\Rightarrow \tau \propto -\theta$

or

$\alpha \propto -\theta$

$\Rightarrow \tau = -C\theta$

Restoring torque const

$C = \frac{\tau}{\theta}\left(\frac{N\text{-}m}{rad}\right)$

$\Rightarrow I\alpha = -C\theta$

$\alpha = -\frac{C}{I}\theta$

$$\boxed{\frac{d^2\theta}{dt^2} + \frac{C}{I}\theta = 0}$$

$\Rightarrow \theta = \theta_0\sin\omega t$

$\frac{d^2x}{dt^2} = -\omega^2\theta_0\sin\omega t$

$$\boxed{\frac{d^2x}{dt^2} + \omega^2\theta = 0}$$

Stand. diff. equ^n of angular SHM

Comparg.

$\omega^2 = \frac{C}{I}$

$\omega = \sqrt{\frac{C}{I}}$

$n = \frac{1}{2\pi}\sqrt{\frac{C}{I}}$

$$\boxed{T = 2\pi\sqrt{\frac{I}{C}}}$$

$T = 2\pi\sqrt{\frac{I}{PE}}$

$T = 2\pi\sqrt{\frac{I}{MB}}$

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Q/ A particle is performing SHM acc. to eqn

$9\frac{d^2x}{dt^2} + 16x = 0$, then find its freq. of motion

coefficient must be 1

$\cancel{\omega^2 = 16}$

$\cancel{\omega = 4}$

$\cancel{2\pi f = 4}$

$\cancel{f = \frac{4}{2\pi} = \frac{2}{\pi}}$

$\frac{d^2x}{dt^2} + \frac{16}{9}x = 0$

comparing with

$\frac{d^2x}{dt^2} + \omega^2 x = 0$

$\omega^2 = \frac{16}{9}$

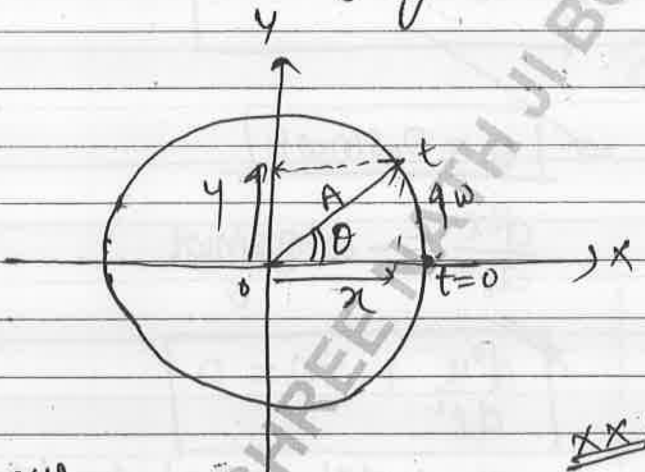
$\omega = \frac{4}{3}$

$2\pi f = \frac{4}{3}$

$f = \frac{4}{2\pi 3}$

$\boxed{f = \frac{2}{3\pi}}$

XXX Geometric meaning of SHM

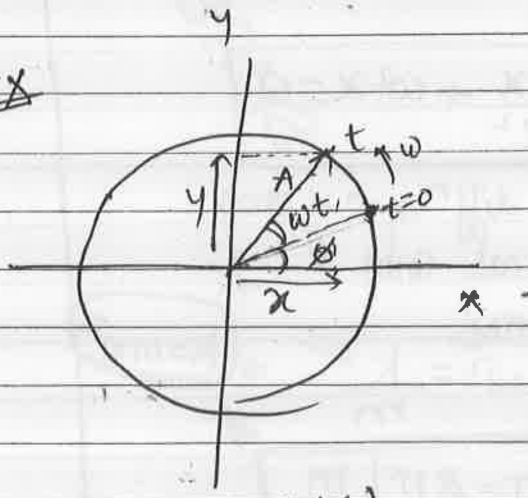


$\frac{\theta}{t} = \omega \qquad \theta = \omega t$

$x = A\cos\theta = A\cos\omega t$

$y = A\sin\theta = A\sin\omega t$

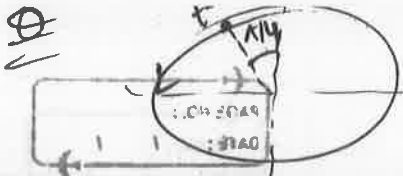
XXX



$x = A\cos(\omega t + \phi)$

$y = A\sin(\omega t + \phi)$ → phase const. or initial phase angle

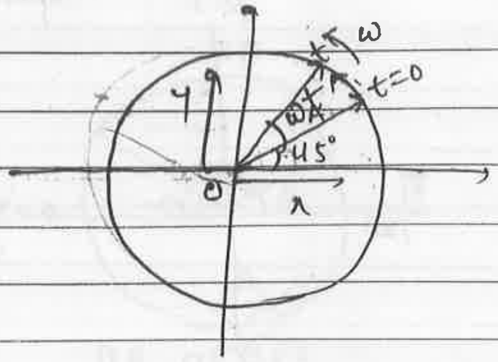
Phase angle at time t



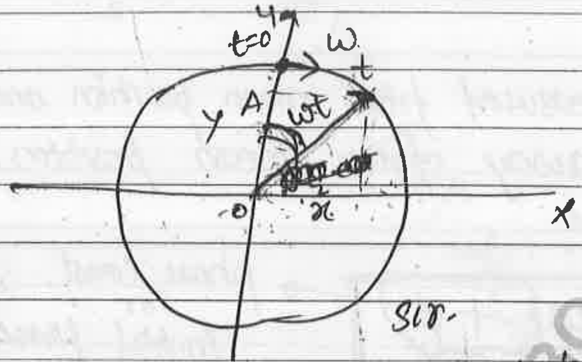
Rad = 5m
T = 4 sec

$\sin(-\theta) = -\sin\theta$
$\cos(-\theta) = \cos\theta$

Q Find disp eq^n on x-axis and y-axis in following condition.

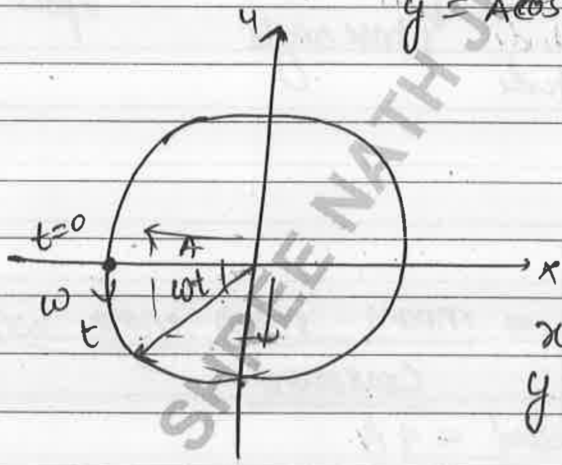
① 

$x = +A\cos(\omega t + 45^\circ)$
$y = +A\sin(\omega t + 45^\circ)$

② 

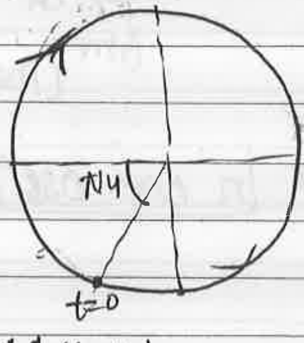
$$\boxed{\begin{aligned} x &= A\cos\left(\tfrac{\pi}{2} - \omega t\right) \\ y &= A\sin\left(\tfrac{\pi}{2} - \omega t\right) \end{aligned}}$$

sir.
$x = A\sin\omega t$
$y = A\cos\omega t$

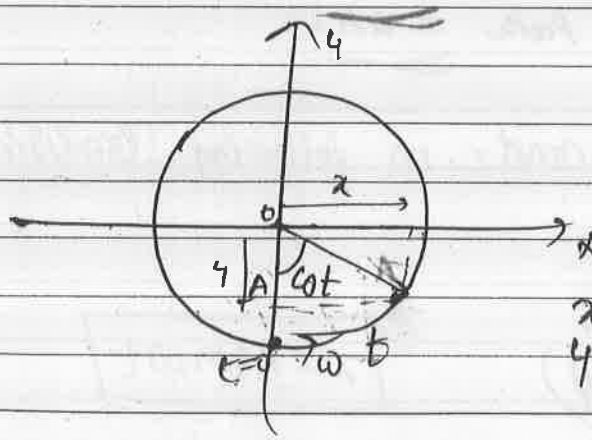
③ 

$x = -A\cos\omega t$
$y = -A\sin\omega t$

AA
R = 2m
T = 8 sec.



x & y disp.

④ 

$x = A\sin\omega t$
$y = -A\cos\omega t$

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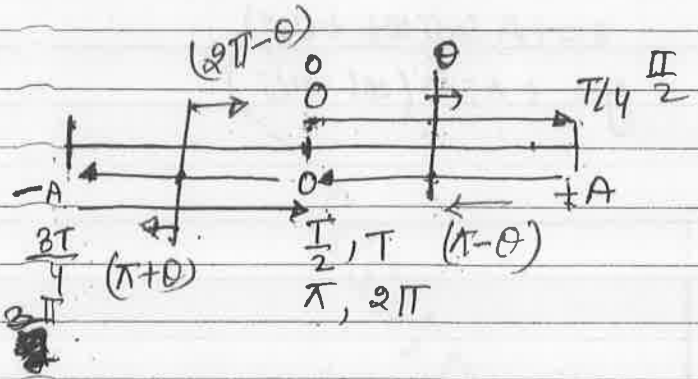
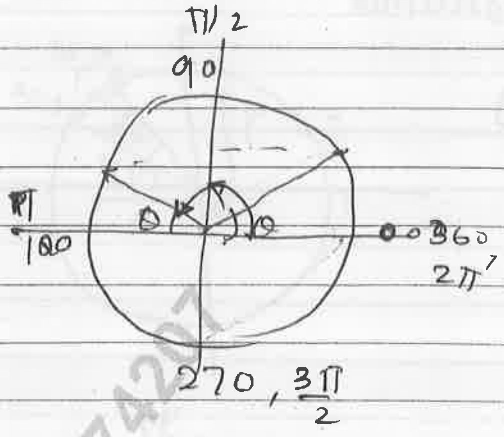
Displacement in SHM :→





* Displacement of particle is measured from mean position and it's dir^n is always away from mean position.
* It's General disp. equ^n →

$$\boxed{x = A\sin(\omega t + \phi)}$$

disp at 't' time (inst. disp) — Amplitude — phase angle — phase const or Initial phase angle or epoch

* In one ossillation →

Net disp = 0

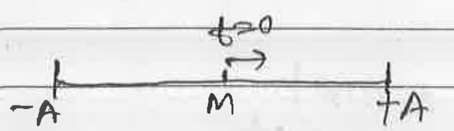
workdone = 0 ⇒ means mech. energy will be conserved

Distance travelled = 4A

Length of Path = 2A ✓

Q: find disp. equ^n & phase const. in following Conditions.

(1)



using $x = A\sin(\omega t + \phi)$

at $t = 0$, $x = 0$

$0 = A\sin(0 + \phi)$

$\sin\phi = 0 \Rightarrow \boxed{\phi = 0}$

$$\boxed{x = A\sin\omega t}$$

$\frac{A}{2} \Rightarrow \frac{\pi}{6}$ $\frac{A}{\sqrt{2}} = \frac{\pi}{4}$ $\frac{\sqrt{3}A}{2} = \frac{\pi}{3}$

$x \rightarrow A$

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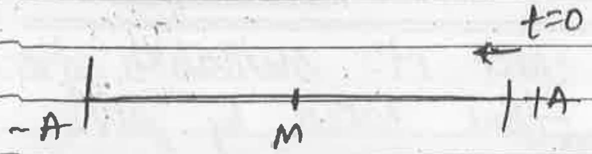
(2)



$x = A\sin(\omega t + \phi)$

$t=0, \quad x = A$

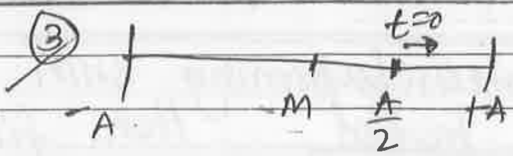
$A = A\sin(0+\phi)$

$1 = \sin\phi \Rightarrow \boxed{\phi = \pi/2}$

* $x = A\sin(\omega t + \frac{\pi}{2})$

$\boxed{x = A\cos\omega t}$

(3)



using

$x = A\sin(\omega t + \phi)$

$\frac{A}{2} = A\sin(0+\phi)$

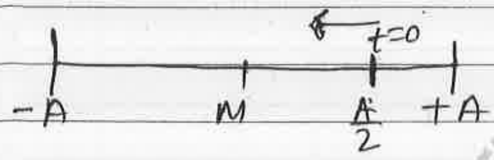
$\frac{1}{2} = \sin\phi$

$\phi = \pi/6$

Now,

$\boxed{x = A\sin(\omega t + \frac{\pi}{6})}$

(4)

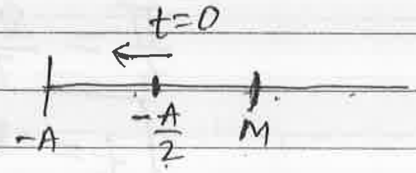


$\theta = \pi/6$

$\phi = (\pi - \theta) = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$

$x = A\sin(\omega t + \frac{5\pi}{6})$

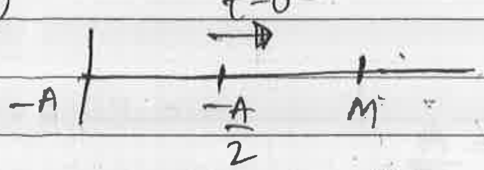
(5)



$\phi = \pi + \theta = \pi + \frac{\pi}{6} = \frac{7\pi}{6}$

$x = A\sin(\omega t + \frac{7\pi}{6})$

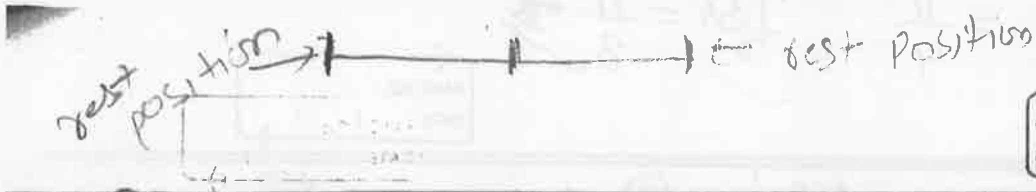
(6)



$\phi = 2\pi - \pi/6 = 11\pi/6$

$x = A\sin(\omega t + 11\pi/6)$

$\boxed{\text{rest position} = \text{Extreme}}$



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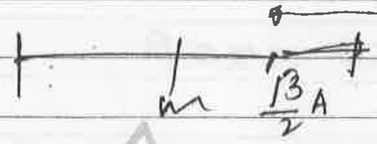
Q. A particle is performing SHM it start it's oscillation from rest position, then find $_{min}$ time taken by particle to reach $\frac{\sqrt{3}}{2}$ time's of Amplitude position.

Sol^{n}

Rest position = Extreme position



using $x = A\cos\omega t$

$$\frac{\sqrt{3}}{2}A = A\cos\left(\frac{2\pi}{T}t\right)$$

$$\frac{\sqrt{3}}{2} = \cos\left(\frac{2\pi}{T}t\right)$$

$$\cos\left(\frac{\pi}{6}\right) = \cos\left(\frac{2\pi}{T}t\right)$$

$$\frac{\pi}{6} = \frac{2\pi}{T}t$$

$$\boxed{t = \frac{T}{12}}$$

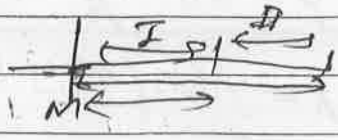
Q. A particle is performing SHM it start it's oscillation from mean position if time period of motion is 8 seconds. Then find ratio of distance travelled by particle in 1 second to 2nd second.

using

$$x = A\sin\omega t$$

$$x_1 = x_I = A\sin\left(\frac{2\pi}{8}\times 1\right) = \frac{A}{\sqrt{2}}$$

$$x_2 = A\sin\left(\frac{2\pi}{8}\times 2\right) = A$$



$$x_{II} = x_2 - x_1$$

$$= A - \frac{A}{\sqrt{2}} = \frac{A}{\sqrt{2}}(\sqrt{2}-1)$$

$$\frac{x_1}{x_2} = \frac{\frac{A}{\sqrt{2}}}{\frac{A}{\sqrt{2}}(\sqrt{2}-1)} = \frac{1}{\sqrt{2}-1} = \sqrt{2}+1$$

Que: A ball is performing SHM in a hollow cylinderical tube of length 4 cm. it completes 100 osscillation in 4 second. if it's starts it's osscillation from a point having distance 70.7% of Amplitude from mean position. Then find displacement eqn.

Sol

50% of Amp $= \frac{50}{100}A = \frac{A}{2}$

70.7% of Amp $= \frac{70.7}{100}A = 0.707A = \frac{A}{\sqrt{2}}$

86.6% of Amp $= \frac{\sqrt{3}}{2}A$

$\Rightarrow \phi = \frac{\pi}{4}$

$\Rightarrow n = \frac{\text{No. of osscill}}{\text{time}} = \frac{100}{4} = 25\,Hz$

$\Rightarrow$ length of path $= 2A = 4\,cm$

$A = 2\,cm$

$x = A\sin(\omega t + \phi)$

$x = 2\sin\left(50\pi t + \frac{\pi}{4}\right)$

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ϕ Phase diff. (lead/lag)

* Velocity in SHM

$$x = A\sin(\omega t+\phi)$$

$$v = \frac{dx}{dt} = A\omega\cos(\omega t+\phi)$$

**

$$\boxed{v = A\omega\cos(\omega t+\phi)}$$

$$\boxed{V_{max} = A\omega}$$

$$V = \pm A\omega\sqrt{1-\sin^2(\omega t+\phi)}$$

$$V = \pm A\omega\sqrt{1-\frac{x^2}{A^2}}$$

*

$$\boxed{V = \pm\omega\sqrt{A^2-x^2}}$$

* Accl^n in SHM → imp.

$$x = A\sin(\omega t+\phi) = A\sin(\omega t+\phi)$$

$$V = A\omega\cos(\omega t+\phi) = A\omega\sin(\omega t+\phi+\pi/2)$$

$$a = -\omega^2 A\sin(\omega t+\phi) = \omega^2 A\sin(\omega t+\phi+\pi)$$

$$\boxed{a = -\omega^2 x}$$

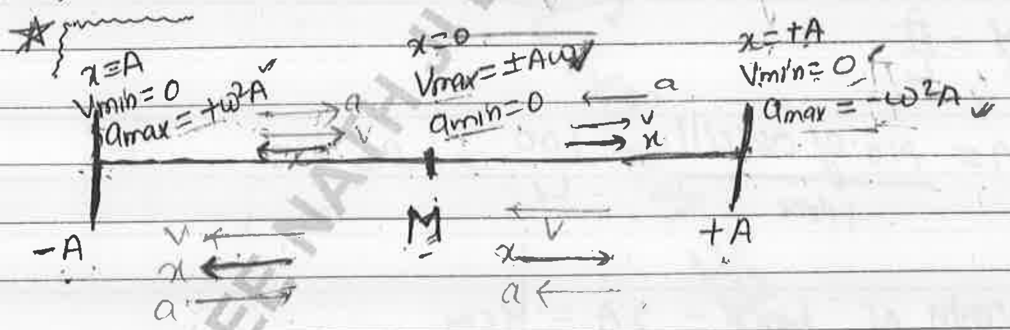
if $x=0$ (Mean) then $a_{min}=0$

$x=\pm A$ then $a_{max} = \mp\omega^2 A$

if $x=0$, (mean) then $V_{max} = \pm A\omega$

$x = \pm A$ (Extreme) than $V_{min}=0$

*



* Dir^n of velocity changes at extreme positions but dir^n of 'x' 'a' and 'F' change at mean position.

Q. A particle is performing SHM acc. to eq^n $x = A\sin(\omega t+\phi)$ if it's initial displacement is 1cm and initial velocity is π cm/sec, then find amplitude and phase const(ϕ).

$$x = A\sin(\omega t+\phi)$$

at $t=0$ $x=1$cm

$$1 = A\sin(0+\phi)$$

$$\boxed{1 = A\sin\phi} \quad (1)$$

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Now $v = \frac{dx}{dt} = A\pi\cos(\pi t + \phi)$

at $t=0 \Rightarrow V = \pi$ cm/s.

$\pi = A\pi \cos(0+\phi)$

$1 = A\cos\phi$ — (II)

from (I) & (II)

$\frac{1}{1} = \frac{A\sin\phi}{A\cos\phi}$

$\tan\phi = 1 \Rightarrow \phi = \pi/4$

$A = \sqrt{2}$ cm

OR

$V = \pm\omega\sqrt{A^2 - x^2}$

This Que take time to solve.

$\pi = \pm\pi\sqrt{A^2 - 1^2}$

$1 = A^2 - 1 \Rightarrow \boxed{A = \sqrt{2}\text{cm}}$

by diff eq. $\phi = \pi/4$

Que A particle is perf SHM of Time period 6sec. If at t=1sec it is at mean position and at t=2 sec it's velocity is $\frac{1}{4}$ m/s. then find Amplitude of motion.

Soln using,

$x = A\sin(\omega t + \phi)$

$t = 1$ sec $x = 0$.

$0 = A\sin\left(\frac{2\pi}{6}\times 1 + \phi\right)$

$0 = \sin\left(\frac{\pi}{3} + \phi\right)$

$\frac{\pi}{3} + \phi = 0$

$\phi = -\pi/3$

Now,

$x = A\sin\left(\frac{\pi}{3}t - \frac{\pi}{3}\right)$

Now

$V = \frac{dx}{dt} = A\frac{\pi}{3}\cos\left(\frac{\pi}{3}t - \frac{\pi}{3}\right)$

at $t = 2$sec $\Rightarrow V = \frac{1}{4}$ m/s

$\frac{1}{4} = A\frac{\pi}{3}\cos\left(\frac{\pi}{3}\times 2 - \pi/3\right)$

$\frac{1}{4} = \frac{A\pi}{3}\times\frac{1}{2}$

$A = \frac{3}{2\pi}$ m

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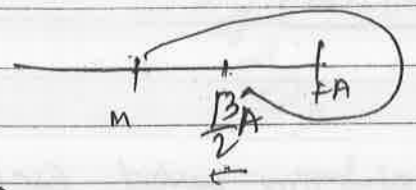
Q A particle is performing SHM, it's phase angle at an instant is 840° then find
① Dirⁿ of motion
② Inst. displacement
③ total distance.

$840^\circ = 720 + 120$
⇓
2 oscillation

① from +ve extreme towards mean.

② $x = A\sin(\omega t + \phi)$ $(\omega t + \phi = 120^\circ)$
$x = A\sin(120)$
$x = \frac{\sqrt{3}}{2}A$

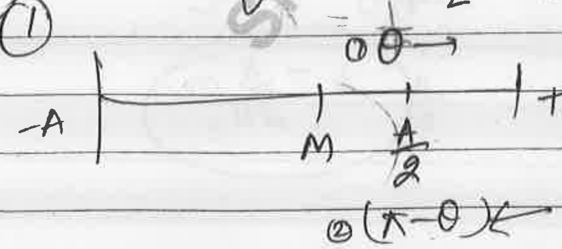


③ Distance $= 4A + 4A + A + \left(A - \frac{\sqrt{3}}{2}A\right)$
$= 10A - \frac{\sqrt{3}}{2}A$

amb ★★
AIPMT

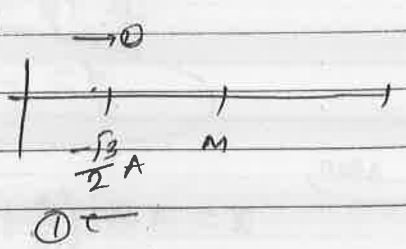
Q Two particles are performing SHM on two ∥ lines which with same amplitude & same frequency. during the oscillation they cross each other moving in opp. dirⁿ. find phase difference b/w them if they cross each other at ① $\frac{A}{2}$ position,
② $-\frac{\sqrt{3}}{2}A$ position

①



① $\rightarrow \theta = \pi/6$
② $\rightarrow \pi - \theta = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$
$\Delta\phi = \frac{5\pi}{6} - \frac{\pi}{6} = \frac{2\pi}{3}$ rad

②



① $\rightarrow \pi + \theta = \pi + \frac{\pi}{3} = \frac{4\pi}{3}$
② $\rightarrow 2\pi - \theta = 2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$
$\Rightarrow \Delta\phi = \pi/3$

~~Revision~~

~~HT Revision~~

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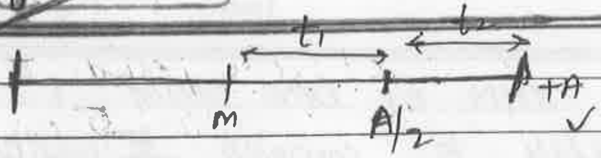
Q



find ratio of

$\frac{t_1}{t_2}$

Soln $t_1 < t_2$ → speed ↓ toward ward

using $x = A\sin\omega t$

$\frac{A}{2} = A\sin\omega t$

$\sin(\frac{\pi}{6}) = \sin(\frac{2\pi}{T}t_1)$

$\frac{\pi}{6} = \frac{2\pi}{T}t_1$

$\boxed{t_1 = \frac{T}{12}}$

(I) Now

$t_2 = \frac{T}{4} - t_1$

$= \frac{T}{4} - \frac{T}{12}$

$\boxed{t_2 = \frac{T}{6}}$

(II) using $x = A\cos(\frac{2\pi}{T}t_2)$

$\frac{A}{2} = A\cos(\frac{2\pi}{T}t_2)$

$\cos\frac{\pi}{3} = \cos(\frac{2\pi}{T}t_2)$

$t_2 = \frac{T}{6}$

(III) using, assum start from A/2

$x = A\sin(\omega t + \frac{\pi}{6})$

$A = A\sin(\frac{2\pi}{T}t_2 + \pi/6)$

$\sin\pi/2 = \sin(\frac{2\pi}{T}t_2 + \frac{\pi}{6})$

$t_2 = T/6$

~~Query~~ (IV) $\boxed{2\pi = T}$ $\Rightarrow \frac{\Delta\theta}{2\pi} = \frac{\Delta T}{T}$

* $\frac{\pi/6}{2\pi} = \frac{\Delta T}{T} \Rightarrow \Delta T = \frac{T}{12}$

Now, $\frac{\pi/3}{2\pi} = \frac{\Delta T}{T} \Rightarrow \Delta T = T/6$

ratio of phase diff = time

Ans $\frac{t_1}{t_2} = \frac{1/12}{1/6} = \frac{1}{12} \times \frac{6}{1} = \frac{1}{2}$ $\boxed{1:2}$

ex-1

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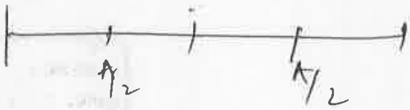
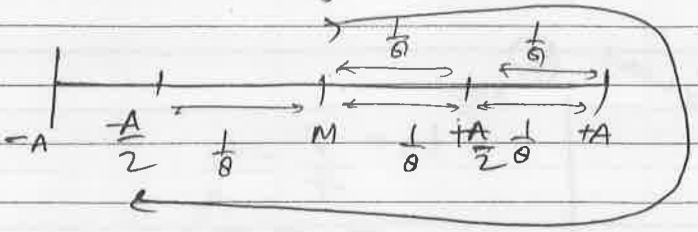


Q. A particle is performing SHM of time period 'T'. find time taken by particle to complete $\frac{5}{8}$ oscillation.



Sol^n

$$t = \frac{T}{4} + \frac{T}{4} + \frac{T}{12} = \frac{7T}{12}$$

or

$$\frac{7\pi/6}{2\pi} = \frac{\Delta T}{T} \Rightarrow \Delta T = \frac{7T}{12}$$

AIIMS

Q. A particle is performing SHM if it's velocities 10 cm/sec and 8 cm/sec at displacement 4cm & 5cm respectively then find Amplitude and time period of motion.

using

$$V = \pm\omega\sqrt{A^2 - x^2}$$

$$10 = \pm\omega\sqrt{A^2 - 16} \quad \text{—(I)}$$

$$8 = \pm\omega\sqrt{A^2 - 25} \quad \text{—(II)}$$

Squaring,

$$100 = \omega^2 A^2 - 16\omega^2$$

$$64 = \omega^2 A^2 - 25\omega^2$$

$$- \quad - \quad +$$

$$36 = 9\omega^2$$

$$\omega = 2 \text{ rad/sec}$$

$$\Rightarrow \frac{2\pi}{T} = 2 \Rightarrow T = \pi \text{ sec}$$

from eq(I)

$$10 = \pm\omega\sqrt{A^2 - 16}$$

$$10 = \pm 2\sqrt{A^2 - 16}$$

$$\boxed{A = \sqrt{41}}$$

Q. A particle is performing SHM of Time period 12 sec. it's start it's oscillation from mean position, then find min^m time after which velocity of particle becomes $\frac{\sqrt{3}}{2}$ times of max^m velocity. (1 sec)

sol^n

$$V = A\omega \cos \omega t$$

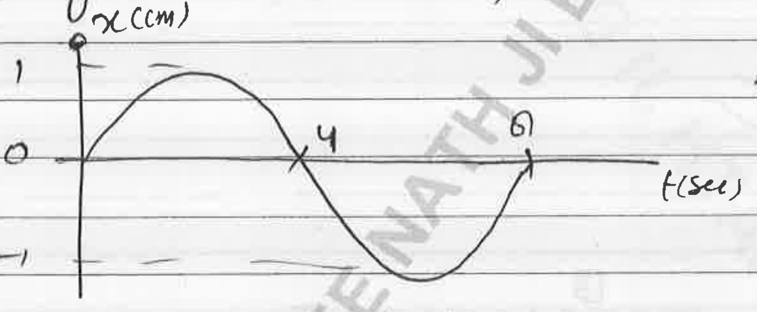
$$\frac{\sqrt{3}}{2} A\omega = A\omega \cos \omega t$$

$$\cos \frac{\pi}{6} = \cos \omega t$$

$$\frac{\pi}{6} = \frac{2\pi}{12} \times t$$

$$\boxed{t = 1 \text{ sec}}$$

Q. A particle is performing SHM it's displacement-time graph is given below. find acc^n of particle at $t = \frac{4}{3}$ sec



at $t = 4/3$ sec

soln,

$$x = A \sin \omega t$$

$$x = 1 \sin \left(\frac{2\pi}{8} t\right)$$

$$x = 1 \sin \left(\frac{2\pi}{8} t\right)$$

$$x = \sin\left(\frac{\pi}{4} t\right)$$

$$\text{acel}^n = \frac{d^2x}{dt^2} = -\frac{\pi^2}{16} \sin\left(\frac{\pi}{4} t\right)$$

$t = \frac{4}{3}$ sec,
$$a = -\frac{\pi^2}{16} \sin\left(\frac{\pi}{4} \times \frac{4}{3}\right)$$
$$= -\sqrt{3}\pi^2/32 \text{ cm/sec}^2$$

Q. A particle is performing SHM. ~~plot~~ if it start it's oscillation from mean position then plot (1) $(x-t)$, (2) $(v-t)$, (3) $(a-t)$, (4) $(F-t)$, $(v-x)$, $(a-x)$, and $(F-x)$ graph.

* $x = A\sin\omega t$
* $v = A\omega\cos\omega t$
* $a = -\omega^2 A\sin\omega t \Rightarrow \boxed{a = -\omega^2 x}$

* $F = ma = -m\omega^2 A\sin\omega t = -KA\sin\omega t$

$\left(\omega^2 = \frac{k}{m}\right) \Rightarrow \boxed{F = -m\omega^2 x = -kx}$

* $v = \pm\omega\sqrt{A^2 - x^2}$

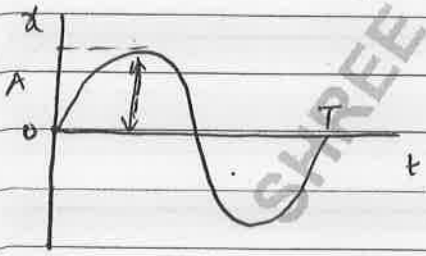
$v^2 = \omega^2 A^2 \; \omega^2 x^2$

$v^2 + \omega^2 x^2 = \omega^2 A^2$

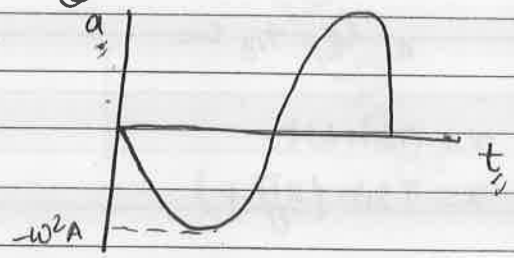
divide by $\omega^2 A^2$

$$\boxed{\frac{v^2}{A^2\omega^2} + \frac{x^2}{A^2} = 1}$$

(1)



(3)



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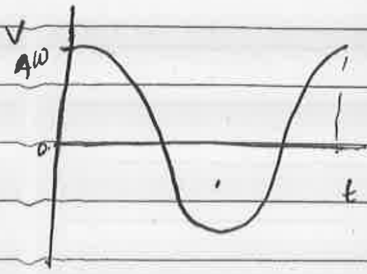

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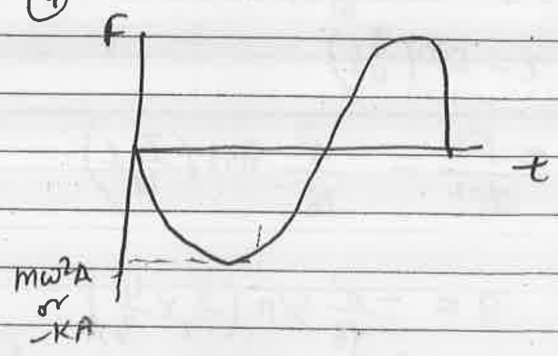

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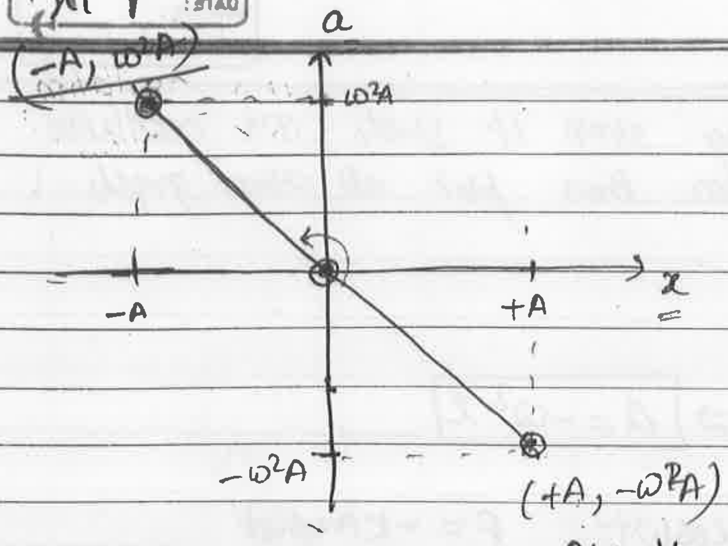
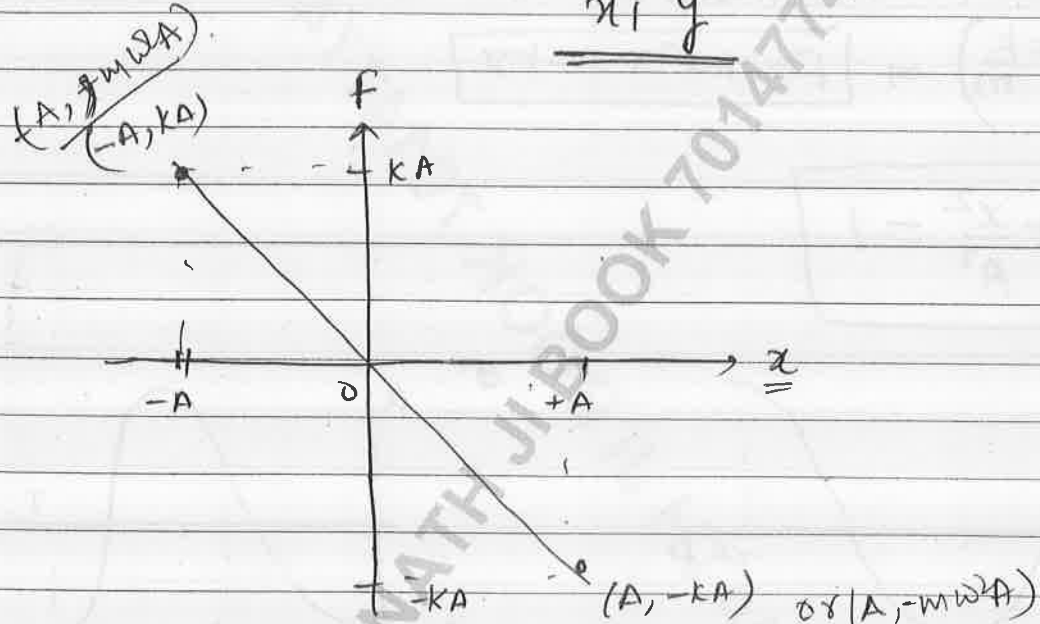
x, y

(5)



x, y

(6)



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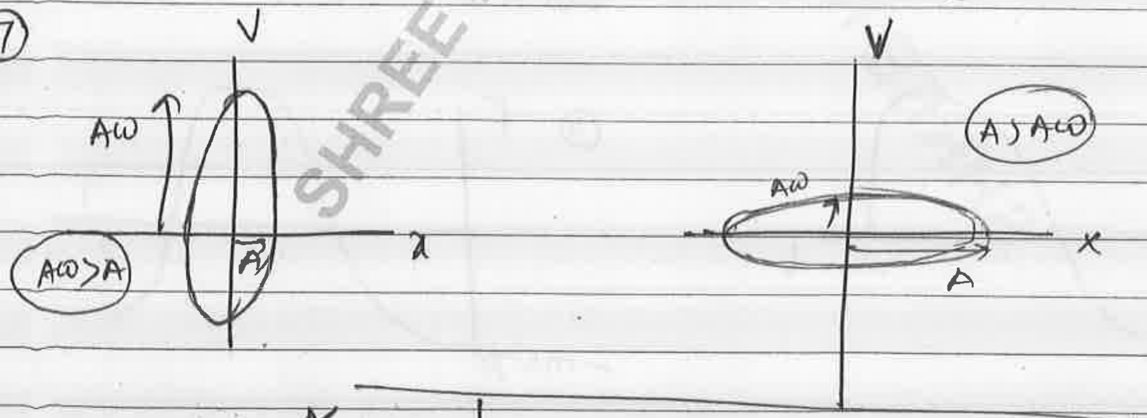
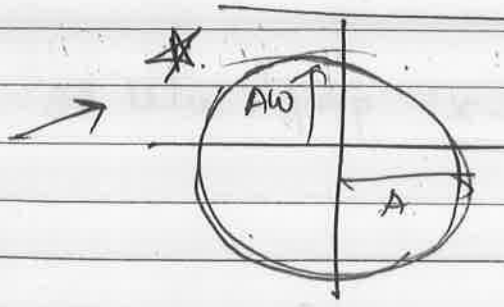




$A\omega = A \Rightarrow$ circle

$\Rightarrow \omega = 1$ rd/sec

$\Rightarrow T = 2\pi = 6.28$ sec

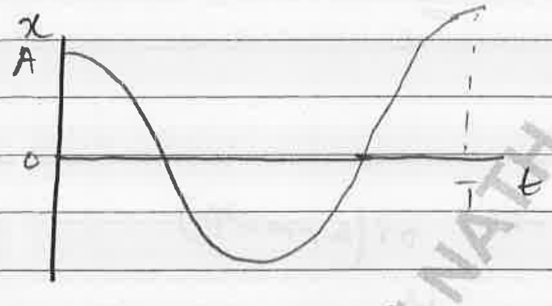
Q A particle is performing SHM. It starts it's oscillation from extreme position then plot all above graphs.

* $x = A\cos\omega t$
* $V = -A\omega \sin\omega t$
* $a = -\omega^2 A\cos\omega t \Rightarrow \boxed{a = -\omega^2 x}$

$F = ma = -m\omega^2 A\cos\omega t \qquad F = -kA\cos\omega t$

$\left(\omega^2 = \frac{k}{m}\right) \Rightarrow \boxed{F = m\omega^2 x = -kx}$

* $\boxed{\frac{V^2}{(A\omega)^2} + \frac{x^2}{A^2} = 1}$

(1)


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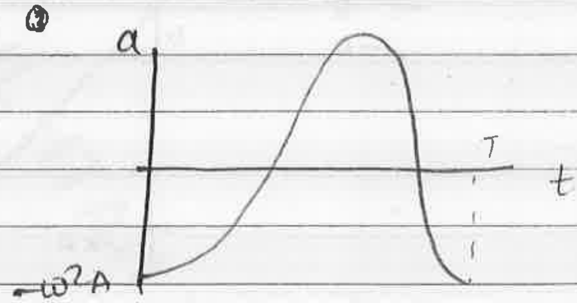

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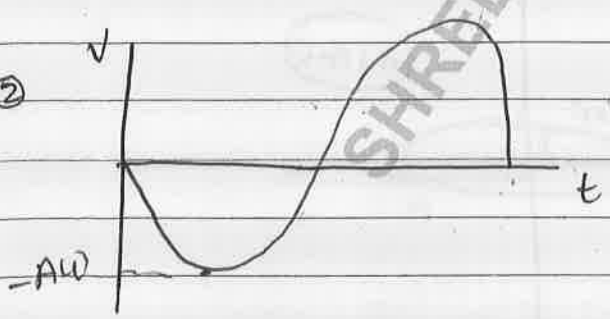

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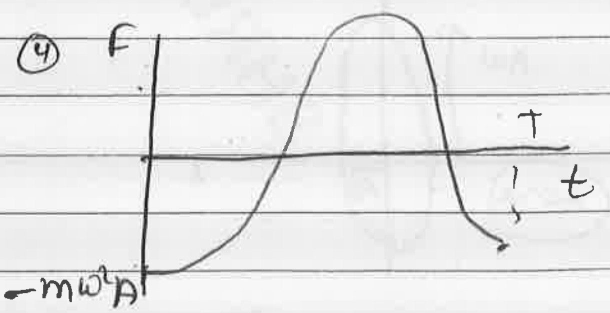
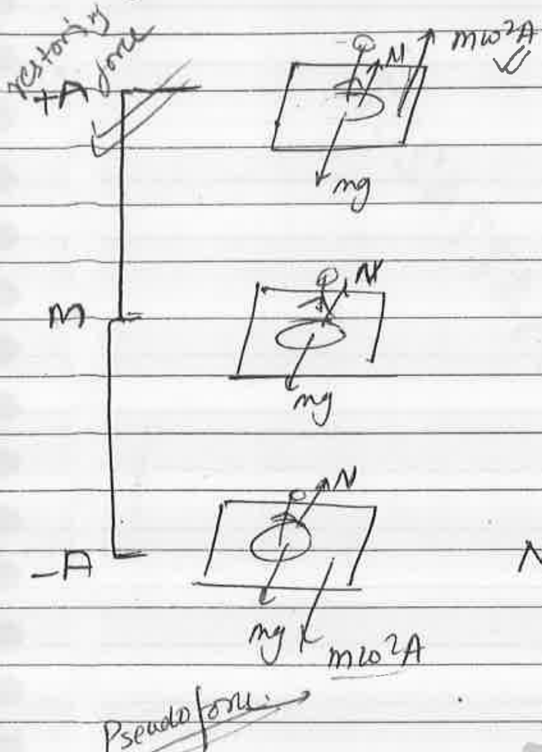


But 'V-x', 'a-x' and 'F-x' graph will be same as before.

Q. A person of 'm' stands on a weighing machine which is placed on a vertically oscillating platform. Platform is performing vertical SHM of angular freq 'ω' and amplitude "A". During the oscillation reading of weighing machine is regularly changed.



$N_{min} = mg - m\omega^2 A$

$N = mg$

$N_{max} = mg + m\omega^2 A$

Condition for lossing contact :

$m\omega^2 A \geq mg$

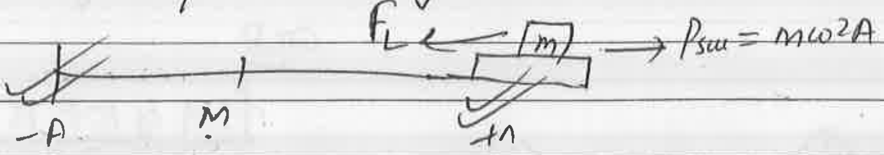
$\omega^2 A \geq g$

$\omega \geq \sqrt{\frac{g}{A}}$

$\boxed{n \geq \frac{1}{2\pi}\sqrt{\frac{g}{A}}}$

$\boxed{T \leq 2\pi\sqrt{\frac{A}{g}}}$

Q. A body is resting on a horizontal slab which is moving with Horizontal SHM of freq. 2 Hz. If coeff. of static friction b/w body and slab is 0.5 then find max^m amp. of motion upto which body does not slip along the slab.



condition for not slipping

$f_L \geq m\omega^2 A$

$\mu mg \geq m\omega^2 A$

$\mu g \geq \omega^2 A \Rightarrow$

$A \leq \frac{\mu g}{\omega^2}$

$A_{max} = \frac{\mu g}{\omega^2} = \frac{0.5 \times 10}{4\pi^2 \times 4}$

$= \frac{1}{32}$ m

Energy in SHM :

* K.E in SHM (K) →

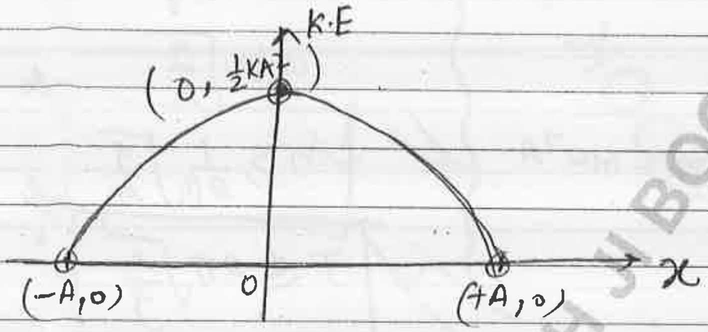
$$K.E = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\pm\omega\sqrt{A^2-x^2}\right)^2$$

$$= \frac{1}{2}m\omega^2(A^2-x^2)$$

$$\boxed{K.E = \frac{1}{2}k(A^2-x^2)}$$

$x=0$ (mean) then $K.E_{max} = \frac{1}{2}KA^2$

$x=\pm A$ (extreme then $K.E_{min} = 0$



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* $K.E = \frac{1}{2}mv^2 = \frac{1}{2}m(A\omega\cos\omega t)^2$

$$= \frac{1}{2}m\omega^2A^2\cos^2\omega t$$

$$\boxed{K.E = \frac{1}{2}KA^2\cos^2\omega t}$$

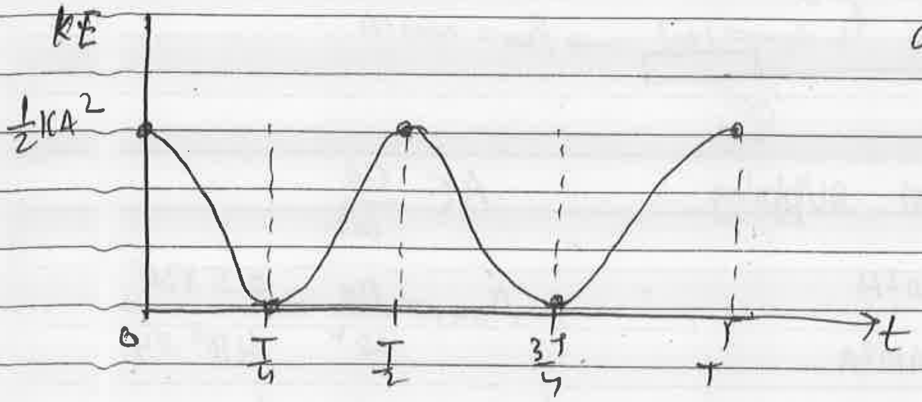
$x = A\sin\omega t$

$V = A\omega\cos\omega t$

$a = -\omega^2A\sin\omega t$

$F = -m\omega^2A\cos\omega t$

$K.E = \frac{1}{2}KA^2\left(\frac{1+\cos 2\omega t}{2}\right)$



conb

freq. of K.E is double of freq. of velocity or disp

P.E in SHM (U) →

$$F = -\frac{dU}{dx}$$

$$\boxed{F\,dx = -dU}$$

conservative

$$-\int kx\,dx = -\int dU \quad (F = -kx)$$

$$\boxed{U = \frac{1}{2}kx^2 + C}$$

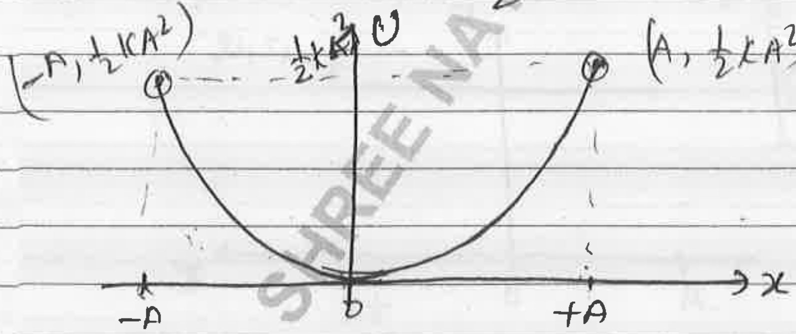
* if $x=0 \Rightarrow U=U_0 \Rightarrow C=U_0$ then $U = \frac{1}{2}kx^2 + U_0$ ✓

if $x=0 \Rightarrow U=0 \Rightarrow C=0$ then $U = \frac{1}{2}kx^2$ ✓

* $U = \frac{1}{2}kx^2$

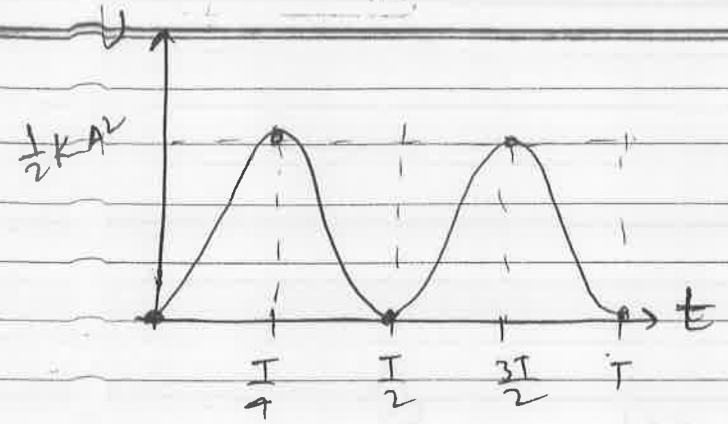
if $x=0 \Rightarrow U_{min}=0$

$x = \pm A \Rightarrow U_{max} = \frac{1}{2}kA^2$



* $U = \frac{1}{2}kx^2 = \frac{1}{2}k(A\sin\omega t)^2$

$$\boxed{U = \frac{1}{2}kA^2\sin^2\omega t}$$



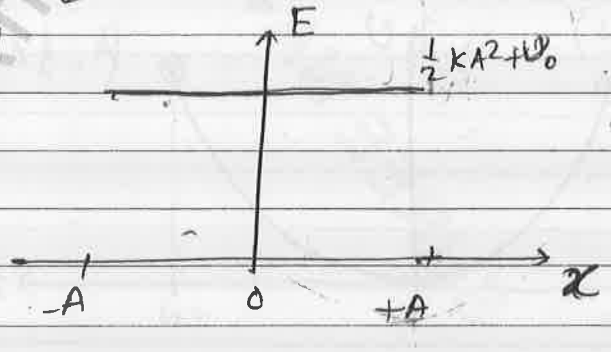
* freq. of P.E is double of freq. of disp. of velocity.

⇒ Total Mechanical Energy (T.E or E)

$E = U + K$

$= \frac{1}{2}kx^2 + U_0 + \frac{1}{2}k(A^2 - x^2)$

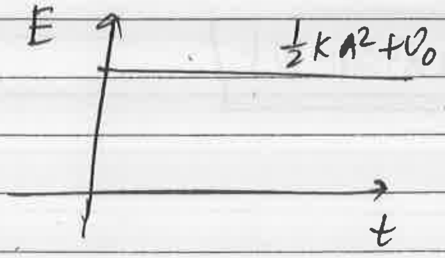
$$\boxed{E = \frac{1}{2}KA^2 + U_0}$$



* $E = U + K$

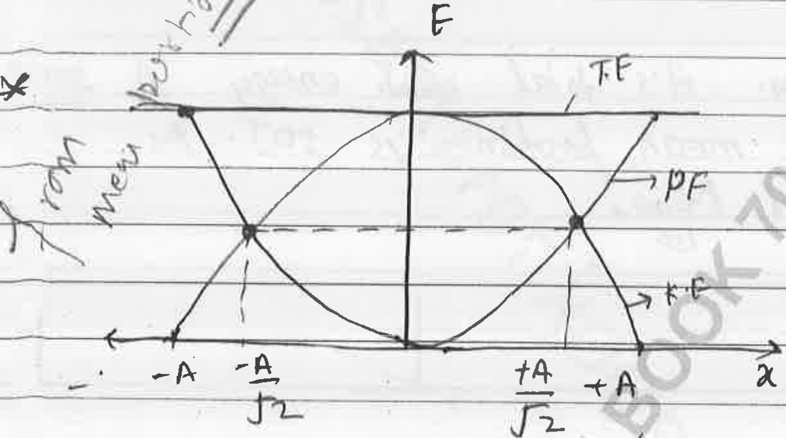
$= \frac{1}{2}KA^2\sin^2\omega t + U_0 + \frac{1}{2}KA^2\cos^2\omega t$

$$\boxed{E = \frac{1}{2}KA^2 + U_0}$$



• At mean position → total mech. energy present in form max^m K.E $\left(\frac{1}{2}KA^2\right)$ and min^m P.E (U_0 or may be zero)

• At extreme position :→ total mech energy present in form of 'PE' $\left(\frac{1}{2}KA^2 + U_0 \text{ or may be zero}\right)$ and it is c/a max^m P.E.



(JTT) from extreme or mean. $U=\frac{1}{2}Kx^2$ reverse of it accordant

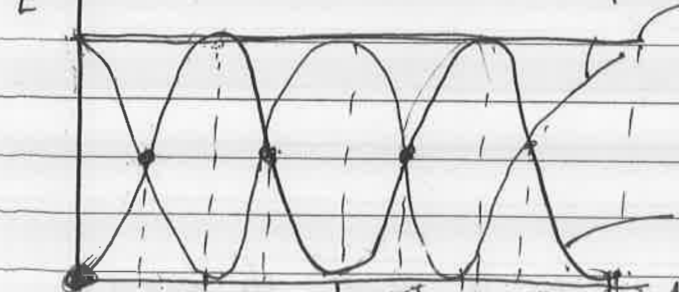
$U = K$

$\frac{1}{2}Kx^2 = \frac{1}{2}K(A^2 - x^2)$

$2x^2 = A^2$

$\boxed{x = \frac{A}{\sqrt{2}}}$

U v/s mean



at which K.E = P.E

$\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

$t = \frac{T}{8}, \frac{3T}{8}, \frac{5T}{8}, \frac{7T}{8}$

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$\rightarrow$ $T.E = \frac{1}{2} K \underset{\uparrow}{A^2}$ max^m disp. [T.E independent 'x']

$= \frac{1}{2} m\omega^2 A^2$

$= 2\pi^2 n^2 m A^2$

$= \frac{2\pi^2 m A^2}{T^2}$

Q. A particle is performing SHM. it's total mech energy is 200J if P.E of particle at mean position is 50J. the find U_{max}, U_{min} and $K.E_{max}$
50 150

Sol^n $U_{min} = U_0 = 50J$

$U_{max} = T.E.E = 200J$

$K_{max} = 150J$.

Q. A simple harmonic ossilator of force const 3×10^6 N/m and amplitude 0.01m has total mechanical energy 210J, then find U_{min}, U_{max} & $K.E_{max}$.

$K.E_{max} = \frac{1}{2} KA^2$

$= \frac{1}{2} \times 3\times10^6 \times (0.01)^2 = 150J$

$U_{max} = T.M.E = 210$

$\Rightarrow U_{min} = 60J$

Que. A particle is performing SHM if it starts it's ossilation from extreme position, then find ratio of potential and K.E at $\frac{T}{6}$.

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$$\Rightarrow \frac{P.E}{K.E} = \frac{\frac{1}{2}K(A^2\cos^2\omega t)}{\frac{1}{2}K(A^2\sin^2\omega t)} = \cot^2\omega t$$

$$= \cot^2\left(\frac{2\pi}{T}\times\frac{T}{6}\right)$$

$$= \left(\frac{1}{\sqrt{3}}\right)^2 = \frac{1}{3}$$

or

$$x = A\cos\left(\frac{2\pi}{T}\times\frac{T}{6}\right) = \frac{A}{2}$$

$$P.E = \frac{\frac{1}{2}K\left(\frac{A}{2}\right)^2}{\frac{1}{2}K\left(A^2 - \frac{A^2}{4}\right)} = \frac{1}{3}$$

Que A particle of mass 1kg is performing SHM along x-axis it's potential energy is given by $U = (x^2 - 4x + 5)$ J then find.

① Mean position ($F=0$, or U_{min} or partial too zero)
(II) Time period of motion
(III) If max^m speed of particle at mean position is $3\sqrt{2}$ m/s then amplitude.
(IV) U_{min}
(V) U_{max}
(VI) $K.E_{max}$.

① $F = -\frac{dU}{dx} = \frac{-d(x^2-4x+5)}{dx}$

$$\boxed{F = -2x + 4}$$

at mean position $F=0$

$-2x+4=0$

$x = 2m$ → mean position

② $F = -2x+4$

$$\boxed{F = -2(x-2)}$$

Comparing with $F = -kX$

$k = 2$ N/m

$X = (x-2) = 0$

$x = 2m$ — mean pos.

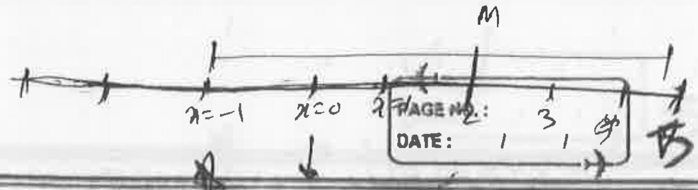
Now

$K = m\omega^2 = 2$

$\omega^2 = 2$

$\omega = \sqrt{2}$ rad/sec

$\Rightarrow T = (\sqrt{2}\pi)$ sec



(3) $A\omega = 3\sqrt{2}$

$A(\sqrt{2}) = 3\sqrt{2}$

$\Rightarrow A = 3m$

(4) $U_{min} \Rightarrow$ at mean,

$U_{x=2m}$

$= x^2-4x+5$

$= (2)^2-4\times2+5 = 1\ J$

(5) $U_{max} \Rightarrow$ at extreme

$U_{x=5m} = U_{x=-1m}$

$U_x = 25-4\times5+5$

$= 10\ J$

$U_{x=-1} = 10\ J$

(6) $K_{max} = \frac{1}{2}kA^2$

$= \frac{1}{2}\times2\times(3)^2$

$= 9\ J$

Q. A particle is performing SHM along x-axis, it Potential energy is given by

$$U = 20+(x-2)^2\ J$$

if total mech. energy is 36 J then find -

(1) Mean position (2) Force const (3) U_{min} (4) U_{max} (5) K.Emax (6) Amp.

$F = -\frac{dU}{dx}$

$\boxed{F = -2x+4}$

K

At mean $F=0$

$x = 2m$ — mean position.

(2) $K = 2\ N/m$

(3) $U_{min} = U_{x=2m} = 20+(2-2) = 20\ J$

(4) $U_{max} = TME = 36\ J$

(5) $K_{max} = 36 - 20 = 16J$

(6) $\frac{1}{2}kA^2 = 16$

$A = 4m$

A particle is performing SHM, it's max^m velocity is V_0 then find avg. speed and avg. velocity of particle per cycle.

$$\vec{V}_{avg} = \frac{\text{Disp}}{\text{time}} = \frac{0}{T} = 0$$

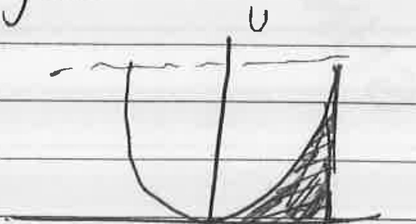
$$V_{avg} = \frac{\text{Dist}}{\text{time}} = \frac{4A \times \omega}{T \times \omega} = \frac{4V_0}{2\pi} = \frac{2V_0}{\pi}$$

time f^n , dist^n

$$\frac{\int v\,dt}{\int dt} = V_{av} \qquad \frac{\int v\,dx}{\int dx} = V_{av}$$

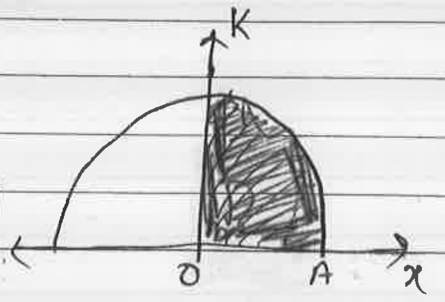
Avg. energy per cycle

$$\langle U_x \rangle = \frac{\int_0^A U\,dx}{\int_0^A dx} = \frac{\int_0^A \frac{1}{2}kx^2\,dx}{\int_0^A dx} = \frac{\frac{1}{2}k\left[\frac{x^3}{3}\right]_0^A}{A} = \boxed{\frac{1}{6}kA^2 + U_0}$$



$$\langle K.E_x \rangle = \frac{\int_0^A \frac{1}{2}k(A^2 - x^2)\,dx}{\int_0^A dx}$$

$$= \frac{\frac{1}{2}k\left(A^2x - \frac{x^3}{3}\right)_0^A}{A} = \boxed{\frac{1}{3}kA^2}$$



$\langle \sin^2\omega t\rangle = \frac{1}{2}$

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$$\boxed{\langle K.E_x\rangle = 2\langle U_x\rangle}$$

& $\langle T.E\rangle = \langle U_x\rangle + \langle K.E_x\rangle = \frac{1}{2}KA^2 + U_0$

$\Rightarrow \langle U_t\rangle = \langle \frac{1}{2}KA^2\sin^2\omega t\rangle = \frac{1}{2}KA^2\langle \sin^2\omega t\rangle = \frac{1}{2}KA^2\times\frac{1}{2} = \frac{1}{4}KA^2$

$\langle K.E_t\rangle = \langle \frac{1}{2}KA^2\cos^2\omega t\rangle = \frac{1}{2}KA^2\times\frac{1}{2} = \frac{1}{4}KA^2$

$\boxed{\langle U_t\rangle = \langle K.E_t\rangle = \frac{1}{4}KA^2}$

* $\langle TE\rangle = \langle K.E_t\rangle + \langle U_t\rangle = \frac{1}{2}KA^2 + U_0$

Que. A particle is performing SHM it's mean K.E is 40J then find K.E of particle at mean.

Sol.

Mean (Avg) K.E $= \frac{1}{4}KA^2 = 40$

K.E at mean $= KE_{max}$
$= \frac{1}{2}KA^2$
$= 2\times 40 = 80J$

Formulas

$K.E = \frac{1}{2}K(A^2 - x^2)$

$K.E_{max} = \frac{1}{2}KA^2$

$K.E = \frac{1}{2}KA^2\cos^2\omega t$

$P.E = \frac{1}{2}Kx^2$

$P.E = \frac{1}{2}KA^2\sin^2\omega t$

$T.M.E = \frac{1}{2}KA^2$

$\langle U_x \rangle = \frac{1}{6}KA^2$

$\langle K.E_x \rangle = \frac{1}{3}KA^2$

$\boxed{\langle K.E_x \rangle = 2\langle U_x \rangle}$

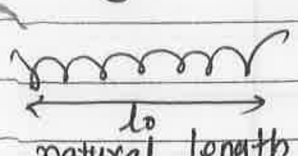
$T.M.E = \frac{1}{2}KA^2$

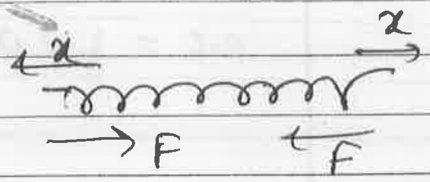
$\langle U_t \rangle = \frac{1}{4}KA^2$

$\langle K.E_t \rangle = \frac{1}{4}KA^2$

$T.M.E = \frac{1}{2}KA^2$

Spring → Potential energy stored





under elastic limit →

$F \propto -x$

*

$$\boxed{F = -kx}$$

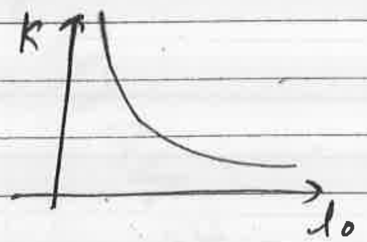
spring force ↓ ↓ spring const

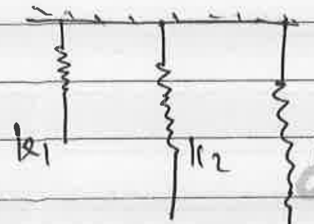
*

$$K = \frac{\eta r^4}{4 l_0 n R^3}$$

*

$$\boxed{k \propto \frac{1}{l_0}}$$



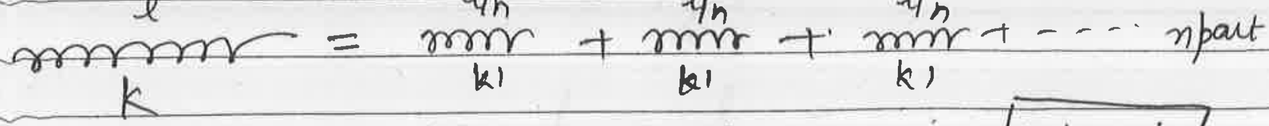


$K_1 > K_2 > K_2$

* assume massless

$$\boxed{W = U = \frac{1}{2}kx^2 = \frac{F^2}{2k}}$$

*



$$k \propto \frac{1}{l} \Rightarrow \frac{k'}{k} = \frac{l}{l'} = \frac{l}{l/n} = n \Rightarrow \boxed{k' = nk}$$

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* $\underset{k}{\overset{l}{\text{~~~}}} + \underset{k}{\overset{l}{\text{~~~}}} + \underset{k}{\overset{l}{\text{~~~}}} \ldots n = \underset{k'}{\overset{nl}{\text{~~~~~~}}}$

$$\frac{k'}{k} = \frac{l}{nl} \Rightarrow k' = \frac{k}{n}$$

Q. $\underset{K}{\overset{l}{\text{~~~}}} = \underset{k_1}{\overset{l/5}{\text{~~~}}} + \underset{K_2}{\overset{4l/5}{\text{~~~}}}$

find k_1 & K_2

Soln

$$\frac{K_1}{K} = \frac{l}{l/5} = 5 \Rightarrow k_1 = 5k$$

$$\frac{K_2}{k} = \frac{l}{\frac{4l}{5}} = \quad k_2 = \frac{5}{4}k$$

Q. A spring of const K is cut into two parts in such that length of one part is 'n' times of other part, then find const of each part.

Ans

$\underset{l}{\overset{K}{\text{~~~}}} \Rightarrow \underset{\underset{nl}{k_1}}{\overset{l_1}{\text{~~~}}} + \underset{\underset{l}{K_2}}{\overset{l_2}{\text{~~~}}}$

$n l + l$

$l(n+1)$

Given, $l_1 = n l_2$

$$\frac{K_1}{K} = \frac{l(n+1)}{nl}$$

$$K_1 = \left(\frac{n+1}{n}\right)K$$

$$\frac{K_2}{k} = \frac{l(n+1)}{l}$$

$$K_2 = (n+1)K$$

$k_1 = \left(\frac{n+1}{n}\right)K$

$K_2 = (n+1)K$

$K_1 = \frac{K_2}{n}$

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Medical (MDPT)

Ques. Two springs of const K_1 & K_2 ($K_1 > K_2$) are given find ratio of work done for both spring if
short long

(i) both are streched by equal displacement.
(ii) " " " " " force

(i) $\frac{W_1}{W_2} = \frac{\frac{1}{2}k_1 x^2}{\frac{1}{2}k_2 x^2} = \frac{k_1}{k_2} \Rightarrow (W_1 > W_2)$

$\Downarrow$

$(W_{short} > W_{long})$

(ii) $\frac{W_1}{W_2} = \frac{\frac{F^2}{2k_1}}{\frac{F^2}{2k_2}} = \frac{k_2}{k_1} \Rightarrow (W_2 > W_1)$

$\Downarrow$

$(W_{long} > W_{short})$

Ques. Two spring are compressed in such that stored P.E of both spring is equal, if const of one spring is 100% more than the const of other spring in find the ratio of restoring forces.

Soln

$k_1 = k_2 + 100\% \text{ of } k_2$

$k_1 = k_2 + k_2$

$k_1 = 2k_2$

$\frac{k_1}{k_2} = \frac{2}{1}$,

Now, $U_1 = U_2$

$\frac{F_1^2}{2k_1} = \frac{F_2^2}{2k_2}$

$\frac{F_1^2}{F_2^2} = \frac{k_1}{k_2}$

$\frac{F_1}{F_2} = \sqrt{\frac{k_1}{k_2}}$

$= \sqrt{\frac{2}{1}}$

$= \sqrt{2} : 1$

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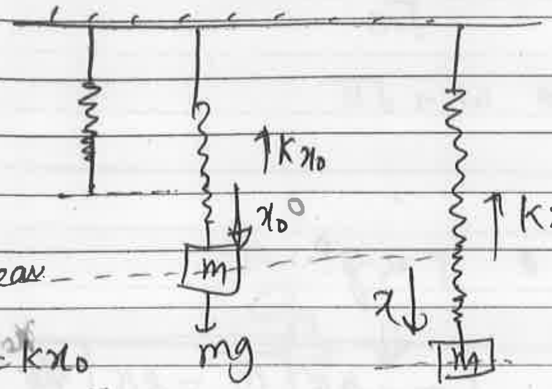
Spring pendulum



$mg = kx_0$

Measured / not natu...

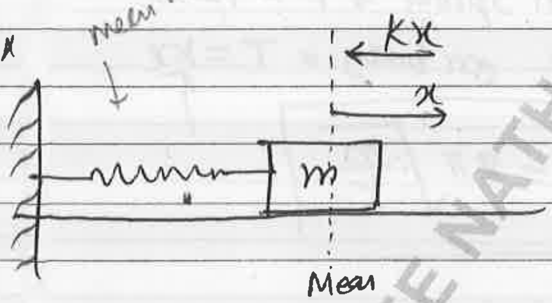
Restoring force in spring or on body

$$F = -kx$$
$$ma = -kx$$
$$a = \frac{-k}{m}x$$

compare with $a = -\omega^2 x$

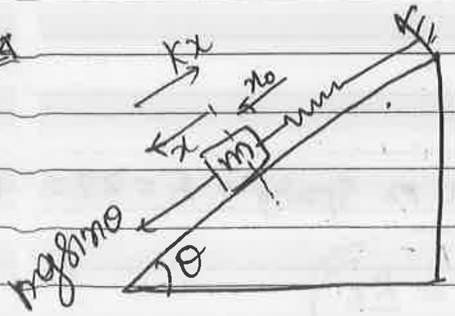
$$\omega^2 = \frac{k}{m}$$
$$\omega = \sqrt{\frac{k}{m}}$$
$$n = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$
$$\boxed{T = 2\pi\sqrt{\frac{m}{k}}}$$

mean to natural length



$F = -kx$

Hence, $\boxed{T = 2\pi\sqrt{\frac{m}{k}}}$



$\Rightarrow F = -kx$

Hence $\boxed{T = 2\pi\sqrt{\frac{m}{k}}}$

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Imp point

*** • $T \propto \sqrt{m} \Rightarrow n \propto \frac{1}{\sqrt{m}} \Rightarrow \omega \propto \frac{1}{\sqrt{m}}$

• $T \propto \frac{1}{\sqrt{K}} \Rightarrow n \propto \sqrt{K} \Rightarrow \omega \propto \sqrt{K}$

• $T \propto \sqrt{l_0}$
$\left(K \propto \frac{1}{l_0}\right)$

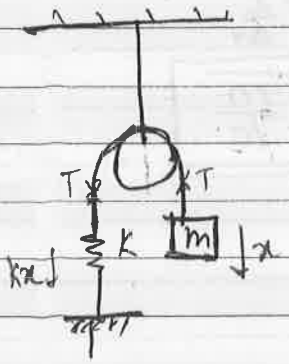
* $T \propto g^0$

$$\boxed{T = 2\pi\sqrt{\frac{m}{K}} = 2\pi\sqrt{\frac{x_0}{g}}}$$

ratio const.

Time period does not depends on 'g'.

Que: find 'T' of given system

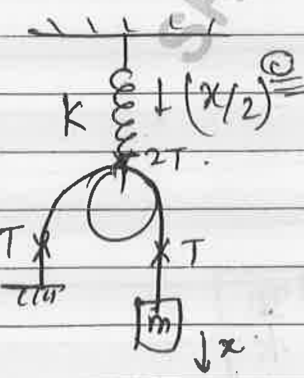


Sol^n

Restoring force in spring → $F = T = kx$
Restoring force on body → $T = kx$

then, $\boxed{T = 2\pi\sqrt{\frac{m}{K}}}$

Que: find 'T' of system



Sol^n

* $T \times x = \text{const}$
* $T \times V = \text{"}$
* $T \times a = \text{"}$

Restoring force in spring → $F = 2T = k\left(\frac{x}{2}\right)$

$\Rightarrow T = \frac{kx}{4}$

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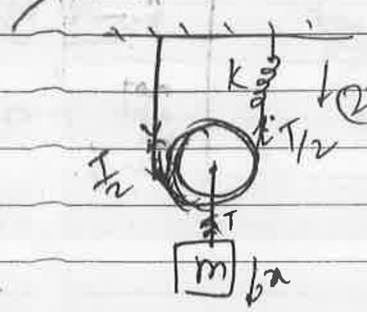
Restoring force on body → $T = \frac{K}{4}x$

then $T = 2\pi\sqrt{\frac{m}{K/4}} = 2\pi\sqrt{\frac{4m}{K}}$

Que: find (T) period of given system



Restoring force in spring ⇒ $F = \frac{T}{2} = K(2x)$

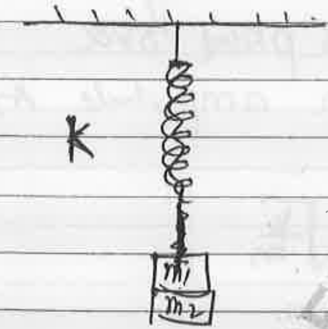
$T = 4Kx$

Restoring force on body ⇒ $T = 4Kx$

then,

$$T = 2\pi\sqrt{\frac{m}{4K}}$$

Q



Initially system is in eq^m. if mass "m_1" is removed without disturbing the system then find amplitude & Timeperiod of motion.

Sol^n

$m_1 g = KA$

$$\boxed{A = \frac{m_1 g}{K}}$$

Now,

$T = 2\pi\sqrt{\frac{m_2}{K}}$ ✓ Ans

$x_1 = m$

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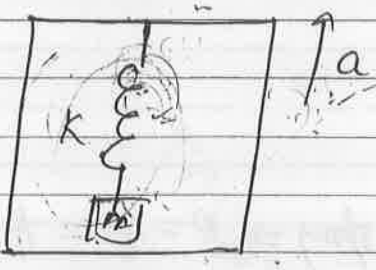
Q. Suddenly lift starts to move up with accⁿ 'a' then find amplitude & Time period of motion.



Psd

$ma = KA$

$$\Rightarrow \boxed{A = \frac{ma}{K}}$$

$$\Rightarrow T = 2\pi\sqrt{\frac{m}{K}}$$

$x = \frac{m(g+a)}{k}$ not cas bcz,

downward with a = g, $x_1 = \frac{mg}{K}$

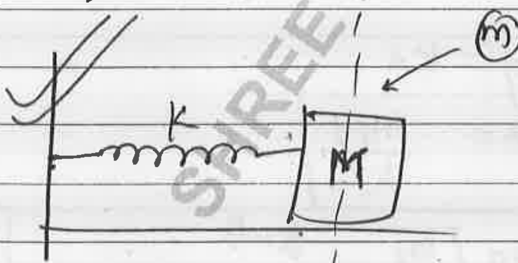
$x_2 = \frac{m(g+a)}{k}$

$$Amp = x_2 - x_1 = \frac{mg}{K} + \frac{ma}{K} - \frac{mg}{K} = \frac{ma}{K}$$

Q. a block of mass 'M' attached to a horizonal spring of spring cost 'k' is performing SHM of angular frequency ω_1 and amplitude A_1 when it passes through it's mean position. A smaller mass 'm' is placed over it and both masses move together with amplitude A_2 and freq. ω_2, then find ratio of $\frac{\omega_1}{\omega_2}$ & $\frac{A_1}{A_2}$

Soln $\omega = \sqrt{\frac{k}{m}}$

$\omega \propto \frac{1}{\sqrt{m}}$



$$\boxed{\frac{\omega_1}{\omega_2} = \sqrt{\frac{m_2}{m_1}} = \sqrt{\frac{M+m}{M}}}$$

At mean position → $f = 0$

Applying momentum conservation →

$$MV_1 = (M+m)V_2$$

$$M(A_1\omega_1) = (M+m)(A_2\omega_2)$$

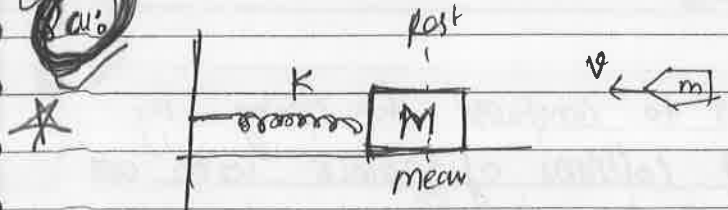
$$\frac{A_1}{A_2} = \frac{(M+m)}{M}\left(\frac{\omega_2}{\omega_1}\right)$$

$$\frac{A_1}{A_2} = \frac{M+m}{M}\left(\sqrt{\frac{M}{M+m}}\right)$$

$$\boxed{\frac{A_1}{A_2} = \sqrt{\frac{M+m}{M}}}$$

Q no.

A bullet of mass 'm' moves with velocity v and strikes the block of mass 'M'. It remains embedded in the block then find velocity of block imediately after collision, amplitude & Time period of motion.



For t=0,

COLM

$$mv = (M+m)V'$$

$$\boxed{V' = \frac{mv}{M+m}}$$

Now,

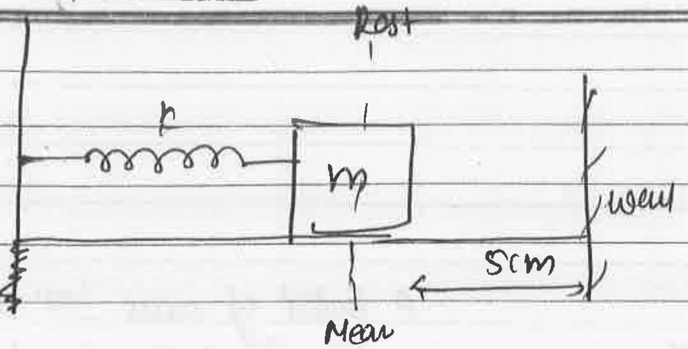
$$A\omega = \frac{mv}{M+m}$$

$$A = \frac{mv}{(M+m)\omega} = \frac{mv}{(M+m)}\sqrt{\frac{M+m}{k}}$$

$$\boxed{A = \frac{mv}{\sqrt{k(M+m)}}}$$

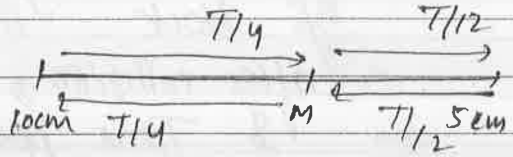
$$T = 2\pi\sqrt{\frac{m+M}{k}}$$

Ques



Block of Mass 'M' is moved to compress the spring by '10cm' and released. if collisions of block with wall are elastic then find time period of motion.

Soln



$$T = \frac{T}{4} + \frac{T}{4} + \frac{T}{12} + \frac{T}{12} = \frac{2T}{3} = \frac{2}{3}\left(2\pi\sqrt{\frac{m}{k}}\right)$$

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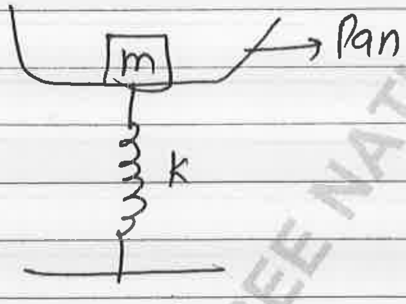
Q



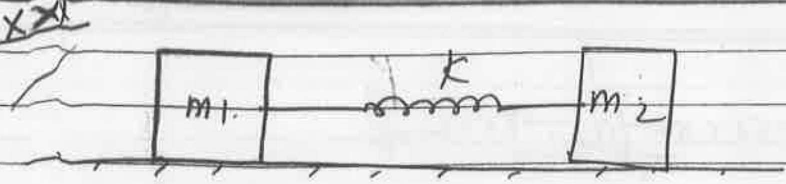
find min^m amplitude of motion at which body of mass 'm' gets detached from the pan.

condition for detaching,

$$m\omega^2 A \geq mg$$

$$A \geq \frac{g}{\omega^2}$$

$$A = \frac{g}{\omega^2} = \frac{gm}{K} = \frac{mg}{K}$$

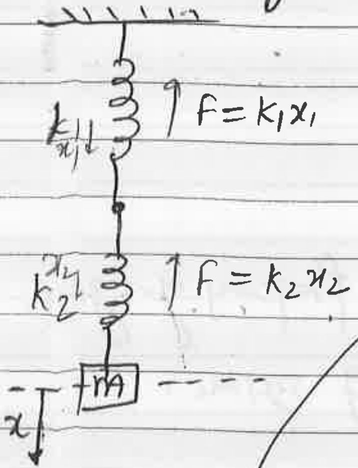


$$\frac{1}{m_{red}} = \frac{1}{m_1} + \frac{1}{m_2}$$

$$\boxed{m_{red} = \frac{m_1 m_2}{m_1 + m_2}}$$

then $$\boxed{T = 2\pi \sqrt{\frac{m_1 m_2}{(m_1+m_2)K}}}$$

series arrangment :



$$x = x_1 + x_2$$

$$\frac{F}{k_{eq}} = \frac{F}{k_1} + \frac{F}{k_2}$$

$$\boxed{\frac{1}{k_{eq}} = \frac{1}{k_1} + \frac{1}{k_2}}$$

$$\boxed{k_{eq} = \frac{k_1 k_2}{k_1 + k_2}} \downarrow$$

$k_{eq} \downarrow \; T_{eq} \uparrow \Rightarrow n_{eq} \downarrow \Rightarrow \omega_{eq} \downarrow$

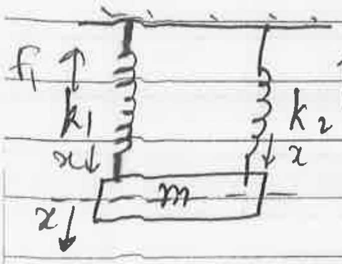
$$\frac{1}{m\omega^2_{eq}} = \frac{1}{m\omega_1^2} + \frac{1}{m\omega_2^2}$$

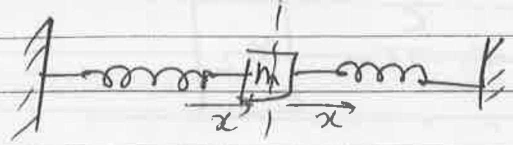
$$\frac{1}{\omega^2_{eq}} = \frac{1}{\omega_1^2} + \frac{1}{\omega_2^2} \quad \downarrow$$

$$\boxed{\frac{1}{n^2_{eq}} = \frac{1}{n_1^2} + \frac{1}{n_2^2}} \downarrow$$

$$\boxed{T^2_{eq} = T_1^2 + T_2^2} \uparrow$$

* Parallel →





$$F = F_1 + F_2$$

$$K_{eq} x = K_1 x + K_2 x$$

$$\boxed{K_{eq} = K_1 + K_2} \uparrow$$

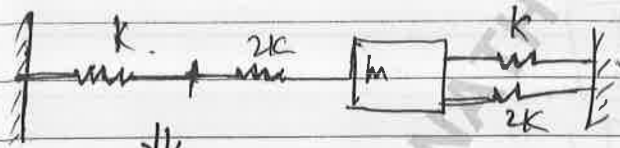
$T_{eq} \downarrow \Rightarrow n_{eq} \uparrow \Rightarrow \omega_{eq} \uparrow$

$$\left[\omega_{eq}^2 = \omega_1^2 + \omega_2^2 \right] \uparrow$$

$$\left[n_{eq}^2 = n_1^2 + n_2^2 \right] \uparrow$$

$$\left[\frac{1}{T_{eq}^2} = \frac{1}{T_1^2} + \frac{1}{T_2^2} \right] \uparrow$$

Q



find frequency of given spring system.

$\Downarrow$ $\frac{K \times 2K}{3K}$ $\quad\quad$ $\Downarrow$ $3K$

$$k_{eq} = \frac{2k}{3} + 3K$$

$$= \frac{11K}{3}$$

$$n = \frac{1}{2\pi}\sqrt{\frac{11K}{3m}}$$

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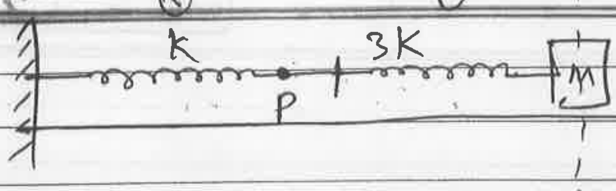
Ques ②



Block of mass 'M' is performing SHM of amplitude "A" as shown in figure" then find amplitude of 'P' point.

Sol^n In series → $F = \text{const}$

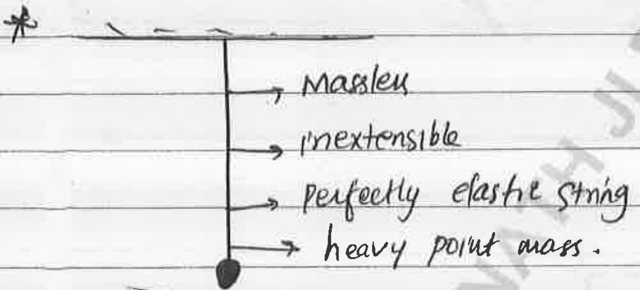
$$\boxed{k_1A_1 = k_2A_2 = k_{eq}A}$$

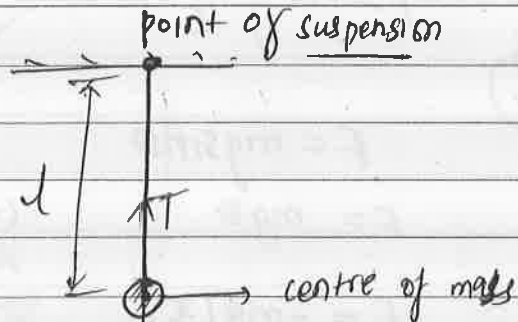
$\Rightarrow k_1A_1 = k_{eq}A$

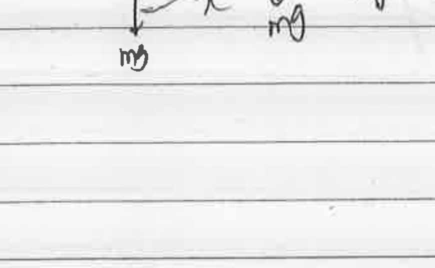
$$KA_1 = \frac{3}{4}KA$$

$$A_1 = \frac{3}{4}A \qquad A_2 = \frac{A}{4}$$

Simple Pendulum (S.P) →







$$\tau = (mg\sin\theta)l$$

$$\tau = -(mgl)\sin\theta$$

if θ is very small then

$\sin\theta \simeq \theta \quad (\theta \le 10°)$

$$\boxed{\tau = -(mgl)\theta}$$

$$\boxed{\alpha = -\left(\frac{mgl}{I}\right)\theta}$$

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comparing, $\alpha = -\omega^2 \theta$

$$\omega^2 = \frac{mgl}{I} = \frac{mgl}{ml^2} = \frac{g}{l}$$

$$\omega = \sqrt{\frac{g}{l}}$$

$$n = \frac{1}{2\pi}\sqrt{\frac{g}{l}} \Rightarrow \boxed{T = 2\pi\sqrt{\frac{l}{g}}}$$

(II)

$$\boxed{\tau = -\underbrace{(mgl)}_{C}\,\theta}$$

$$T = 2\pi\sqrt{\frac{I}{C}} = 2\pi\sqrt{\frac{ml^2}{mgl}} = 2\pi\sqrt{\frac{l}{g}}$$

(III)

$F = mg\sin\theta$

$F = mg\theta \quad (\sin\theta \approx \theta)$

$F = -mg\left(\frac{x}{l}\right) \quad \theta = \frac{\text{arc}}{\text{radius}} = \frac{x}{l}$

$F = -\left(\frac{mg}{l}\right)x$

$\Rightarrow k = \frac{mg}{l}$ then $T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{m}{mg}l}$

$$\boxed{T = 2\pi\sqrt{\frac{l}{g}}}$$

Imp points

* $\boxed{T = 2\pi\sqrt{\frac{l}{g}}}$

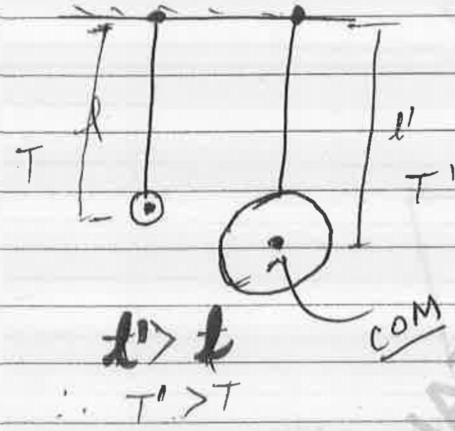
* $T \propto \theta^0$
* $T \propto m^0$
* $T \propto \frac{1}{\sqrt{g}}$

$g_{poles} > g_{equator}$

$\therefore T_{poles} < T_{equator}$.

* $T \propto \sqrt{l}$

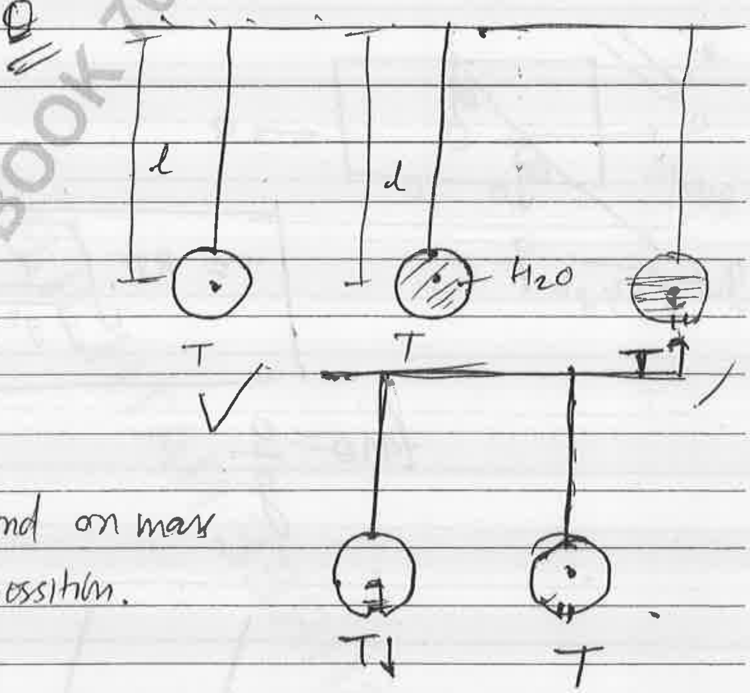
Q



$l' > l$

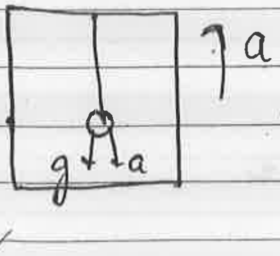
$\therefore T' > T$

Q



* Timeperiod of s.p does not depend on mass but depend on C.O.M possition.

* Suspend point, move,



$g_{eff} = g + a \uparrow$

$T = 2\pi\sqrt{\frac{l}{g+a}} \downarrow$

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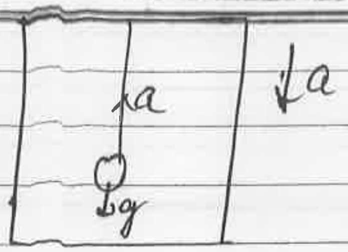
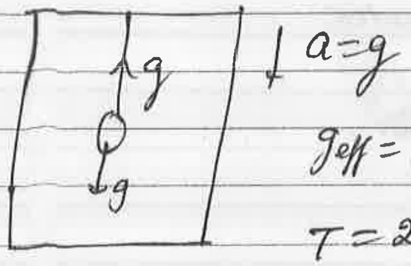


$g_{eff} = g - a \downarrow$

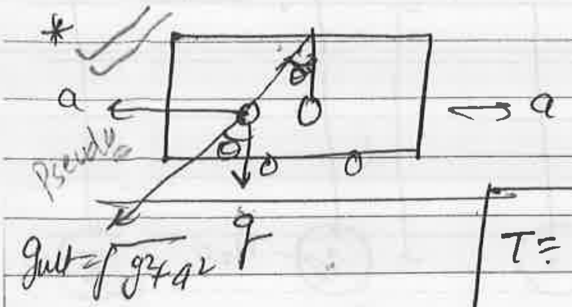
$T = 2\pi\sqrt{\frac{l}{g-a}} \uparrow$



$g_{eff} = g - g = 0$

$T = 2\pi\sqrt{\frac{l}{0}} = \infty \Rightarrow n = 0$

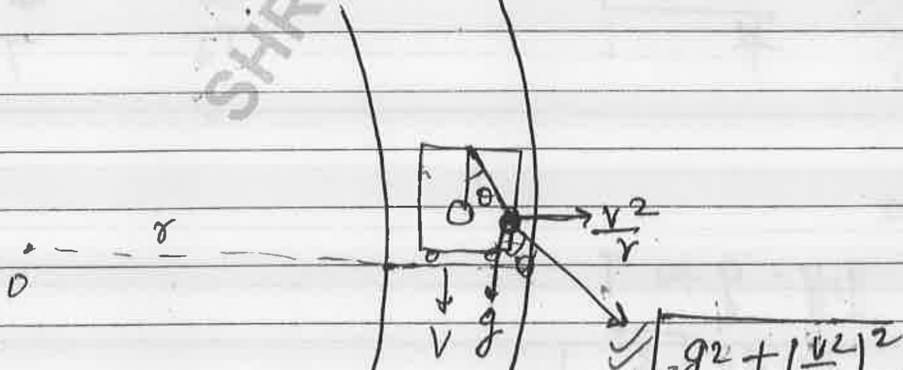
$\Rightarrow$ does not ossillation occur.



$$\boxed{T = 2\pi\sqrt{\frac{l}{\sqrt{g^2+a^2}}}}$$

$\tan\theta = \frac{a}{g}$

realtenbrne = same.



$$T = 2\pi\sqrt{\frac{l}{\sqrt{g^2+\left(\frac{v^2}{r}\right)^2}}}$$

$$\boxed{\tan\theta = \frac{v^2}{rg}}$$

p-1-150
Q-89. (89) $y = kt^2$

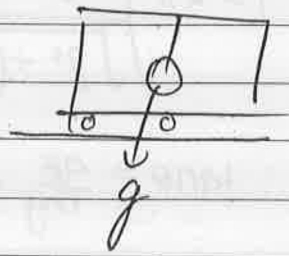
$v = 2kt$

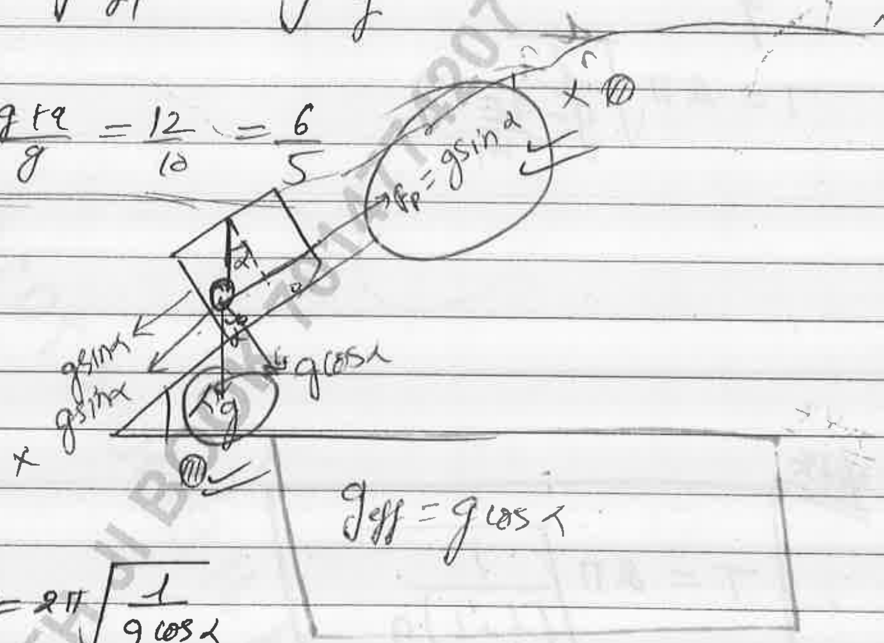
$a = 2k \ m/s^2$

$T \propto \frac{1}{\sqrt{g}} \Rightarrow \frac{T_1}{T_2} = \sqrt{\frac{g_2}{g_1}} = \sqrt{\frac{g+a}{g}}$

$\Rightarrow \frac{T_1^2}{T_2^2} = \frac{g+a}{g} = \frac{12}{10} = \frac{6}{5}$

(88)





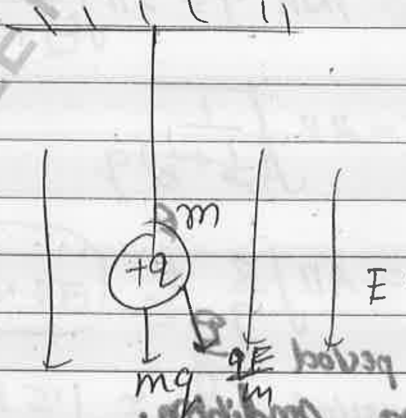
$g_{eff} = g\cos\alpha$

$T = 2\pi\sqrt{\frac{l}{g\cos\alpha}}$

Q $F = qE$

$ma = qE$

$a = \frac{qE}{m}$



$T = 2\pi\sqrt{\frac{l}{g + \frac{qE}{m}}}$

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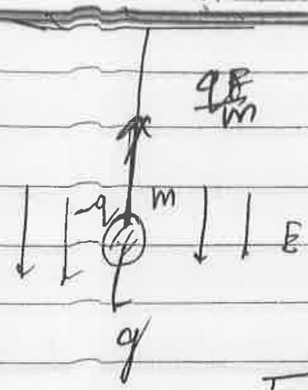
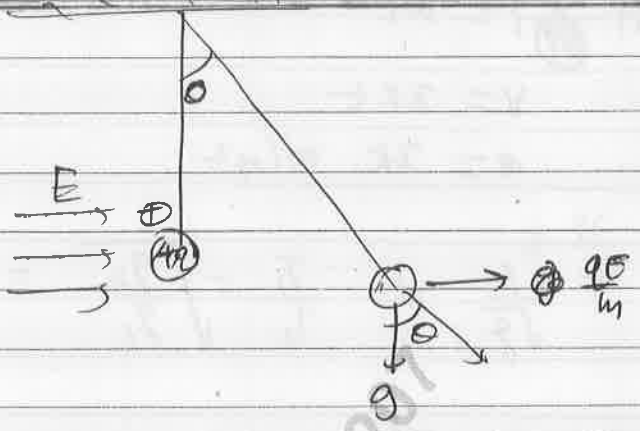


$$T = 2\pi\sqrt{\frac{l}{g - \frac{qE}{m}}} \uparrow$$



$$g_{eff} = \sqrt{g^2 + \left(\frac{qE}{m}\right)^2}$$

$$T = 2\pi\sqrt{\frac{l}{\sqrt{g^2 + \left(\frac{qE}{m}\right)^2}}}$$

$$\tan\theta = \frac{qE}{mg} = \frac{F_e}{F_g}.$$

***

$$\boxed{T = 2\pi\sqrt{\frac{1}{\left(\frac{1}{l}+\frac{1}{R}\right)g}}}$$

* if $l <<< R \Rightarrow \frac{1}{l} >>> \frac{1}{R}$ then $T = 2\pi\sqrt{\frac{1}{\left(\frac{1}{l}+0\right)R}} \Rightarrow 2\pi\sqrt{\frac{l}{g}}$

* if $l = \infty$, then $T = 2\pi\sqrt{\frac{1}{\left(\frac{1}{\infty}+\frac{1}{R}\right)g}}$

$$= 2\pi\sqrt{\frac{R}{g}} \underset{=}{\overset{\theta}{\simeq}} 84.6 \text{ min}$$

it is the max^m time period $\simeq 1.5$ hr.
of s.p in ossilating condition.

* if $l = R$ then $T = 2\pi\sqrt{\frac{1}{\left(\frac{1}{R}+\frac{1}{R}\right)g}} = 2\pi\sqrt{\frac{R}{2g}}$

$\simeq 59.8$ min

$\simeq$ 1 hrs.

Second's pendulum :-

$$T = 2\pi\sqrt{\frac{l}{g}}$$

Time period of Second's pendulum is const 2 Second but it's length can be changed.

$$T = 2\,\text{second} = \text{const}$$

$$\Rightarrow l \propto g$$

Que Find time period and length's of second pendulum at the surface of earth & moon.

i) $T = 2$ second const

ii) length

$l_e = ?$

$$T = 2\pi\sqrt{\frac{l_e}{g}}$$

$$\boxed{g = \pi^2\ \text{m/sec}^2}$$

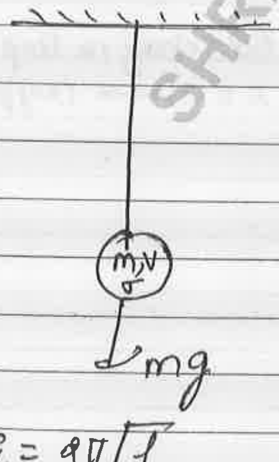
$$2 \simeq 2\pi\sqrt{\frac{l_e}{\pi^2}}$$

$$1 \simeq \sqrt{l_e} \Rightarrow l_E \simeq 1\text{m}$$

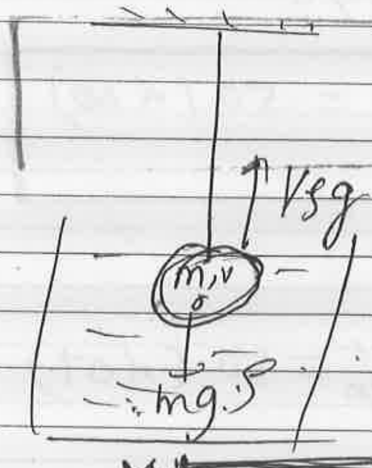
No,

$$l_m = \frac{l_E}{6} \simeq \frac{1}{6}\text{m} \quad (l \propto g)$$

**



$$T_{air} = 2\pi\sqrt{\frac{l}{g}}$$



$$F_{net} = mg - V\sigma g$$

$$m g_{net} = mg - V\sigma g$$

$$g_{net} = g - \frac{V\sigma g}{m}$$

$$= g - \frac{\cancel{V}\sigma g}{\cancel{V}\rho}$$

$$g_{net} = g\left(1 - \frac{\sigma}{\rho}\right)$$

$$T_{liq} = 2\pi\sqrt{\frac{l}{g\left(1-\frac{\sigma}{\rho}\right)}} = \frac{T_{air}}{\sqrt{1-\frac{\sigma}{\rho}}}$$

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Q. Time period of S.P is 5 sec. if it is merged in a liquid whose densities 64% density of bob. Then find New time period.

$\rho = 64\%$ of σ

$\frac{\rho}{\sigma} = \frac{64}{100}$

$T_{liq} = \frac{T_{air}}{\sqrt{1-\frac{\rho}{\sigma}}} = \frac{5}{\sqrt{1-\frac{64}{100}}} = \frac{25}{3}$ sec.

$T = 2\pi\sqrt{\frac{l}{g}}$

$T \propto \sqrt{l}$

$\boxed{\frac{\Delta T}{T} \times 100 = \frac{1}{2}\frac{\Delta l}{l} \times 100}$ up to 5% change.

Thermal strain $\frac{\Delta l}{l} = (\alpha \Delta \theta)$

$\frac{\Delta T}{T} \times 100 = \frac{1}{2} \times (\alpha \Delta \theta) \times 100$

$\boxed{\text{\% change in 'T' of S.P} = 50\,(\alpha \Delta \theta)}$

$\Delta\theta$ = change in temp.
α = Thermal coefficient of mate.

$\frac{\Delta T'}{T} \times 100 = 50\,(\alpha \Delta T)$

Same phase $\Rightarrow 0, 2\pi, 4\pi, 6\pi, \ldots$
In phase condition

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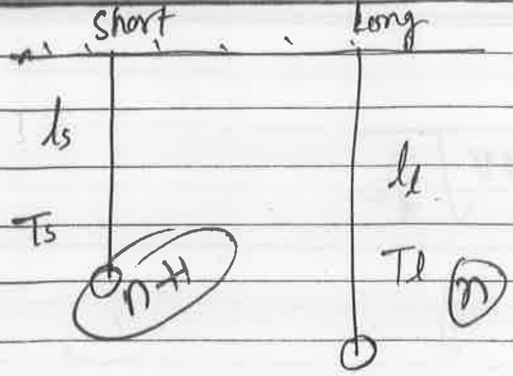

at $t=0 \Rightarrow \Delta\phi = 0$

$t = \uparrow \Rightarrow \Delta\phi = \uparrow$

$t = t_{min} \Rightarrow \Delta\phi = 2\pi$ rad, means one ossillation difference.

When both are in same phase $\rightarrow$ $\boxed{t_{min} = n\,T_l = (n+1)\,T_s}$

$n = ?$

no. of ossillations $\Rightarrow$ $n\,T_l = (n+1)\,T_s$

$$n\left[2\pi\sqrt{\frac{l_l}{g}}\right] = (n+1)\left[2\pi\sqrt{\frac{l_s}{g}}\right]$$

$$\boxed{n\sqrt{l_l} = (n+1)\sqrt{l_s}}$$

Que: Two simple pendulums of lengths 1m & 1.21m starts ossillation simultaneously from the same position. find no. of ossillation & min^m time after which both pendulum will be in same phase.

$$n\sqrt{l_l} = (n+1)\sqrt{l_s}$$

$$n\sqrt{1.21} = (n+1)\sqrt{1}$$

$$1.1n = (n+1)$$

$\boxed{n = 10}$ ossillation $\rightarrow$ long

$n+1 = 11$ ossillation $\Rightarrow$ short.

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$t_{min} = nT_l = (n+1)T_s$

$$10\left[2\pi\sqrt{\frac{[illegible]}{\pi^2}}\right] = (n+1)\,2\pi\sqrt{\frac{1}{\pi^2}}$$

$= 22$ second

same phase

$t = 0$, 22 sec, 44 sec, 66 sec, ... (in same phase)

Ans

* Types of osscillations:

* free oscillation.
* Damped "
* Forced "
* Resonant "

Que: In a damped oscillator amplitude of ossilation becomes half of initial after completing 100 osillation, then find amplitude of motion after completing 400 osillation.

$N = N_0 e^{-\lambda t}$

So, using,

$$A_{(t)} = A_0 e^{-\gamma t}$$

$$\frac{A_0}{2} = A_0 e^{-\gamma t}$$

$$\frac{1}{2} = e^{-\gamma t}$$

Now,

$$A(t) = A_0 e^{-\gamma(4t)}$$

$$A(t) = A_0 \left(e^{-\gamma t}\right)^4$$

$$= A_0 \left[\frac{1}{2}\right]^4$$

$$A_{(t)} = \frac{A_0}{16}$$

Waves → disturbance, energy

- waves is a disturbance which propogate Energy, *momentum, or information from one place to another place in medium without any actual transport of matter.

Note:→ Wind blow is not an example of wave.

Classification of waves :-

(a)* on basis of medium requirement

(1) Mechanical waves — (2) Non-mechanical waves

(1) Mechanical waves

* For these wave propagation materal medium is essential.
* Essential property of materal medium are Inertia & Elasticity.
* these waves are governed by Newton's law's of motion.
* These " " also called Elastic waves.

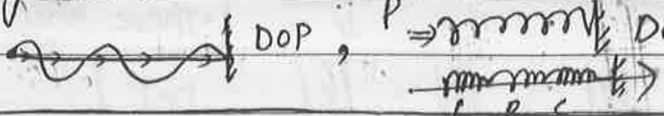
(2) Non-mechanical waves.

* For these wave propagation materal medium is not essential means can propagates in solid, liquids, gases and vaccum also.

ex EM waves, heat radiation, light waves.

* These are of Transverse nature and so these wave can be Polarised.

(b)* on basis of dirn of propagation :

→ 1-D wave *eg :  DOP , P → DOP C R C

→ 2-D wave eg.  Ripple wave (wave on water surface)

→ 3-D wave eg. Sound wave, light wave etc

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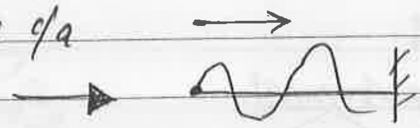
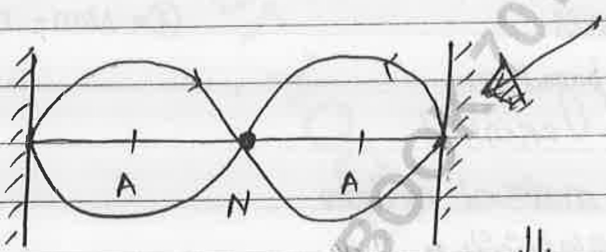
(c) on the basis of energy propagation:

* Progressive/travelling wave
↳ waves in which energy propagate in a medium with finite speed is c/a Progressive wave.
eg: sound wave, light wave etc.



* Non-progressive/stationary/Standing wave
↳ In these waves energy exist in a limited bound medium, energy does not propagate.
• Speed of wave is zero/not finite
eg: waves in musical intrument, organ pipes Sonameter wire etc.



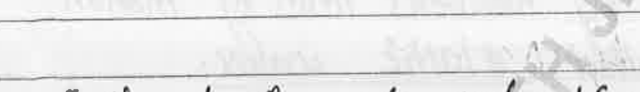
⇓
nature of wave from sound source to listner ⇒ Progressive

(d) on the basis of vibration of particle
[further classification of *Mechanical wave)

* longitudinal
* VOP ∥ DOP
vibration of particle

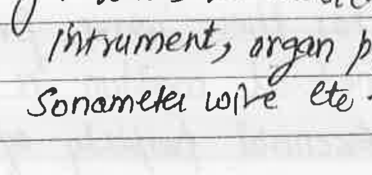
eg.


eg: sound wave in gas



#. These waves are c/a Pressure wave or P-waves.
These waves can propagate in Solid, liquid, gases

* Transverse mech. wave
* VOP ⊥ DOP

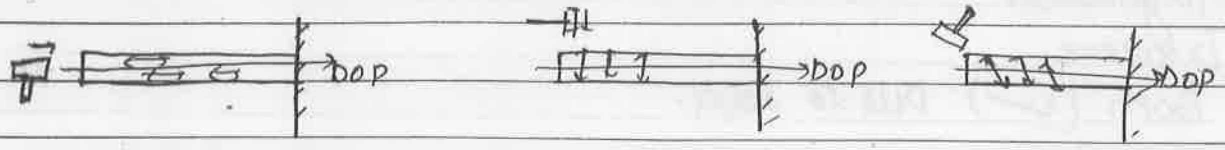
eg


• These waves are c/a SHEAR-wave or S-wave also.
• these wave can propagate only rigid medium. such as solids & liquid surface (surface tension)
• Can be Polarised.

• these waves can't be Polarised

Some mixed examples of longitudinal & transverse mech. waves :-

①

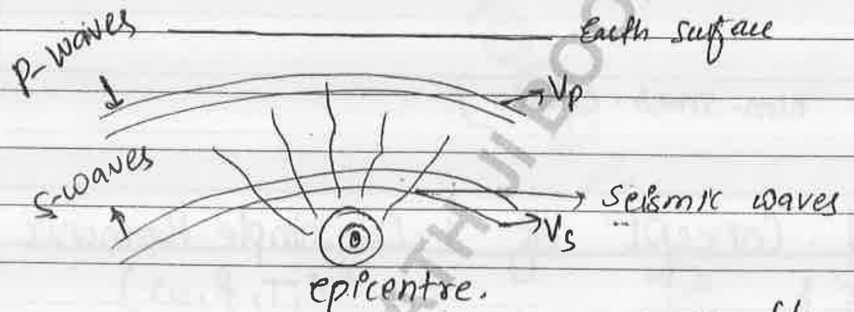


longitudinal wave — Transeverse wave — **Both**

But here $\boxed{V_p > V_s}$

② Seismic waves produced by an earthquake, are nature of both longitudinal & transverse mech. waves.

(Solid ⇒ Both)



$\Delta t = t_s - t_p$

$$\boxed{\Delta t = \frac{d}{V_s} - \frac{d}{V_p}}$$

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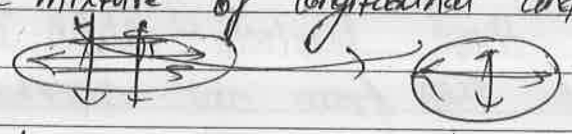
③ wave on water surface is c/a as ripple waves
It is the mixture of longitudinal and transverse waves



④ Nature of ocean waves is also both long. and trans. mech waves

⑤ Nature of waves in long and flexible spring is also both longitudinal and transverse mech. waves.

(i) spring ⇒ longi (ii) long spring ⇒ both long & Transever.

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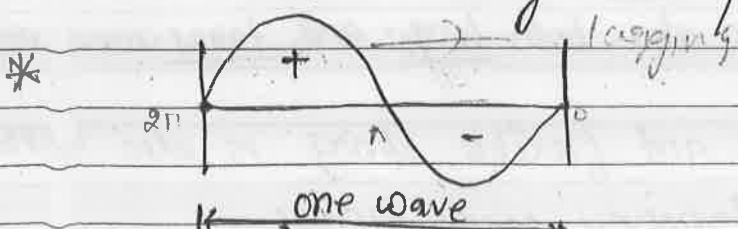
Q. Nature of sound wave in a solid medium may be
a. longitudinal
b. Transere
c. Both (✓) Due to solid.
d. None

Q. Nature of wave in a gaseous medium may be
→ (Both) [reason Due to Non-mech. consideration]

Q. A Transevere wave can propogate in
1) Solid
2) Liquid
3) Gases
4) All ✓ [Due to Non-mech. Consider]

Fundamental Concept of 1-D simple Harmonic Mech. wave : (A, T, f, ω)

★⇒ In Ideal condition, during mech. wave propogation all medium particle excecute simple harmonic motion of same type means have same, Amplitude time period, frequeny & angular frequeny but have different phase angle of vibration [except those particle which are λ distance away from one another]



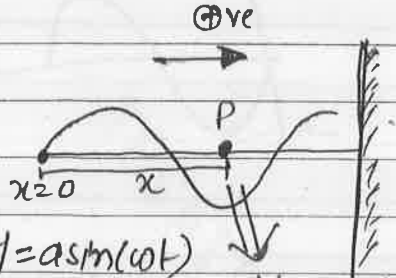
one wave
$\lambda \equiv 2\pi \equiv T$

A rhthymic of vibrating partcl in the direction of energy propagation is c/a wave

* Amplitude, T, f, & ω of particle is equal to A, T, f & ω of wave. (waves are made of particle)

Mathematical analysis :

$$y = a\sin(\omega t \pm kx)$$



$y = a\sin(\omega t)$

$y_p = a\sin(\omega t - kx)$

$\therefore$ In 'λ' distance, phase difference $= 2\pi$

$\therefore$ In 1 unit " ——— " $= \frac{2\pi}{\lambda}$

$\therefore$ In "x" dist ——— " $= \frac{2\pi}{\lambda}x = kx$

K = Propagation const or Angular wave no.

* $\left(K = \frac{2\pi}{\lambda}\right)$

* Disp. of particle 'P' at time 't' → $y = a\sin(\omega t - kx)$ and it is c/a as equⁿ of progressive wave propagating in ⊕x-axis

* $y = a\sin(\omega t - kx)$ → ⊕x-axis propagation

$y = a\sin(\omega t - (-kx)) = a\sin(\omega t + kx) \Rightarrow$ -ve x-axis propagation.

* General wave equⁿ :

$$y = a\sin(\omega t \pm kx)$$

$$y = a\sin\left(\frac{2\pi}{T}t \pm \frac{2\pi}{\lambda}x\right)$$

$$y = a\sin 2\pi\left(\frac{t}{T} \pm \frac{x}{\lambda}\right)$$

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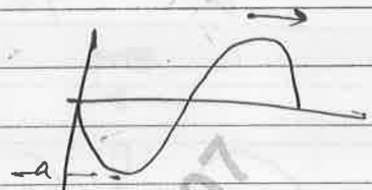
Q A wave eq^n $y = a\sin(kx - \omega t)$ is given then find - DOP, VOP and nature of wave.

Sol^n

$$y = a\sin(-(\omega t - kx))$$

$$\boxed{y = -a\sin(\omega t - kx)}$$



DOP = +ve x-axis

VOP = y-axis

nature = Transverse

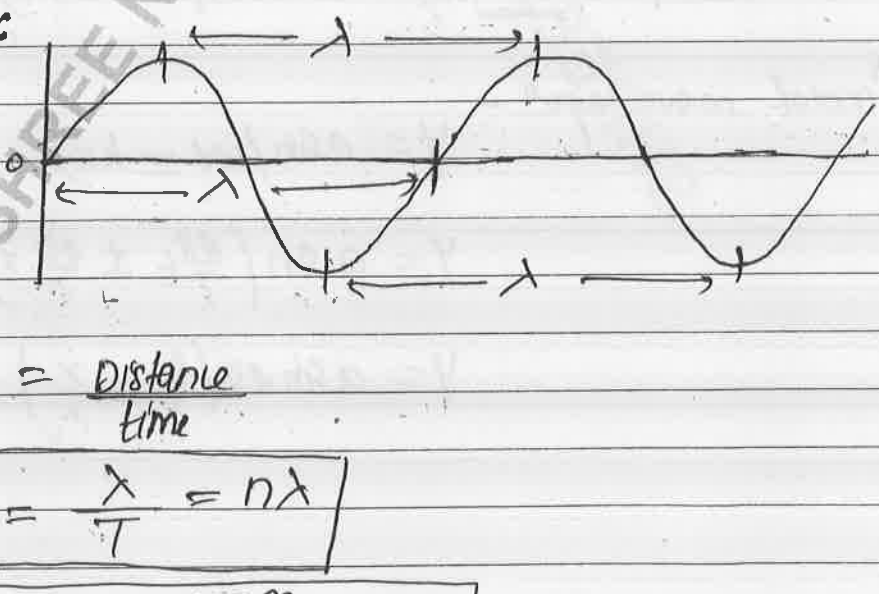
Hint: If ωt & kx both are of same sign then wave is propagation in -ve x-axis otherwise +ve x-axis.

Q $z = a\sin(-\omega t - kz)$ find DOP, VOP,

DOP $\Downarrow$ -ve z-axis, VOP $\Downarrow$ z-axis

nature = longitudinal.

★ **Wavelength (λ):**



Wave speed (v) = $\frac{\text{Distance}}{\text{time}}$

$$\boxed{v = \frac{\lambda}{T} = n\lambda}$$

$$\boxed{v = \frac{\omega}{k} = \frac{2\pi n}{\frac{2\pi}{\lambda}} = n\lambda}$$

same velocity

Sound wave = 330

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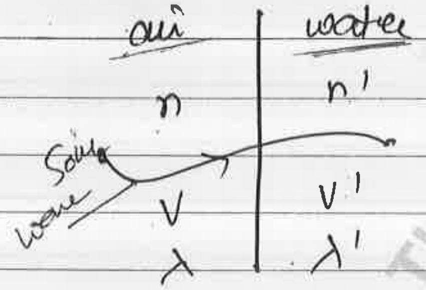
* $k = 2\pi\left(\frac{1}{\lambda}\right) = \frac{2\pi}{\lambda}$ = wave propagation const or Ang. wave number

Note: Speed of Mech. wave is medium dependent quantity
- It depends on property of medium, density, Elasticity
 • inertia etc.
- For a given medium it is const quantity. It does not depend on ν and λ.

Note: frequency of wave is the characteristic of it's Source. it is decided by it's Source.
 • If a wave of particular freq. propagate from one medium to another medium then it's frequency remains const.

Q



$\Rightarrow$ $n = n'$
$V \neq V'$
$\lambda \neq \lambda'$

* Particle velocity (V_p) and accln (f) :→

$$Y = a\sin(\omega t \pm kx)$$
$$\frac{\partial Y}{\partial t} = a\omega\cos(\omega t \pm kx)$$

$$\boxed{V_p = a\omega\cos(\omega t \pm kx)}$$

$$\boxed{f = \frac{\partial V_p}{\partial t} = -\omega^2 a\sin(\omega t \pm kx)}$$

D.P.

$\boxed{(V_p)_{max} = a\omega}$ $\quad$ $\boxed{f_{max} = \omega^2 a}$

particle velocity

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Relation b/w wave velocity & particle velocity →

$$y = a\sin(\omega t \pm kx)$$

$$\frac{\partial y}{\partial t} = a\omega\cos(\omega t \pm kx) \quad \text{—(1)}$$

$$\frac{\partial y}{\partial x} = \pm a\,k\cos(\omega t \pm kx) \quad \text{—(2)}$$

from (1)/(2)

$$\frac{\frac{\partial y}{\partial t}}{\frac{\partial y}{\partial x}} = \frac{\omega}{\pm k}$$

$$\frac{V_p}{\left(\frac{\partial y}{\partial x}\right)} = \pm V$$

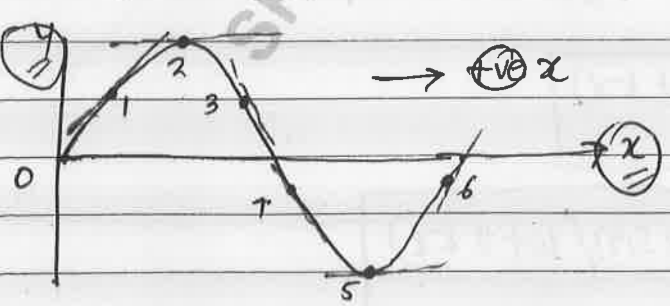
BHO

$$\boxed{V_p = \pm V\left(\frac{\partial y}{\partial x}\right)}$$

where, (+) ⟶ (−) x-axis propagation
(−) ⟶ (+) x-axis — " —

$\frac{\partial y}{\partial x}$ = wave slope or wave strain

$$\boxed{V_p = \pm V\left(\text{slope or strain}\right)}$$

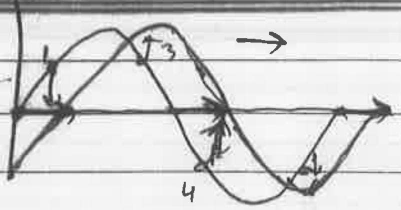
Q. 

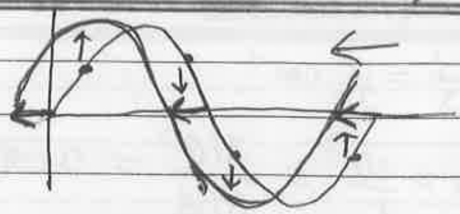
find dirn of motion of marked particles.

Particle	V_p
1	−ve ↓
2	0
3	+ve ↑
4	(+) ↑
5	0
6	(−) ↓

solⁿ using, $\boxed{V_p = -V\,(\text{slope})}$

Imp





Relation b/w Phase diff ($\Delta\phi$), time diffⁿ (ΔT) and path diffⁿ (Δx) →

$$\boxed{2\pi \equiv T \equiv \lambda}$$

$$\Downarrow$$

★ $$\boxed{\frac{\Delta\phi}{2\pi} = \frac{\Delta T}{T} = \frac{\Delta x}{\lambda}}$$

Q NO. 1
(B.B-1)
P-9 162

$$\frac{\Delta\phi}{2\pi} = \frac{\Delta x}{\lambda}$$

$$\frac{\pi/3}{2\pi} = \frac{\Delta x}{\left(\frac{300}{50}\right)}$$

$$\Delta = \frac{1}{10} = 0.1\,m$$

Q. A progressive wave equⁿ is given by $y = 5\sin\frac{\pi}{4}\left(\frac{t}{2} - \frac{x}{4} + 1\right)$ where,

$x \Rightarrow cm$, $y \Rightarrow mm$ then find ①
$t \rightarrow sec$

① a
② n
③ λ
④ $\vec{V}$ (wave no.)
⑤ V
⑥ $(V_p)_{max}$ ⑦ f_{max} ⑧ ϕ
accⁿ

$$\Downarrow$$

$$y = 5\sin\left[\frac{\pi t}{8} - \frac{\pi x}{16} + \frac{\pi}{4}\right]$$

$$y = a\sin\left[\omega t - kx + \phi\right]$$

(i) $a = 5\,mm$

(ii) $2\pi f = \frac{\pi}{8}$

$f = \frac{1}{16}\ sec^{-1}$ or Hz $\qquad T = 16\,sec$

③ $\lambda = \frac{2\pi}{K} = \frac{2\pi}{\pi/16} = 32\,cm$

$\lambda \rightarrow$

(4) $\bar{\nu} = \frac{1}{\lambda} = \frac{1}{32}\ cm^{-1}$

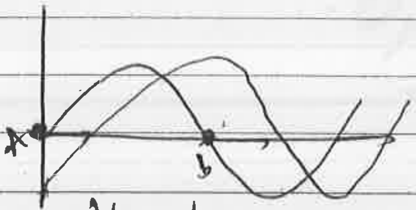
(5) $V = \frac{\omega}{k} = \frac{\pi/8}{\pi/16} = 2\ cm/sec$ ✓

(6) $(V_p)_{max} = a\omega = 5 \times \frac{\pi}{8} = \frac{5\pi}{8}\ mm/sec$

(7) $f_{max} = \omega^2 a = \frac{\pi^2}{64} \times 5 = \frac{5\pi^2}{64}\ mm/sec^2$

(8) $\phi = \frac{\pi}{4}$ rad.

$\frac{Q \cdot 10}{p \cdot 207}$

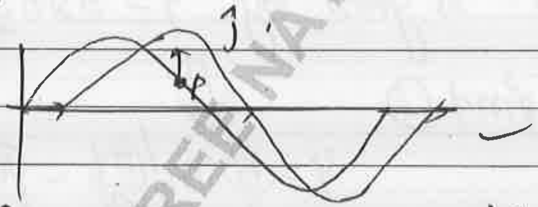


$V_A = \perp$

(pashe angle)$_A$ > (phase an B)

lagging

Q.15
p.no. 207



direct
by direction

$\vec{V_p} = (\omega\sqrt{A^2 - x^2})\hat{j}$

$= \frac{2\pi V}{\lambda}\sqrt{A^2 - x^2}$

$= \frac{2\pi \times 10}{50}\sqrt{10^2 - 5^2}$

$= 2\sqrt{3}\pi\ cm/sec\ \hat{j}$

$= \frac{\sqrt{3}\pi}{50}\hat{j}$

Fun = 1

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Q. 11 mam^m transverse string speed $= (v_p)_{max}$

(1) → $v = \frac{\omega}{k} = \frac{6}{3} = 2$

$v_p = a\omega = 2 \times 6 = 12$

(2) $v = \cancel{\frac{4}{12}} = \frac{1}{3} = \frac{12}{4} = 3$

$v_p = a\omega = \cancel{3 \times \frac{1}{3} = 1} = v_{p_{max}} = 12 \times 3 = 36$

(3) $v = \frac{11}{5} = 2.2$ $(v_p)_{max} = 11 \times 4 = 44$ (4)

$\frac{\Delta\phi}{2\pi} = \frac{x}{\lambda}$

Q. 7
p-206
(chhat)

$v = \frac{Dist}{time} = \frac{600}{2} = 300$

$n = 500$

$\lambda = \frac{v}{n} = \frac{300}{500} = \frac{3}{5}$ m

No. of wavelength $= \frac{Dist}{\lambda}$

$= \frac{600}{3/5} = 1000$

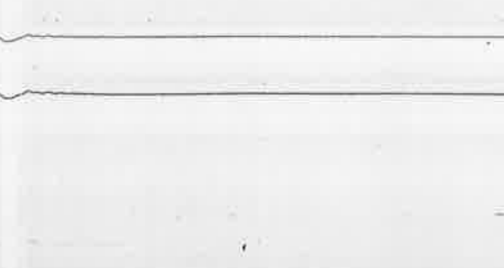
graph

Q $y = a\sin(\omega t - kx)$

* at $t=0$ → $(y-x)$ graph

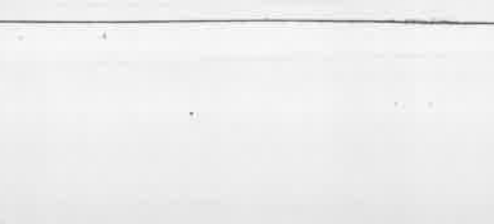
$y = a\sin(0 - kx)$

$\boxed{y = -a\sin(kx)}$



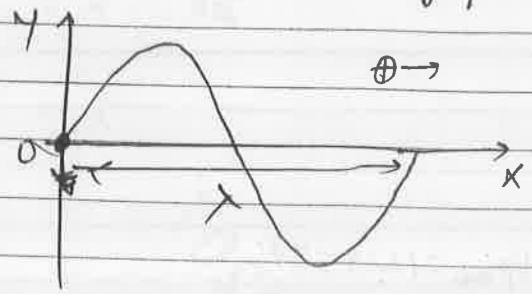
* at $x=0$ → $(y-t)$ graph

$y = a\sin(\omega t)$

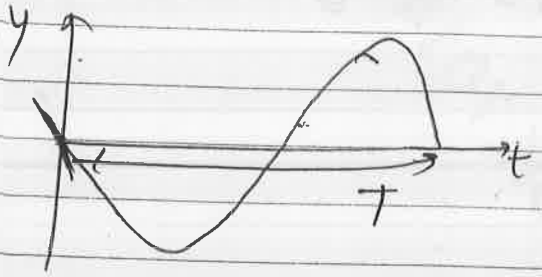


Q $y = a\sin(kx - \omega t)$

* at $t=0 \rightarrow$ $(y-x)$ graph



* at $x=0$, $(y-t)$ graph



* General eqn of travelling wave $\longrightarrow$ $\boxed{y = f(ax \pm bt)}$

but here funtn 'y' should have always finite value for all 'x' and 't' values.

$\Rightarrow$

$x=0, \quad t=0 \Rightarrow y \neq \infty$

$x=\infty, \quad t=\infty \Rightarrow y \neq \infty$

$x=-\infty, \quad t=-\infty \Rightarrow y \neq \infty.$

II→ B.B-2

3.

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Intensity of wave (I) ⇒ from module (pg No. 164) →

$$\boxed{I = \frac{E_{max}}{A\,t}} = \frac{P}{A}$$

A, l, ρ, t, E_{max}

★★★

$$\boxed{I = 2\pi^2 a^2 n^2 \rho V}$$

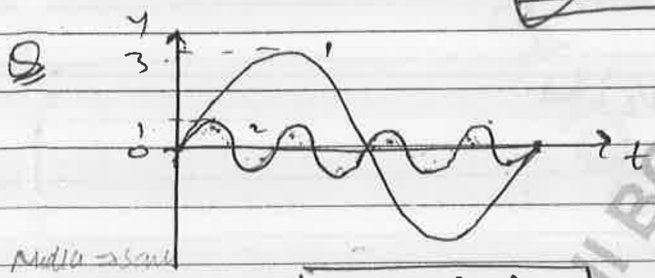
Result: * for a given medium → Medium const ⇒ ρ, V = const

then $I \propto a^2 n^2$

* for a given medium, for a given source ⇒ ρ, V & n = const

$$\boxed{I \propto a^2}$$

Q



find $\frac{I_1}{I_2}$ =

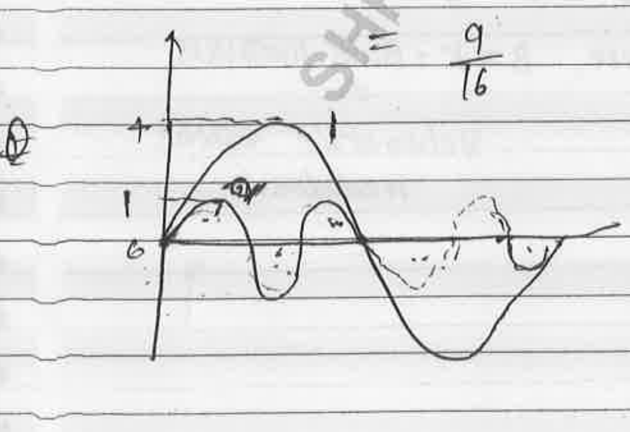
$$\boxed{I \propto a^2 n^2}$$

$$\frac{I_1}{I_2} = \left(\frac{a_1}{a_2}\right)^2 \left(\frac{n_1}{n_2}\right)^2$$

$$= \left(\frac{3}{1}\right)^2 \left(\frac{\cancel{7}}{\cancel{4}}\right)^2$$

$$= \frac{9}{16}$$

Q



$$\frac{I_1}{I_2} = \left(\frac{a_1}{a_2}\right)^2 \left(\frac{n_1}{n_2}\right)^2$$

$$= \left(\frac{4}{1}\right)^2 \left(\frac{1}{3}\right)^2$$

$$= \frac{16}{9}$$

→ wave front:

Q. A person stand at dist 'd' from a point sound source, when he moves 50 m dist toward point sound source, then Intensity becomes 4 times of initial. find 'd'

$I \propto \frac{1}{r^2}$ $\frac{I_1}{I_2} = \left(\frac{r_2}{r_1}\right)^2$

$\frac{I}{4I} = \left(\frac{d-50}{d}\right)^2$

$\frac{1}{2} = \frac{d-50}{d}$ $\Rightarrow$ $d = 2d - 100$

$\boxed{d = 100}$

* Speed of Mech. waves : (medium depends)

[1] Speed of longitudinal wave (V_L) →

⇒ Newton's formula - $V_L = \sqrt{\frac{E}{\rho}}$

where, E = Elastic modulus of medium

ρ = density of medium

(A) In solids →

$\boxed{V_L = \sqrt{\frac{Y}{\rho}}}$

Y = Youngs modulus

(B) In liquids

$V_L = \sqrt{\frac{B}{\rho}} = \sqrt{\frac{K}{\rho}}$

where, B = K = Bulk modulus or Volumetric elastic modulus.

imp

(C) In gaseous medium (sound waves)

$E_{gas} = \frac{stress}{strain} = \frac{\Delta P}{\frac{\Delta V}{V}} = -V\frac{\Delta P}{\Delta V}$

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Slow | Fast

Isothermal ($PV = const$) | Adiabatic ($PV^{\gamma} = C$)

$$\boxed{E_{iso} = -V\frac{\Delta P}{\Delta V} = P}$$

$$\boxed{E_{adia} = \frac{-V\Delta P}{\Delta V} = \gamma P}$$

* Newton's concept
⇒ Isothermal const
⇒ $V_L = \sqrt{\frac{E_{iso}}{\rho}} = \sqrt{\frac{P}{\rho}}$

* In air medium

$$V = \sqrt{\frac{P}{\rho}} = \sqrt{\frac{1\,atm}{1.29\,kg/m^3}}$$

$\simeq 280\,m/s$ ✗

But. $V_{practical} = 330\,m/s$

* Laplace correction / γ - correction
⇒ Adiabatic condⁿ
⇒ $V_L = \sqrt{\frac{E_{adi}}{\rho}} = \sqrt{\frac{\gamma P}{\rho}}$

⇒ In air medium

$$V = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\frac{1.4 \times 1\,atm}{1.29\,kg/m}} \simeq 332\,m/s$$

We know that,

* So, $\boxed{V_L = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\gamma\frac{RT}{M_w}}}$

AIIMS
* A-R

$E_{solid} >>> E_{Liquid} >>> E_{gas}$

$\rho_{solid} > \rho_{liquid} > \rho_{gas}$

$V_{solid} > V_{liquid} > V_{gas}$

Sound pace: $\simeq 5000\,m/s$, $\simeq 1500\,m/s$, $\simeq 330\,m/s$

Main deciding factor of Speed = Elastic modulus (E)

* Factors affecting the Speed in a gaseous medium.

① Effect of Tempʳ (T) →

$$V = \sqrt{\frac{\gamma RT}{M_w}} \Rightarrow \boxed{V \propto \sqrt{T}} \rightarrow \boxed{\frac{V_1}{V_2} = \sqrt{\frac{T_1}{T_2}}}$$

273

$$\begin{array}{r} 227 \\ 273 \\ \hline 5\cancel{1}0 \end{array}$$

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Q. If a gaseous medium of tempʳ is increased by 1000 kalvin then, speed of sound becomes $\sqrt{3}$ times of initial. find initial temp in °C.

$$\frac{V_1}{V_2} = \sqrt{\frac{T_1}{T_2}}$$

$$\frac{V}{\sqrt{3}V} = \sqrt{\frac{T}{T+1000}}$$

$$\frac{1}{3} = \frac{T}{T+1000} \Rightarrow$$

$$3T = T + 1000$$

$$T = 500K$$

or $T = 227°C$

Q. In what tempʳ speed of sound in O_2 medium is equal to speed of sound in H_2 gaes med at 27°C tempʳ.

$$V_{O_2} = V_{H_2}$$

$$\sqrt{\frac{\cancel{\gamma R}T}{32}} = \sqrt{\frac{\cancel{\gamma R} \times 300}{2}}$$

$$\frac{T}{32} = \frac{300}{2} \rightarrow T = 4800\,K$$

or $T = 4527°C$ ✓

Q. Speed of sound in air medium is 330 m/s at 0°c temp, then find speed of sound at 10°c tempʳ.

Soln

$$\frac{V_1}{V_2} = \sqrt{\frac{T_1}{T_2}}$$

$$\frac{330}{V_2} = \sqrt{\frac{273}{283}} \longrightarrow$$

$$V_t = V_0 + 0.61t$$

$$= 330 + 0.61 \times 10$$

$$= 336.1\,m/s$$

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$$\frac{V_{t°C}}{V_{0°C}} = \sqrt{\frac{t+273}{273}}$$

$$\frac{V_t}{V_0} = \left(1+\frac{t}{273}\right)^{\frac{1}{2}}$$

if $t <<< 273$ then,

$$\frac{V_t}{V_0} = \left(1+\frac{t}{2\times 273}\right)$$

$$\boxed{V_t = V_0\left(1+\frac{t}{546}\right)}$$

i; ~~up to 30°c~~

airy med.

* In air medium

$V_0 = 330$

$$V_t = V_0 + \frac{V_0 t}{546}$$

$$V_t = V_0 + \frac{330 \times t}{546}$$

$$\boxed{V_t = V_0 + 0.61t}$$

up to 30°C

② Effect of pressure (P) →

At const temp there is no change effect of Pressure change on speed of sound in a gaseous medium

$PV = \text{const}$

$\Rightarrow P \propto \frac{1}{V} \Rightarrow V \propto \frac{1}{\rho} \Rightarrow P \propto \rho$

$$V = \sqrt{\frac{\gamma P}{\rho}} \;\; \begin{matrix}\downarrow\uparrow \\ \downarrow\uparrow\end{matrix} = \text{const}$$

$$V = \sqrt{\frac{\gamma RT}{M_w}} = \text{const.}$$

Note:① if pressure changing at const volume condition then speed of sound is affected due to change in Tempr.

Note:② In Ques if Pressure & Tempr both are changing then only change tempr change is consider.

③ if no condition of Pressure change is given in question then const tempr condition is accepted. Ans → No change in speed of sound

$$\gamma_{mix} = \frac{f_{mix} + 2}{f_{mix}}$$

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③ Effect of nature of gaseous medium:

$$V = \sqrt{\frac{\gamma R T}{M_w}}$$ Here

$$\boxed{V \propto \sqrt{\frac{\gamma}{M_w}}} \Rightarrow \boxed{\frac{V_1}{V_2} = \sqrt{\frac{\gamma_1}{\gamma_2} \times \frac{(M_w)_2}{(M_w)_1}}}$$

$\Rightarrow \gamma_{Mono} = \frac{5}{3} \longrightarrow C_p = \frac{5}{2}R$
$\longrightarrow C_v = \frac{3}{2}R$

$\Rightarrow \gamma_{dia} = \frac{7}{5} \longrightarrow C_p = \frac{7}{2}R$
$\longrightarrow C_v = \frac{5}{2}R$

* $(M_w)_{mix} = \frac{n_1 M_{w_1} + n_2 M_{w_2}}{n_1 + n_2}$

* $\gamma_{mix} = \frac{n_1 C_{p_1} + n_2 C_{p_2}}{n_1 C_{v_1} + n_2 C_{v_2}}$

Que. Calculate the speed of sound in a gaseous mixture of 1 mole of H_2 & 1 mole of He at 27°C temp.

Sol^n

$$V = \sqrt{\frac{\gamma R T}{M_w}}$$

* $R = 8.314$ J/mol K
* $T = 27°C + 273 = 300$ K
* $M_w = \frac{1 \times 2 + 1 \times 4}{2} = \frac{6}{2} = 3$ gm/mol $= 3 \times 10^{-3}$ kg/mol.
* $\gamma_{mix} = \frac{f_{mix} + 2}{f_{mix}}$ or $\frac{1 \times \frac{7}{2}R + 1 \times \frac{5}{2}R}{1 \times \frac{5}{2}R + 1 \times \frac{3}{2}R} = \frac{12}{8} = 3/2$

$$V = \sqrt{\frac{\frac{3}{2} \times 8.314 \times 300}{3 \times 10^{-3}}} = \approx 1100 \text{ m/s}$$

Q. Calculate ratio of speed of sound in H_2 to He gaseous medium at $27^\circ C$ temp.

$$\frac{v_1}{v_2} = \sqrt{\frac{\gamma_1}{\gamma_2} \times \frac{(M_{w_2})}{(M_{w_1})}} = \sqrt{\frac{7/5}{5/3} \times \frac{4}{2}} = \sqrt{\frac{42}{25}}$$

Q. speed ratio = ? T = cons

$$\frac{V_{O_2}}{V_{O_2+N_2}} = ?$$

1 : 4

$$\sqrt{\frac{\gamma_{O_2}}{\gamma_{mix}} \times \frac{(M_{w_2})}{(M_{w_1})}} = \sqrt{\frac{7/5}{7/5} \times \frac{28.8}{32}} = \sqrt{\frac{28.8}{32}}$$

$$= \sqrt{\frac{9}{10}}$$

$$(M_w)_{mix} = \frac{1\times 32 + 4\times 28}{1+4}$$

$$= \frac{144}{5} = 28.8 = (M_w)_{air}$$

4. Effect of density (Humidity) →

$$V = \sqrt{\frac{\gamma P}{\rho}} \Rightarrow V \propto \frac{1}{\sqrt{\rho}}$$

$\Rightarrow \boxed{\rho_{dry\ air} > \rho_{humid\ air}}$

So, $\boxed{V_{humid\ air} > V_{dry\ air}}$

* $\rho \propto M_w$

$(M_w)_{dry\ air} > (M_w)_{Humid\ air}$

28.8 ; dry air + H_2O

28.8 ; 18

= b/w 18 and 28.8

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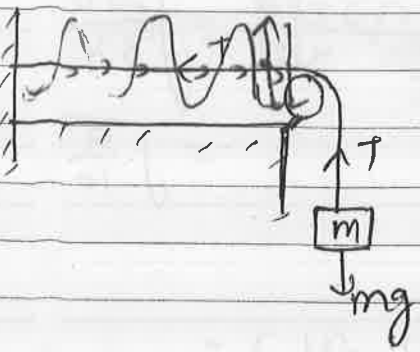
(5) Effect of Wind (V_w) →

[2] Speed of transerse Mech. wave (V_T) :→

* Speed of transverse wave on a stretched wire :→



$$\boxed{V_T = \sqrt{\frac{T}{\mu}}}$$

where, T = tension in wire

$\mu = m$ = mass per unit length of wire

or

Linear mass density

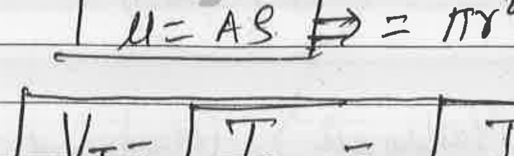
or

linear density

$$\mu = \frac{M_{wire}}{L} = \left(\frac{kg}{m}\right) = [M^1 L^{-1} T^0]$$

wire,

*



$$\mu = \frac{M_{wire}}{L} = \frac{Vol. \times dens}{L} = \frac{(AL)\rho}{L}$$

$$\boxed{\mu = A\rho} \Rightarrow = \pi r^2 \rho$$

so,

$$\boxed{V_T = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{T}{A\rho}} = \sqrt{\frac{T}{\pi r^2 \rho}}}$$

where, A = cross-section area of wire

r = radius of wire

ρ = density of wire

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* $V_T \propto \sqrt{T}$ ⇒ Tension ↑ wave spd ↑ ...

* $V_T \propto \frac{1}{\sqrt{\mu}}$ ⇒ two wire, same L, one Hollow, Solid

* $V_T \propto \frac{1}{r}$ ⇒ Q* Guitar wire should be hollow and thin.

$$\Rightarrow V_T = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{T}{A\rho}} = \boxed{\sqrt{\frac{stress}{\rho}} = \sqrt{\frac{Y \times strain}{\rho}}}$$

$$V_T = \sqrt{\frac{Y}{\rho}} \times \sqrt{strain}$$

$$\boxed{V_T = V_L \sqrt{strain}} \qquad V_L > V_T$$

AIEEE

Q Calculate speed of longitudinal wave and transverse mech. wave produced in a solid wire of length 1.5 m, if given that $Y = 2.2 \times 10^{11}\ N/m^2$, $\rho = 8.8 \times 10^3\ kg/m^3$, strain = 1%

$$V_L = \sqrt{\frac{Y}{\rho}} = \sqrt{\frac{2.2 \times 10^{11}}{8.8 \times 10^3}} = \frac{1}{2} \times 10000$$

$$= 5000\ m/s$$

$$V_T = V_L \sqrt{strain}$$

$$= 5000 \sqrt{\frac{1}{100}}$$

$$= 500\ m/s$$

B.B.Q
Q. No. 4
pg - 160

$$\mu = \frac{Mwire}{L} = \frac{4.5 \times 10^{-3}\ kg}{2.25\ m} = 2 \times 10^{-3}\ kg/m$$

$$T = 2g = 20N$$

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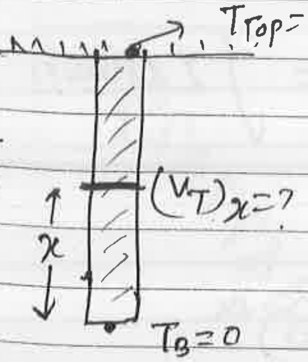
$$V_T = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{20}{2\times10^{-3}}} = 100\, m/s$$

$$t = \frac{L}{V} = \frac{2}{100} = 0.02\, sec.$$

Q.7
Pg.no. 168



$\mu = \frac{M}{L}$

$T_x = \frac{M}{L} x g$

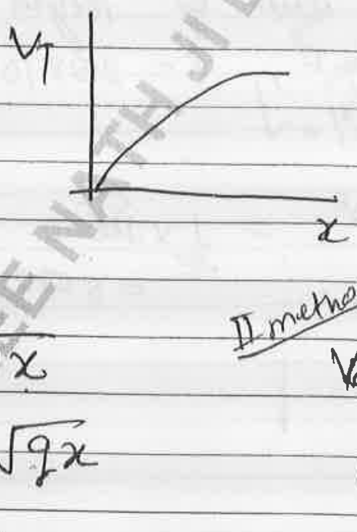
① $(V_T)_x = \sqrt{\frac{T_x}{\mu}}$

$= \sqrt{\frac{\frac{M}{L} x g}{\frac{M}{L}}}$

$$\boxed{V_T = \sqrt{xg}}$$ DR

I/S $x = 0 \Rightarrow V_{Bottom} = 0$

$x = L \Rightarrow V_{Top} = \sqrt{Lg}$



② Time,

$(V_T)_x = \sqrt{gx}$

$\frac{dx}{dt} = \sqrt{gx}$

$$\int_0^L \frac{dx}{\sqrt{gx}} = \int_0^t dt$$

$$t = \frac{1}{\sqrt{g}}\left[\frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}\right]_0^L = \frac{1}{\sqrt{g}}\left(\frac{L^{\frac{1}{2}}}{\frac{1}{2}}\right) = 2\sqrt{\frac{L}{g}}$$

II method

$g/2$ in upward dir^n.
⇑

$V_{avg} = \frac{V_{bottom} + V_{Top}}{2}$ (acc^n const)

$V_{avg} = \frac{0 + \sqrt{Lg}}{2}$

$t = \frac{L}{V_{avg}} = \frac{2L}{\sqrt{Lg}} = 2\sqrt{\frac{L}{g}}$

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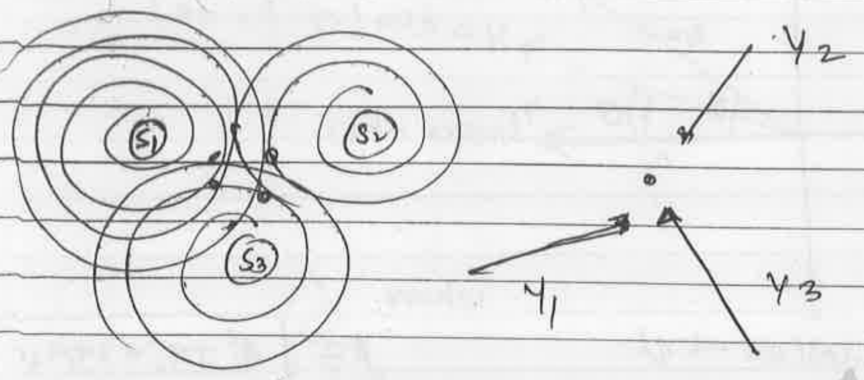
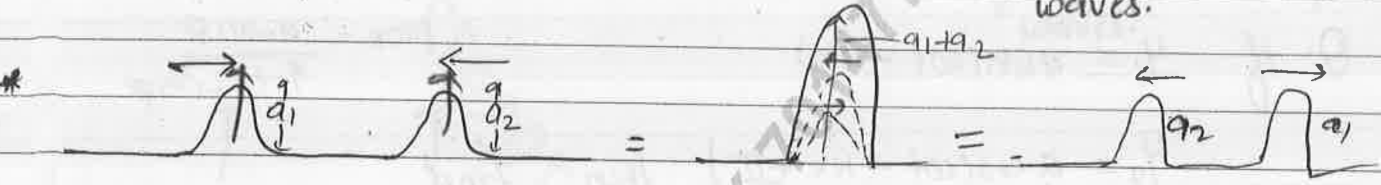
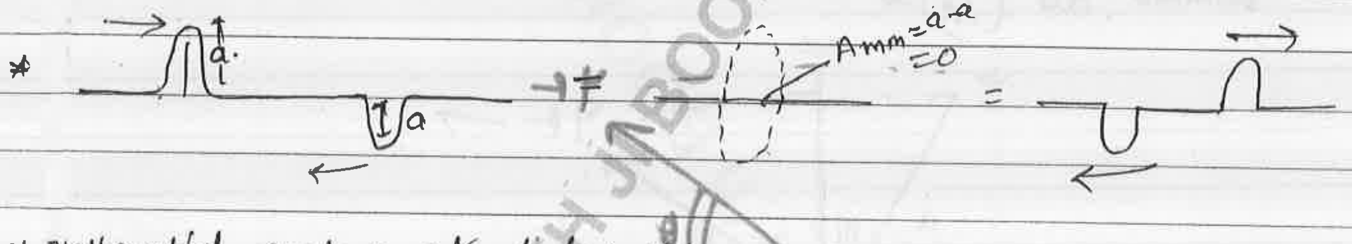
Superposition of waves →



$$\left|\vec{Y} = \vec{Y_1} + \vec{Y_2} + \vec{Y_3}\right|$$

It is called as principle of superposition of waves.





* Mathematical analysis of superposition of waves –

let two waves –

$$Y_1 = a_1 \sin(\omega t - kx)$$

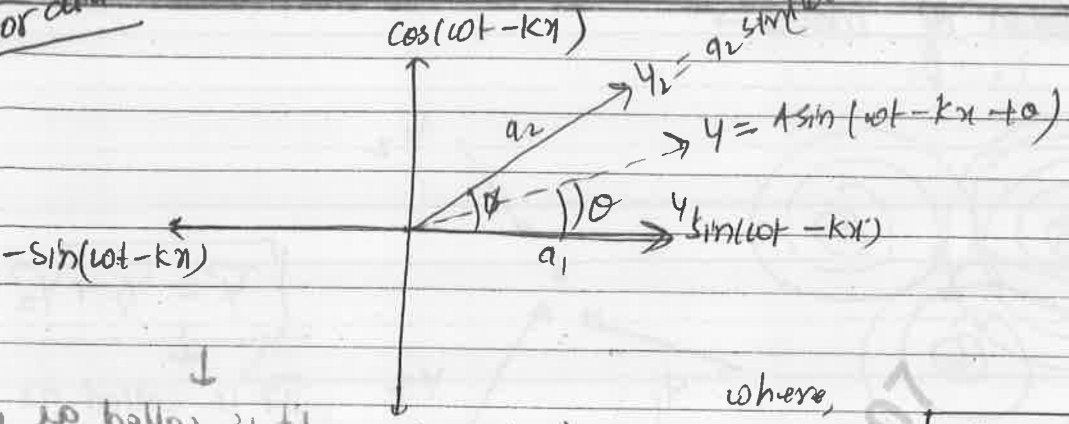
$$Y_2 = a_2 \sin(\omega t - kx + \phi)$$

$$\Delta\phi = \phi$$

then, resultant disp → $\left|\vec{Y} = \vec{Y_1} + \vec{Y_2}\right|$

$$\left|Y = A \sin(\omega t - kx + \theta)\right|$$

Phasor dia



where,

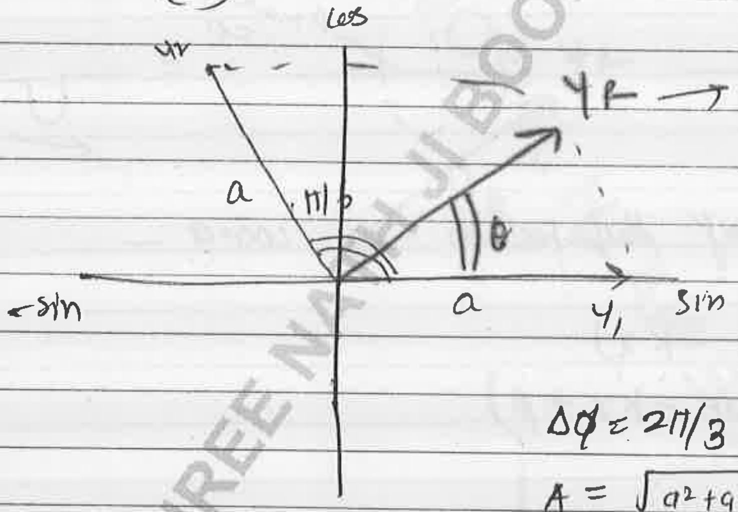
$$A = \sqrt{a_1^2 + a_2^2 + 2a_1a_2\cos\phi}$$

$$\tan\theta = \frac{a_2\sin\phi}{a_1 + a_2\cos\phi}$$

It is called as principle of super position of waves.

Q. If $y_1 = a\sin(\omega t - kx)$

$y_2 = a\cos(\omega t - kx + \frac{\pi}{6})$ then find resultant dis $(y_R) \rightarrow$



$$\Delta\phi = 2\pi/3$$

$$A = \sqrt{a^2 + a^2 + 2a^2 \times \cos\frac{2\pi}{3}} = a$$

$$\tan\theta = \frac{a\sin 2\pi/3}{a + a\cos\frac{2\pi}{3}} = \sqrt{3}$$

$$\theta = \pi/3$$

$$y_R = a\sin(\omega t - kx + \pi/3)$$

- Coherent ⇒ f same | (S1) ~~ ν
(S2) ~~ ν same

they may show interference → in case of light ✗
→ of sound ✓

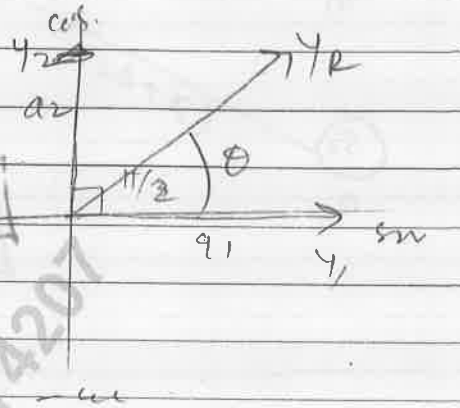
Q. If $y_1 = a_1 \sin(\omega t - kx)$ & $y_2 = a_2 \cos(\omega t - kx)$ find Resultant.

$\Delta\phi = \pi/2$

$A = \sqrt{a_1^2 + a_2^2}$

$\tan\theta = \dfrac{a_2 \sin \pi/2}{a_1 + a_2 \cos\frac{\pi}{2}} = \dfrac{a_2}{a_1}$

$\theta = \tan^{-1}\left(\dfrac{a_2}{a_1}\right)$



$$\boxed{y_R = \sqrt{a_1^2 + a_2^2}\, \sin\left(\omega t - kx + \tan^{-1}\left(\frac{a_2}{a_1}\right)\right)}$$

$\Delta\phi = \frac{\pi}{2}$

$\sqrt{a_1^2 + a_2^2} \sin(\omega -$

Interfeerance of sound wave :

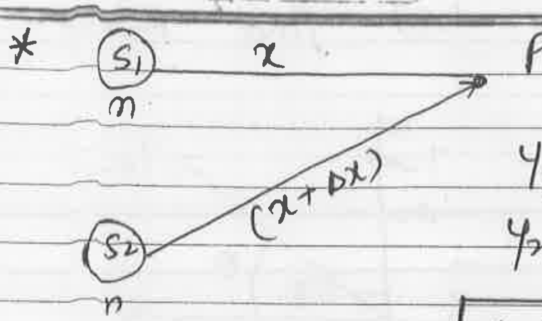
When two or more than 2 coherent wave (Phase diff remains const. w.r.t time) of same frequencies propagate in same dir^n superimpose then Amplitude & Intensity of Resultant wave becomes max^m at some position and min^m at some other position, this phonomena of Intensity variation w.r.t position is c/a Interfeerance.

$$\boxed{I = f(x) \text{ but } I \neq f(t)}$$

- Total energy of wave remains const but here only redistribution of enegy takes place.

- Same Amplitude is not an essential condition. (favourable condition)

*



$$y_1 = a_1 \sin(\omega t - kx)$$
$$y_2 = a_2 \sin(\omega t - k(x+\Delta x))$$

$$\boxed{\Delta\phi = k(\Delta x)}$$ → cons. w.r.t time
⇒ coherent waves
⇒ $n_1 = n_2 = n$
⇒ $\xrightarrow{\oplus x}$ $\xrightarrow{\oplus x}$

then, result disp. at 'P' point → $\vec{y} = \vec{y_1} + \vec{y_2}$

$$\boxed{y = \underline{\underline{A}} \sin(\omega t - kx + \underline{\underline{\theta}})}$$

where,

$$A = \sqrt{a_1^2 + a_2^2 + 2a_1a_2\cos(\Delta\phi)}$$

$$\boxed{A^2 = a_1^2 + a_2^2 + 2a_1a_2\cos(\Delta\phi)}$$

But $I \propto a^2$

$$\boxed{I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos(\Delta\phi)}$$ ☆

Where, $\Delta\phi = k(\Delta x)$ depends on position but does not depend on time

CI

* $\cos(\Delta\phi) = +1$

$\Delta\phi = \underline{0, 2\pi, 4\pi, \dots 2n\pi}$ $(n = 0, 1, 2 \dots)$

$\Delta x = \underline{0, \lambda, 2\lambda, \dots n\lambda}$

* $A_{max} = a_1 + a_2$
* $I_{max} = I_1 + I_2 + 2\sqrt{I_1 I_2}$
$= (\sqrt{I_1} + \sqrt{I_2})^2$

DI

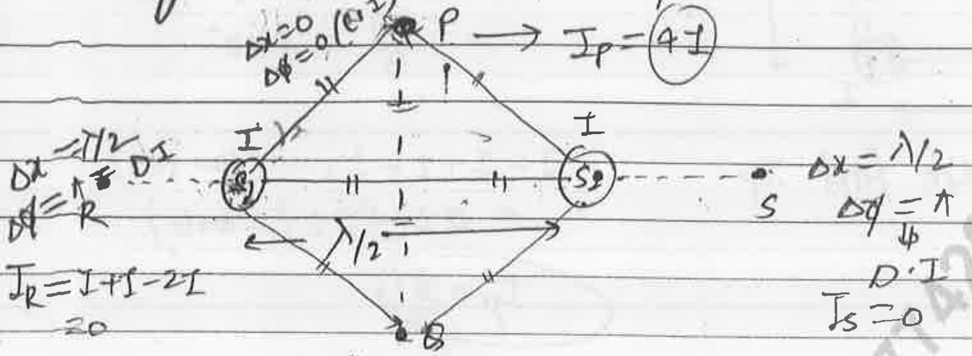
* $\cos(\Delta\phi) = -1$

$\Delta\phi = \pi, 3\pi, 5\pi \dots (2n+1)\pi$ $(n = 0, 1, 2, 3 \dots)$

$\Delta x = \frac{\lambda}{2}, \frac{3\lambda}{2}, \frac{5\lambda}{2} \dots (2n+1)\frac{\lambda}{2}$

* $A_{max} = a_1 \sim a_2$
* $I_{min} = I_1 + I_2 - 2\sqrt{I_1 I_2}$
$= (\sqrt{I_1} - \sqrt{I_2})^2$

Q. S_1 & S_2 are two coherent source produce sound waves of equal Intensity 'I'. find type of interference and resultant Intensity at P, Q, R, S points.

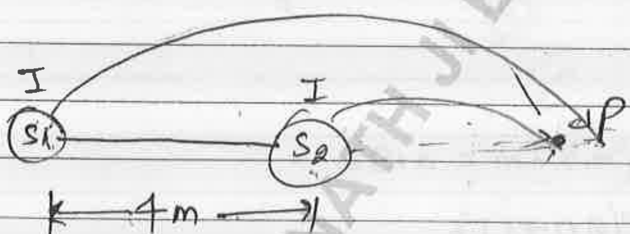


$\Delta x = 0$

$\Delta\phi = 0 \Rightarrow$ C.I

$I_Q = I + I + 2I = 4I$

Q.



$\Delta x = 4m$

$\frac{\Delta\phi}{2\pi} = \frac{\Delta x}{\lambda}$

$\Delta\phi = \frac{4}{3} \times 2\pi = \frac{8\pi}{3}$

If $\lambda = 3m$ then find I_P.

Sol^n $\Delta x = 4m$, $\lambda = 3m$

$\Delta\phi = K(\Delta x)$

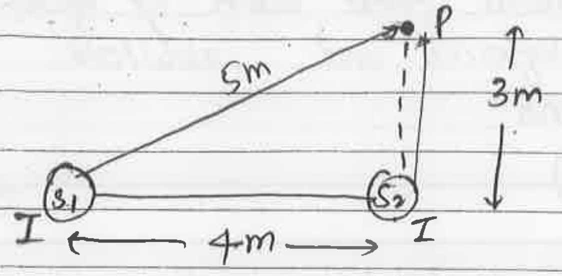
$\Delta\phi = \frac{2\pi}{3} \times 4 = \frac{8\pi}{3} = 480° = 360 + 120°$

Now, $I_P = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos(\Delta\phi)$

$= 2I + 2I\left(-\frac{1}{2}\right)$

$I_P = I$

Q.



$\Delta x = 2m,$

$\Delta\phi = \frac{2\pi}{\lambda}\times 2$

$= \frac{4\pi}{3} = 240^\circ$

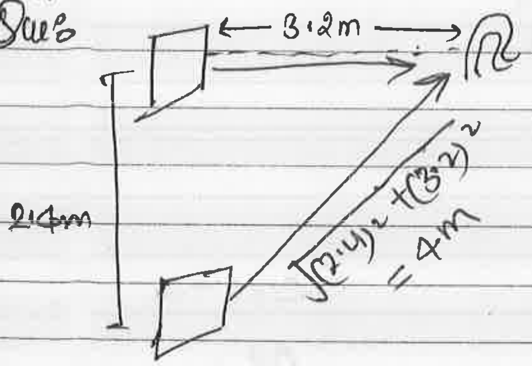
If $\lambda = 3m$ then find I_p

$I_p = I + I + 2I\cos(180+60)$

$= 2I + 2I(-\cos 60)$

$\boxed{I_p = I}$

Que



If speed of sound is 320 m/s then find possible frequencies of sound for which person will hear minimum intensity of sound.

from dia → $\Delta x = 4m - 3.2m = 0.8m$

For D.I → $\Delta x = (2n+1)\frac{\lambda}{2}$

$(2n+1)\frac{\lambda}{2} = 0.8$

$\boxed{\lambda = \frac{1.6}{2n+1}}$

Now, $f = \frac{v}{\lambda} = \frac{320}{1.6}(2n+1) = 200(2n+1)$

$\boxed{f = 200(2n+1)}$

If $n=0 \longrightarrow f_1 = 200\,Hz$

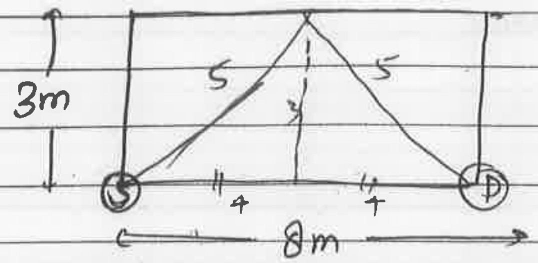
$n=1 \longrightarrow f_2 = 600\,Hz$

Or max^m hear ⇒ C.I

Q17 Janquey (Room) mat imp

Que.

If speed of sound is 320 m/s then find possible freq. of sound for which person will hear max^m intensity of sound.



$\Rightarrow \Delta x' = 10m - 8m = 2m$ — ①

$\Rightarrow \Delta x = n\lambda$ — ②

$n\lambda = 2 \Rightarrow \lambda = \frac{2}{n}$

$f = \frac{V}{\lambda} = \frac{320(n)}{2} = 160n$

$f = 160n$

$n = 0$ ×

$n = 1 \longrightarrow f_1 = 160\,Hz$

$n = 2 \longrightarrow f_2 = 320\,Hz$

* Imp. formula

* $\boxed{\frac{I_1}{I_2} = \frac{a_1^2}{a_2^2}}$

* $\frac{I_{max}}{I_{min}} = \frac{A_{max}^2}{A_{min}^2} = \left(\frac{a_1 + a_2}{a_1 - a_2}\right)^2 = \left(\frac{\sqrt{I_1} + \sqrt{I_2}}{\sqrt{I_1} - \sqrt{I_2}}\right)^2$

* $I_{avg} = \frac{I_{max} + I_{min}}{2}$

$= \frac{I_1 I_2 + 2\sqrt{I_1 I_2} + I_1 + I_2 - 2\sqrt{I_1 I_2}}{2}$

imp

$\boxed{I_{avg} = I_1 + I_2} \Rightarrow$

It shows the total intensity in the medium.

* Degree of interference pattern (degree of hearing)

$\boxed{(f\,\%) = \frac{I_{max} - I_{min}}{I_{max} + I_{min}} \times 100}$, not imp

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if $f\% = 100$ then perfect interferance

* Condition for perfect Interference

(1) $f = 100\%$
(2) $I_{min} = 0$
(3) $I_1 = I_2 = I$
(4) $a_1 = a_2 = a$
(5) $A_{max} = 2a$
(6) $A_{min} = 0$
(7) $I_{max} = 4I$

Q. If degree of interferance is 60%. Then find $\frac{I_{max}}{I_{min}}$ & $\frac{I_1}{I_2}$

(i) $60 = \frac{I_{max} - I_{min}}{I_{max} + I_{min}} \times 100$

$\frac{3}{5} = \frac{I_{max} - I_{min}}{I_{max} + I_{min}}$

$3I_{max} + 3I_{min} = 5I_{max} - 5I_{min}$

$2I_{max} = 8I_{min}$

$\frac{I_{max}}{I_{min}} = \frac{8}{2} = \frac{4}{1}$

(ii) $\frac{I_{max}}{I_{min}} = \left(\frac{\sqrt{I_1} + \sqrt{I_2}}{\sqrt{I_1} - \sqrt{I_2}}\right)^2$

$\frac{2}{1} = \frac{\sqrt{I_1} + \sqrt{I_2}}{\sqrt{I_1} - \sqrt{I_2}}$

$2\sqrt{I_1} - 2\sqrt{I_2} = \sqrt{I_1} + \sqrt{I_2}$

$\sqrt{I_1} = 3\sqrt{I_2}$

$\frac{\sqrt{I_1}}{\sqrt{I_2}} = \frac{3}{1}$

$\left(\frac{I_1}{I_2} = \frac{9}{1}\right)$

Quincke's tube experiment

* It is a practical proof of interferance of sound waves.
* It is used to find speed & wavelength of sound waves in gaseous medium.
* Ultrasonic sound waves are used in it (due to high intensity)

Infrasonic	Sonic	Ultrasonic
$n < 20Hz$	$20Hz$ to $20kHz$ (audible range)	$n > 20kHz$

3mm 6mm = $\frac{\lambda}{2}$

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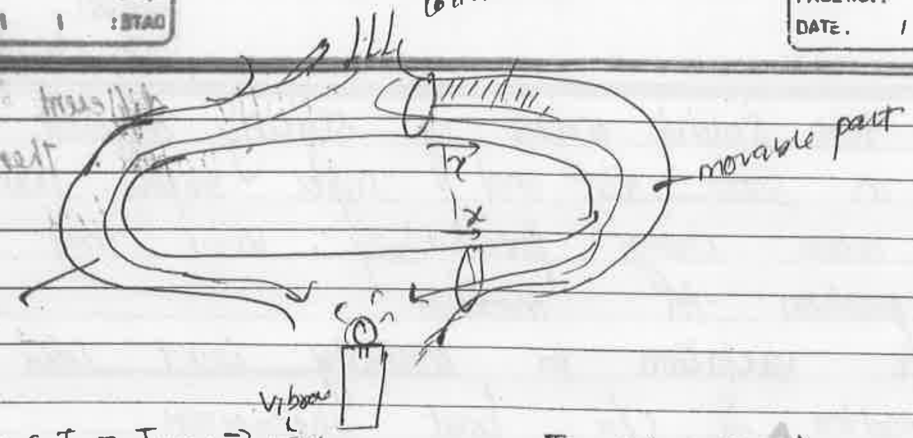

$* \ \Delta x = 0 \Rightarrow C.I = I_{max} \Rightarrow max$

$\Downarrow$

$\Delta x = \frac{\lambda}{2} \Leftarrow D.I \Leftarrow I_{min} \Leftarrow min$

$\Downarrow$

$\Delta x = \lambda \Leftarrow C.I \Leftarrow I_{max} \Leftarrow max$

To calculate λ

① from vibration in flame -

max to max, min to min $\Big\rangle \ \Delta x = \lambda$

max to min, min to max $\Big\rangle \ \Delta x = \frac{\lambda}{2}$

② If sliding amount is 'x' then path difference becomes "$2x$"

Q. No. 5 Pg. 101

Q. No - 4 Pg No. 100

(5) $\Delta x = 2 \times 16.2$

$= 33.2 \text{ cm} = 33.2 \times 10^{-2} \text{ m}$

$\Delta x = \lambda$

$\lambda = 33.2 \times 10^{-2} \text{ m}$

Now,

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Beats when two sound waves of slightly different frequency propagate in same dir^n and superimpose. Then Intensity of wave change periodically w.r.t time at particular position in medium.

This periodic variation in Intensity w.r.t time at a particular position is c/a beat phenomena.

* $\longrightarrow n_1$ $\qquad \Delta n \leq 10\,Hz$

$\longrightarrow n_2$ $\qquad$ or $\quad \Delta n \not> 10\,Hz$

let two sound waves at $x=0$ (meet)

$$y_1 = a_1 \sin \omega_1 t = a_1 \sin 2\pi n_1 t$$
$$y_2 = a_2 \sin \omega_2 t = a_2 \sin 2\pi n_2 t$$

$$\boxed{\Delta\phi = (\omega_2 - \omega_1)t} \rightarrow \text{time dependent.}$$

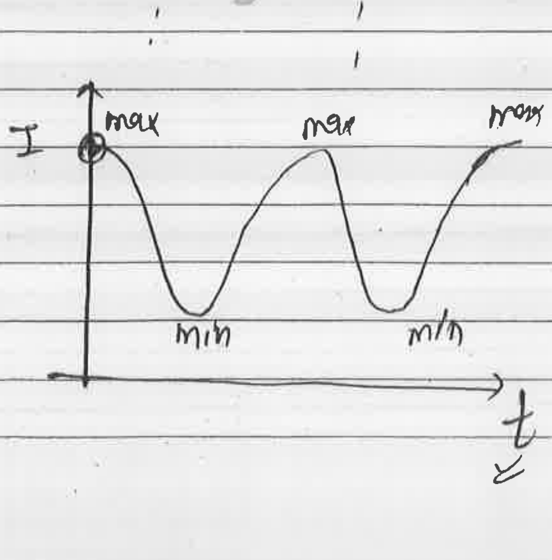
then resultant intensity $\boxed{I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos(\omega_2 - \omega_1)t}$

$\cos(\omega_2-\omega_1)t \rightarrow$ time dependent.

* $t=0 \Rightarrow \Delta\phi = 0 \Rightarrow I_{max}$

$t = \uparrow \Rightarrow \Delta\phi = \pi \Rightarrow I_{min}$

$t = \uparrow \Rightarrow \Delta\phi = 2\pi \Rightarrow I_{max}$



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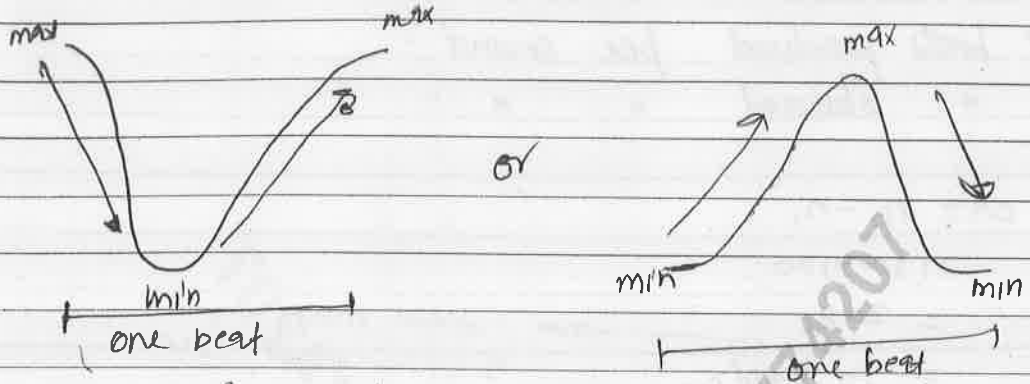
*<u>One beat</u>: one time increment and one time decrement in intensity is c/a as one beat.



Note:- <u>one maxima</u> and <u>one Minima</u> are present in one beat.

* <u>Beat time period (T_b)</u>: Time interval during which <u>phase difference change</u> by amount of <u>2π</u> or <u>Intensity become</u> <u>max to max</u> or <u>min to min</u> is c/a as beat time period.

$$\boxed{\Delta\phi = (\omega_2 - \omega_1)\, t}$$

$\Delta\phi \to 2\pi$, $t \to T_b$

$$\Rightarrow \quad 2\pi = (\omega_2 - \omega_1) T_b$$

$$2\pi = 2\pi (n_2 - n_1) T_b$$

$$1 = (n_2 - n_1) T_b$$

$$\boxed{T_b = \frac{1}{n_2 - n_1} = \frac{1}{\Delta n} = \frac{1}{\text{beat freq (b)}}}$$

+ <u>Beat frequency</u> (b) $\Rightarrow \Delta n = n_2 \sim n_1$

= No. of beats produced / sec

$$\boxed{b = \frac{1}{T_b}}$$

visibility $= \frac{1}{16}$ sec

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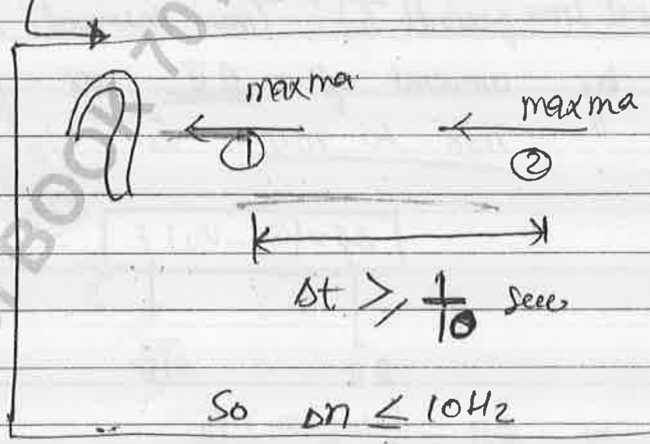
Q. Two sound waves of freq 100 & 120 Hz produced beat phenomena.
1) No. of beats produced per second.
2) " " " observed " " .

Sol^n ① $b = \Delta n = n_2 - n_1$
$= 120 - 100$
$= 20\ Hz$ → max = 20 Hz maxima/sec.
$= 20$ beats/sec — 20 Hz min

② $b = \Delta n = 0$ [due to persistance of hearing]



another exp
$A.C = 50\ Hz$
light.

imp. points

⇒ * freq. of resultant intensity variation = beat freq(b) $= \Delta n = n_2 - n_1$

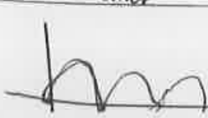
* freq. of resultant amplitude variation (n_{amp}) =

$$\frac{b}{2} = \frac{\Delta n}{2} = \frac{n_2 - n_1}{2}$$

$I \propto a^2$
↓ ↓
$b = 2 n_{amp}$

$n_{amp} = \frac{b}{2}$

$U = \frac{1}{2} k x^2$
$U \propto x^2$
↓ ↓
2ω ω



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* ~~freq.~~ of resultant wave (n_{avg}) $= \frac{n_1+n_2}{2}$

$\longrightarrow n_1$

$\longrightarrow n_{avg} = \frac{n_1+n_2}{2}$

$\longrightarrow n_2$

(2) Two ~~sound~~ waves - $y_1 = 0.3 \sin 596\pi \left(t - \frac{x}{330}\right)$ &

$y_2 = 0.5 \sin 604\pi \left(t - \frac{x}{330}\right)$

produced beat phenomena.

(1) no. of beats produced per minute. $\Rightarrow$ beat freq

$$n_1 = \frac{\omega_1}{2\pi} = \frac{596\pi}{2\pi} = 298\ Hz$$

$$n_2 = \frac{\omega_2}{2\pi} = \frac{604\pi}{2\pi} = 302\ Hz$$

$$\Delta n = n_2 - n_1$$
$$= 302 - 298$$
$$= 4\ Hz$$
$$= 4\ \text{beats/sec}$$
$$= 4 \times 60 = 240\ \text{beats/min}$$

(2) Time taken to complete 5 beats.

Time period of beats per formation

$$\rightarrow T_b = \frac{1}{b} = \frac{1}{\Delta n} = \frac{1}{4}\ sec$$

$$t = 5T_b = \frac{5}{4}\ sec$$

(3) frequency of Resultant intensity variation.

$$b = \Delta n = 4\ Hz$$

(4) frequency of Resultant amplitude variation.

$$n_{amp} = \frac{b}{2} = \frac{\Delta n}{2} = \frac{4}{2} = 2\ Hz$$

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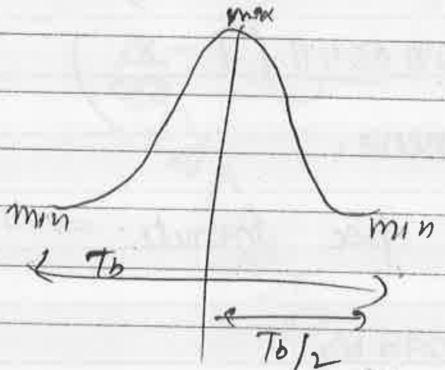
⑤ frequency of Resultant wave

$$n_{avg} = \frac{n_1 + n_2}{2} = \frac{302 + 298}{2} = 300\,Hz$$

⑥ Time interval during which Intensity becomes max^m to min.



$$t = T_b/2 = \frac{1}{4}/2 = \frac{1}{8}\ sec$$

Imp

⑦. waxing and waning intensity $\longrightarrow$ ratio $= \left(\frac{I_{max}}{I_{min}}\right) \rightarrow$

$$\frac{I_{max}}{I_{min}} = \left(\frac{a_1 + a_2}{a_1 - a_2}\right)^2$$

$$= \left(\frac{0.3 + 0.5}{0.3 - 0.5}\right)^2$$

$$= \frac{16}{1}$$

⑧ beats appearence timing.

$b = \Delta n = 4$ beats/sec — equal lim

$$t = \frac{1}{4}\ sec,\ \frac{2}{4}\ sec,\ \frac{3}{4}\ sec,\ \frac{4}{4}\ sec.$$

$b = \text{Maxm freq} - \text{Minm freq}$

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Q Three sound sorces of freq 401 Hz, 403 Hz, & 405 Hz vibrate together and produces beat phenomena. Then find beat freq.

Trick

* 2 beats/sec ⟶ $\frac{1}{2}$ $\frac{2}{2}$
* 2 beats/sec ⟶ $\frac{1}{2}$ $\frac{2}{2}$
* 4 beats/sec ⟶ $\frac{1}{4}$ $\frac{2}{4}$ $\frac{3}{4}$ $\frac{4}{4}$

1 1 1 1

Ans = $b = 4$ beats/sec

Q find beat frequency

300 Hz 302 Hz 305 Hz

2 3

5

2 → $\frac{1}{2}$ $\frac{2}{2}$

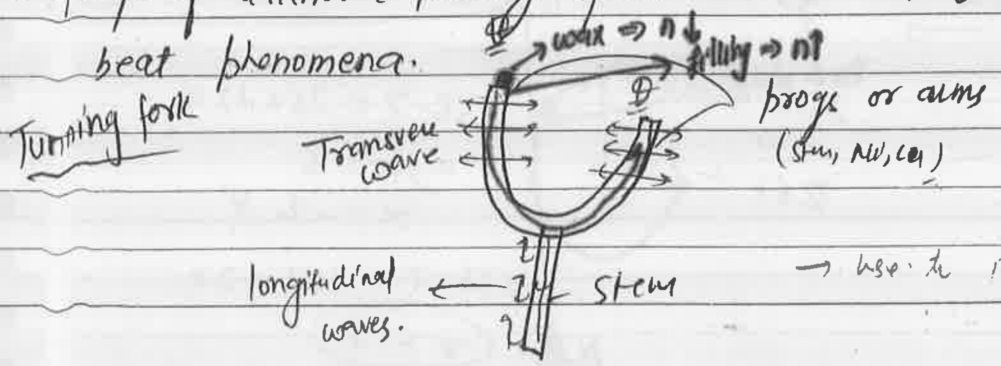
3 → $\frac{1}{3}$ $\frac{2}{3}$ $\frac{3}{3}$

5 → $\frac{1}{5}$ $\frac{2}{5}$ $\frac{3}{5}$ $\frac{4}{5}$ $\frac{5}{5}$

~~5 beat/sec~~ 8 beats/sec $b = 8$ beats/sec

Application of beat Phenomena →

* freq. of unknown tunning fork can be calculated by using beat phenomena.



→ use to produce Sound waves.

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Q. A tunning fork 'A' produce 5 beats/sec with a fork of freq. 200 Hz and produces 10 beats/sec with another fork of freq. 205 Hz. find freq. of fork 'A'.

200 Hz —— 5 —— A

posi'

195 Hz
or
205 Hz

(1)

205 Hz —— 10 beats/sec —— A

195 Hz
or
215 Hz

$n_A = 195\ Hz$

Easy

Q. A tunning fork 'A' produce some beats per sec with a fork of freq 256 Hz and produce double of that beats/sec with another fork of freq. 262 Hz find possible frequency of fork 'A'

256 —— x —— A

$256+x$
or
$256-x$

262 —— $2x$ —— A

$262+2x$
or
$262-2x$

$256+x = 262+2x$

$x = -6$ ✗

$256+x = 262-2x$

$x = 2$ ✓

$\boxed{n_A = 256+2 = 258\ Hz}$

$256-x = 262+2x$

$-3x = 6$

$x = -2$ ✗

$256-x = 262-2x$

$x = 6$

$\boxed{n_A = 256-6 = 250\ Hz}$

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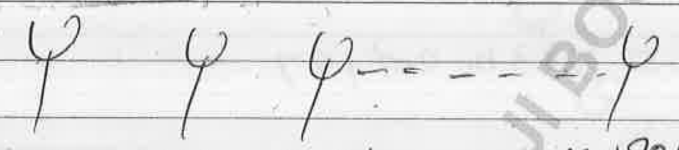
Q: 81 tunning fork are arranged is ascending order in such that each tunning fork produced 4 beats / sec with it's neighbouring fork. if frequency of last fork is octave of freq of 1st fork. then find n_{25} an n_{81}

→ Octave = Double freq. $\left(\frac{n_2}{n_1} = \frac{2}{1}\right)$
or — or
octave higher — two times freq

* unison = same freq. $\left(\frac{n_2}{n_1} = \frac{1}{1}\right)$

① Sa → 24 Hz, Re → 27 Hz, Ga → 30 Hz - - - - ② Sa → 48 Hz



n $\quad n+b \quad n+2b \quad \ldots \quad n+80b$

$2n = n + 80b$

$2n = n + 80 \times 4$

$n = 320\,Hz$

$n_{25} = n + 24b$

$= 320 + 24 \times 4 = 320 + 96$

$= 416\,Hz$

$n_{81} = n + 80b$

$= 320 + 80 \times 4 = 320 + 320$

$= 640\,Hz$

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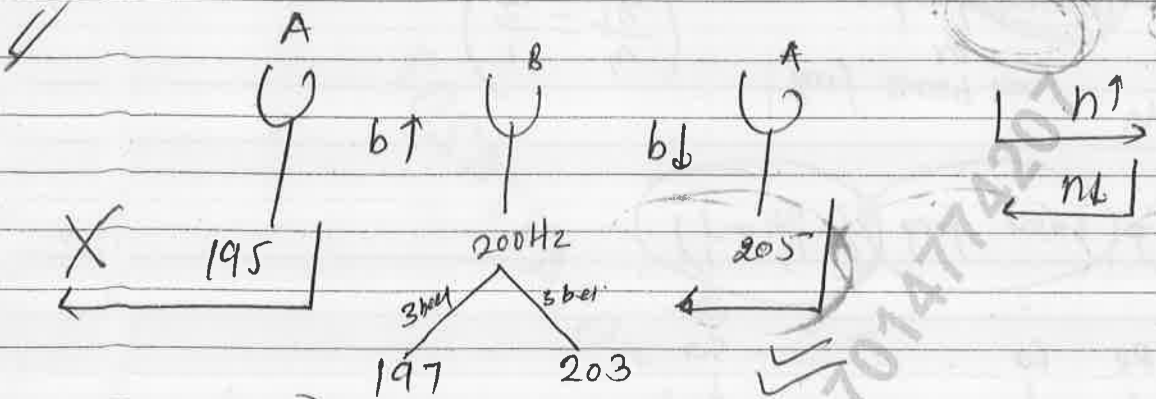
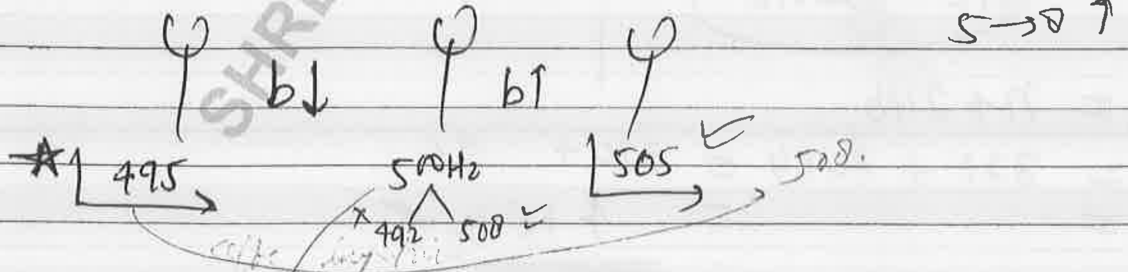
Q. A tunning fork 'A' produced 5 beats/sec with a fork 'B' of freq 200 Hz. Now, after loading wax on fork 'A' and vibrate again with fork 'B', 3 beats/sec produce. Find freq of fork 'A' before loading wax and after loading wax.



Ans ① Before loading = 205 Hz

② After loading = 203 Hz and 197

Q. A tunning fork 'A' produce 5 beats/sec with a fork of 'B' ~~with~~ of freq 500 Hz. Now after filling the fork 'A' and vibrate again fork B. 8 beats/sec produced. Find freq of fork A before & after filling.



Ans → ① Before → 505 Hz and 495 Hz

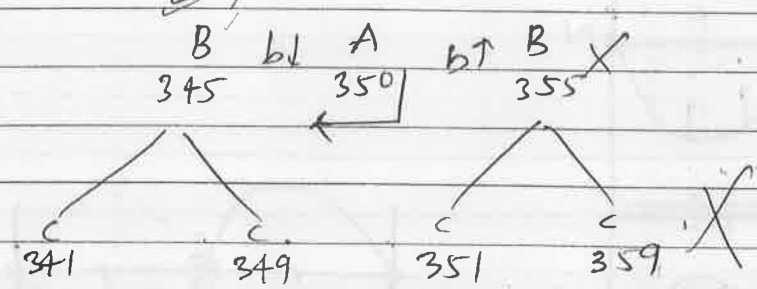
② After → 508 Hz

b = beat freq

• loading
• filling

Q-38

Pg No 209

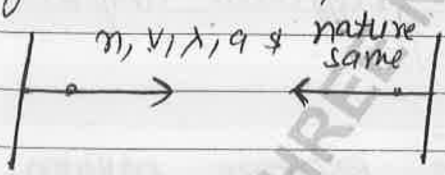


(B) → 345 beats

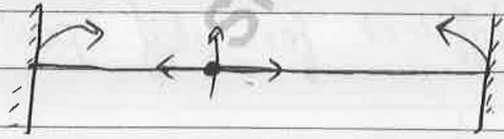
(C) → 341 Hz and 349 Hz

Stationary waves when two exact (n, v, λ, a and nature same) waves propagate in opp. dir^n and superimpose one another in a bounded medium. then, the resultant displacement pattern (loop pattern) collapse and expends at their positions. Such type of resultant dis pattern is c/a Stationary wave pattern.

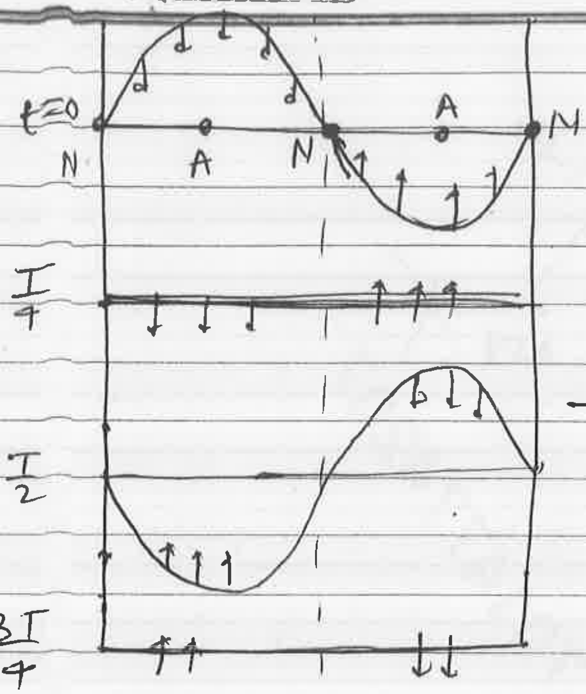
*

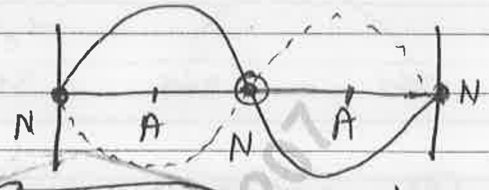


*eg.



St. wave pattern.





$N-N = \lambda/2$

$A-A = \lambda/2$

$N-A = \lambda/4$

$A-N = \lambda/4$

$$(Amp)_{antinode} = a + a = 2a$$

$$(Amp)_{Node} = a - a = 0$$

<u>Properties of Standing waves:</u>

(1) In stationary pattern all medium particle execute SHM of same T and f but have different amplitude. there amp is the f^n of position.

(2) All medium particle reach at their extreme position simultaneously. but H is different for diff particle.

So, $\boxed{(V_p)_{max} = a\omega \text{ different}}$

(3) All medium particle reach at mean position simultaneously and show stationary pattern become straight line 2 time in 1 cycle.

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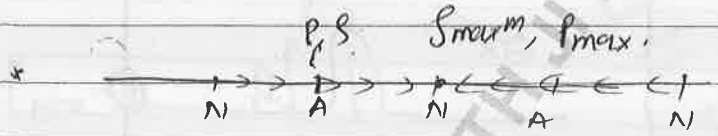
(4) Position's where amp. of vibration is max^m c/a as Antinode & min^m c/a as node.

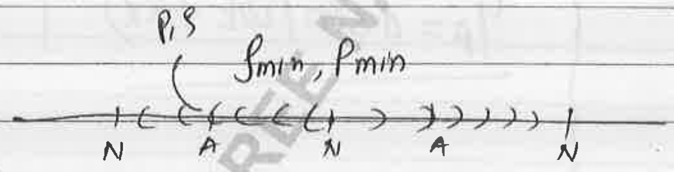
imp* (5) All particle b/w two consecutive node vibrate in same phase but either sides of node particle vibrate in opp. phase.

(6) Due to Node formation energy does not transfer, loop pattern does not transfer and so speed of this wave is Zero.

imp
Some extra properties only for longitudinal stationary waves
not in Trans

(1)* change in Pressure and change in density are max^m at node position & min^m at antinode position.





Neet 13

* $\left.\begin{matrix}\Delta P \\ \Delta \rho\end{matrix}\right\}$ max^m at nodes ✓

* $\left.\begin{matrix}\Delta P \\ \Delta \rho\end{matrix}\right\}$ min^m at antinodes

(2) strain is max^m at node position and min^m at antinode

stress ∝ strain
⇓
ΔP ∝ strain

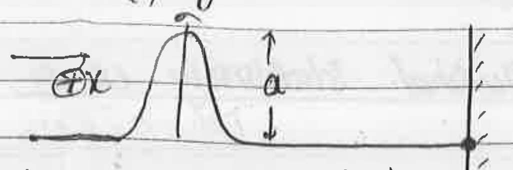
~~Node => max^m~~ node => min^m
Antinode => max^m

★ (13) Displacement node is c/a as pressure antinode and displacement antinode is c/a as pressure node position.

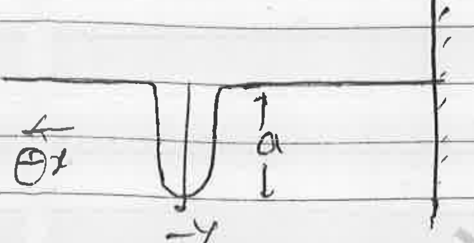
Phase difference in b/w pressure wave & displacement wave is $\pi/2$.

Reflection of wave: ⟶ two type of Reflector

Rigid end reflector



$y_{incid} = a\sin(\omega t - kx)$

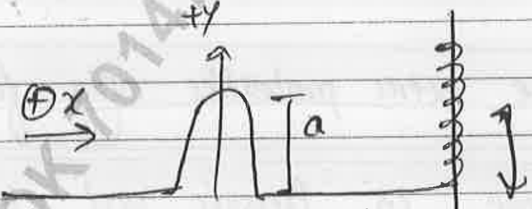


$y_R = a\sin(\omega t + kx + \pi)$

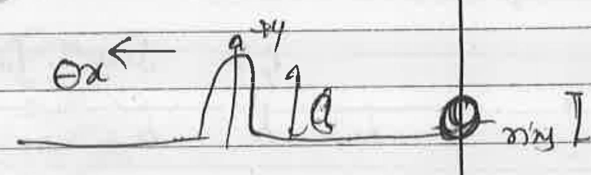
or

$y_R = -a\sin(\omega t + kx)$

free end reflector



$y_{in} = a\sin(\omega t - kx)$



$y_R = a\sin(\omega t + kx)$

Reflection of longitudinal waves

* Rigid end ⟶ compression ⟺ compression, rarefaction ⟺ rarefaction] same phase ★

* Free end ⟶ compression ⟺ rarefaction, rarefaction ⟺ compression] opposite phase ★

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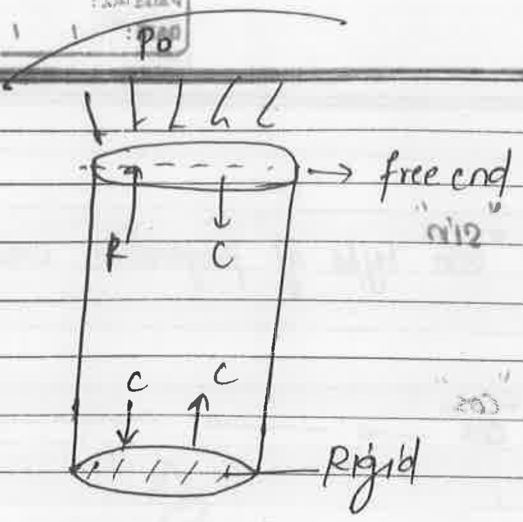

COP [Closed organ Pipe]

Mathematical analysis →

$$Y_{in} = a\sin(\omega t - kx) \quad (\cos)$$

Rigid end

$$Y_R = -a\sin(\omega t + kx) \quad (\cos)$$

$$\vec{Y}_{sw} = \vec{Y}_{in} + \vec{Y}_R$$

$$Y_{sw} = a[\sin(\omega t - kx) - \sin(\omega t + kx)] \quad (\cos)$$

$$\boxed{Y_{sw} = \underbrace{-2a\sin kx}_{\text{Amp}} \; \underbrace{\cos\omega t}_{\text{SHM}}}$$

$$Y_{sw} = 2a\sin kx \sin\omega t$$

free end

$$Y_R = a\sin(\omega t + kx) \quad (\cos)$$

$$\vec{Y}_{sw} = \vec{Y}_{in} + \vec{Y}_R$$

$$Y_{sw} = a[\sin(\omega t - kx) + \sin(\omega t + kx)] \quad (\cos)$$

$$\boxed{Y_{sw} = \underbrace{2a\cos kx}_{\text{Amp.}} \; \underbrace{\sin\omega t}_{\text{SHM}}}$$

$$Y_{sw} = 2a\cos kx \cos\omega t$$

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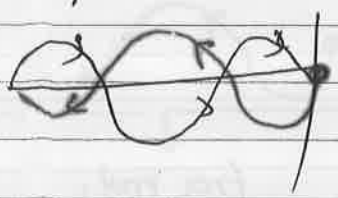
Result

* $y = 2a\sin kx \cos\omega t$
  $y = 2a\cos kx \sin\omega t$] $\rightarrow$ "sin" type of progressive waves ✓

$y = 2a\sin kx \sin\omega t$
$y = 2a\cos kx \cos\omega t$] $\rightarrow$ "cos" —" —" —".

* if reflection is from rigid end then "NODE" and reflection is from freeend then "ANTINODE" is formed at that end.



* $(x=0)$

$x=0 \Rightarrow y=0 \Rightarrow$ Node $\Rightarrow$ Rigid end $(\sin kx)$

$x=0 \Rightarrow y\neq 0 \Rightarrow$ Antinode $\Rightarrow$ free end $(\cos kx)$

Que. A stationary wave eqn is given by –

$$y = 10\cos\left(\frac{\pi}{6}x\right)\cos(60\pi t)$$
$$y = 2a\cos kx \cos\omega t$$

Where $x \Rightarrow cm$, $y \rightarrow cm$, $t \Rightarrow sec$

then ① Eqn of incident & reflecting wave

* cos type
* $x=0 \Rightarrow y\neq 0 \Rightarrow$ Antinode $\Rightarrow$ free

or $\cos kx =$ free end.

Now,

$y_{in} = a\cos(\omega t - kx)$

$= 5\cos(60\pi t - \frac{\pi}{6}x)$ m

$y_R = +5\cos(60\pi t + \frac{\pi}{6}x)$

if dirn of incident wave is not given then assume to be moving in +ve x-dirn.

Q. A stationary wave eqⁿ is given–

$$y = 5\sin\left(\frac{\pi}{3}x\right)\cos(40\pi t)$$

$y = 2a\sin kx \cos\omega t$

where, $x \to$ cm
$y \to$ cm
$t \to$ sec

then find (i) Eqⁿ of incident and reflecting waves if incident wave is propagating in $\ominus$ x-axis.

* 'sin' type.
* $\sin kx \Rightarrow$ Rigid
* $y_{in} = a\sin(\omega t + kx)$
$= 2.5\sin\left(40\pi t + \frac{\pi}{3}x\right)$

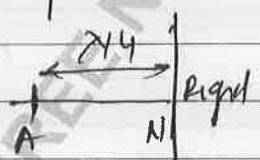
$y_R = -2.5\sin\left(40\pi t - \frac{\pi}{3}x\right)$

ii) Distance b/w two consecutive node.

$k = \frac{2\pi}{\lambda} \Rightarrow \lambda = \frac{2\pi}{k} = \frac{2\pi}{\pi/3} = 6\text{ cm}$

$N\text{–}N = \frac{\lambda}{2} = \frac{6}{2} = 3\text{cm}.$

iii) Minimum distance from end position when amplitude is maximum.



$\frac{\lambda}{4} = \frac{6}{4} = 1.5\text{cm}$

iv) Amp of st. wave at $x = 3$ cm. $\to$ Ans zero
$\Downarrow$
0

$x = 1$ cm

$y = 5\sin\left(\frac{\pi}{3}x\right)$

$A_x = 5\sin\left(\frac{\pi}{3}\times 1\right)$

$= 5\times\frac{\sqrt{3}}{2} = \frac{5\sqrt{3}}{2}$ cm.

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v) Amp. at node and Antinode.

$(Amp)_{antinode} = 2a = 5cm$

$(Amp)_{node} = 0$

(6) velocity of particle at $x = 1.5cm$ and $t = \frac{9}{8}$ sec

$$y = 5\sin\left(\frac{\pi}{3}x\right)\cos(40\pi t)$$

$$\frac{\partial y}{\partial t} = V_p = \left[5\sin\left(\frac{\pi x}{3}\right)\right]\left[-\sin(40\pi t)\right] \times 40\pi$$

$$= \left[5\sin\left(\frac{\pi}{3}\times 1.5\right)\right]\left(-\sin\left(40\pi \times \frac{9}{8}\right)\right) \times 40\pi$$

$$= (-200\pi)(\sin(45\pi))$$

$$\underline{V_p = 0}$$

Q. A incident progressive wave eqn is given by

$y_{in} = a\sin(kx - \omega t)$ if at $\underline{x=0}$, node is formed then find.

(a) eqn of reflecting wave.

$$y_{in} = a\sin(-(\omega t + kx))$$

$$y_{in} = -a\sin(\omega t - kx)$$

$x = 0 \Rightarrow$ Node $\rightarrow$ Rigid end

$$y_R = +a\sin(\omega t + kx) \text{ Ans}$$

$$\boxed{y_R = a\sin(kx + \omega t)} \text{ Ans}$$

★ $\sin(-\theta) = -\sin\theta$

but

$\cos(-\theta) = \cos\theta$

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Applications → musical instrument.

longitudinal st. waves

* Organ pipes
* Resonance tube exp.

Transverse st. waves

* Sonometer wire
* Sonometer exp..

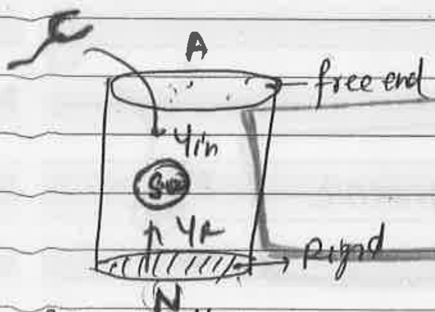
eg.

multiple

40 Hz	60 Hz	80 Hz	100 Hz	120 Hz
fundamental freq. or fundament tone Or I Harmonic	I OT	II OT	III OT	IV OT
	X	II Harmonic	X	III Harmonic

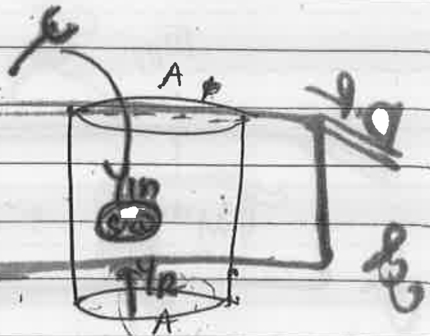
Application of longitudinal stationary waves →

COP



eg. whistle

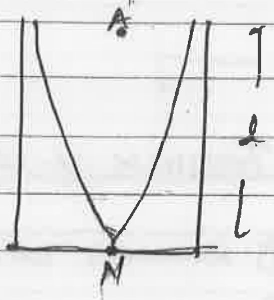
OOP



eg. Flute

COP

n_1



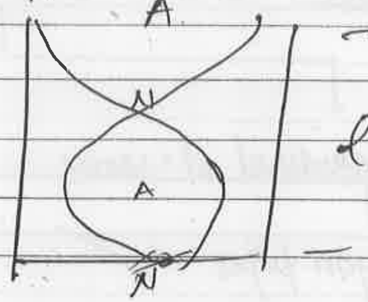
$\frac{\lambda_1}{4} = l \Rightarrow \lambda_1 = 4l$

$$\boxed{n_1 = \frac{V}{\lambda_1} = \frac{V}{4l}}$$

* fundemental freq.
I Harmonic

C

n_2



$\frac{3\lambda_2}{4} = l \Rightarrow \lambda_2 = \frac{4l}{3}$

$n_2 = \frac{3V}{4l}$

III Harmonic
or
I OT

* freq. ratio → 1 : 3 : 5 : 7 : 9 . . .
* OT freq. ratio → 3 : 5 : 7 : 9 - -

* only odd Harmonic are present.

* I OT → $\frac{3V}{4l}$ = III Harmonic

II OT → $\frac{5V}{4l}$ = V Harmon

III OT → $\frac{7V}{4l}$ = VII Harmon

D.R

$$\boxed{M^{th}\ OT \rightarrow (2M+1)\frac{V}{4l} = (2M+1)^{th} \text{ Harmonic.}}$$

COP

A

N

$M = OT \cdot no.$

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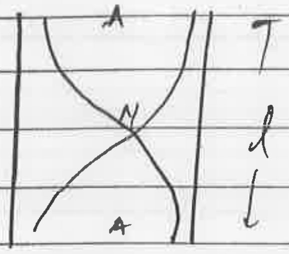
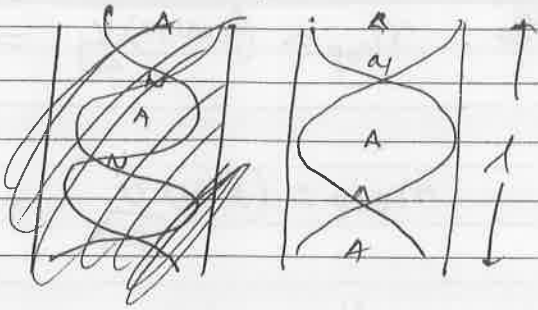
OOP



$\frac{\lambda_1}{2} = l \Rightarrow \lambda_1 = 2l$

$$\boxed{n_1 = \frac{V}{\lambda_1} = \frac{V}{2l}}$$

* fundamental freq.
or
I Harmonic



$\frac{2\lambda_2}{2} = l \Rightarrow \lambda_2 = \frac{2l}{2}$

$$\boxed{n_2 = \frac{V}{\lambda_2} = \frac{2V}{2l}}$$

I OT
or II Harmonic

* frequeny ratio $\rightarrow$ 1 : 2 : 3 : 4 - - -

* OT freq ratio $\rightarrow$ 2 : 3 : 4 : . .

* even and odd Harmonic are present (High quality of sound)

* I OT $\rightarrow \frac{2V}{2l}$ = II Harmonic

II OT $\rightarrow \frac{3V}{2l}$ = III harmonic

III OT $\rightarrow \frac{4V}{2l}$ = IV harmonic

$$\boxed{M^{th}\ OT \rightarrow \frac{(M+1)V}{2l} = (M+1)^{th} \text{ Harmonic}}$$

Q. freq of 5^{th} OT of COP is greter by 100 Hz then the freq. of 2^{nd} O.T of OOP of same length. find fundamental frequeny of both pipe.

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Sol^n $n_{cop} = (2m+1)\frac{V}{4l} = \frac{11V}{4l}$

$n_{oop} = (m+1)\frac{V}{2l} = \frac{3V}{2l}$

$\frac{11V}{4l} - \frac{3V}{2l} = 100$

$\frac{11V - 6V}{4l} = 100$

$\frac{5V}{4l} = 100 \Rightarrow \frac{V}{4l} = 20\,Hz = (n_1)_{cop}$

$\Rightarrow \frac{V}{2l} = 40\,Hz = (n_1)_{oop}$

Q-10 ✓
P-194
Q-11

Q-6

$l \quad \frac{V}{2l} = 1400\,Hz$

$0.7l \quad \frac{V}{4 \times 0.7l} = n'$

$\boxed{n_1' = \frac{V}{2l(1.4)} = \frac{1400}{1.4} = 1000\,Hz}$

P-210
Q 64

425 : 595 : 765
85 : 119 : 153
5 : 7 : 9

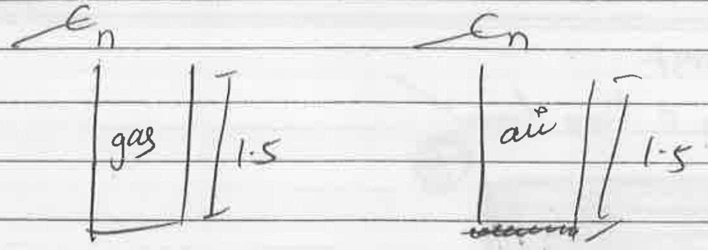
$\frac{V}{4l} = 425$ $\quad \frac{340}{4l} = 425$

COP

$\frac{5V}{4l} = 425$

~~NB~~ Be careful about tempr unit in sound calculating eqn 'I'

(62)



$n = \frac{V_{gas}}{4l}$ $\qquad$ $n = \frac{V_{air}}{2l}$

$\Rightarrow \frac{V_{gas}}{4l} = \frac{V_{air}}{2l}$

$\Rightarrow \boxed{V_{gas} = 2V_{air}}$

At 30°c temp $V_{gas} = 2 \times 360 = 720$ m/s

$V_t = V_0\left(1 + \frac{t}{546}\right)$

$720 = V_0\left(1 + \frac{30}{546}\right)$

A $\qquad V_0 = 683$ m/s

Q. 2 COP of length 50cm & 51cm vibrate together and produce 5 beats per second. find frequency of both C. Organ. Pipe.

$n_1 - n_2 = \Delta n$

$\frac{V}{4(0.50)} - \frac{V}{4(0.51)} = 5$

2nd

let $\frac{V}{4(50)} = n$ — (1)

$\frac{V}{4(51)} = n - 5$ — (2)

from (I) & (II)

$\frac{51}{50} = \frac{n}{n-5}$

$n = 255$ Hz

$n - 5 = 250$ Hz

IInd weh

$n \propto \frac{1}{l} \Rightarrow nl = const$

$\Rightarrow n_{more}\, l_{less} = n_{less}\, l_{more}$

$$\Rightarrow \boxed{\frac{n_{more}}{n_{less}} = \frac{l_{more}}{l_{less}}}$$

$$\frac{n}{n-5} = \frac{51}{50}$$

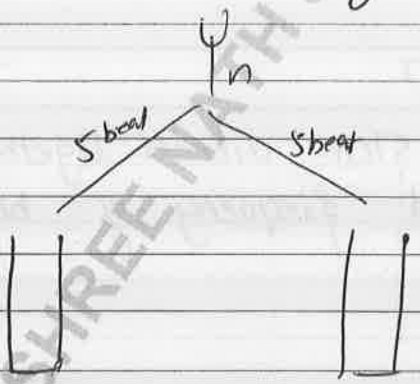
or

$$\frac{n}{n+5} = \frac{50}{51}$$

$n = 255$

$n+5 = 250$

Q. A tuning fork vibrate with 2 COP of length 50cm & 51cm ~~separately~~ and produce 5 beats/sec with each organ pipe. Find freq. of tunning fork and both organ pipe.



$n_1 = n+5$ $\qquad$ $n_2 = n-5$

$$\frac{n+5}{n-5} = \frac{51}{50}$$

$n = n_{TF} = 505$

$n_1 = n+5 = 510$

$n_2 = n-5 = 500$

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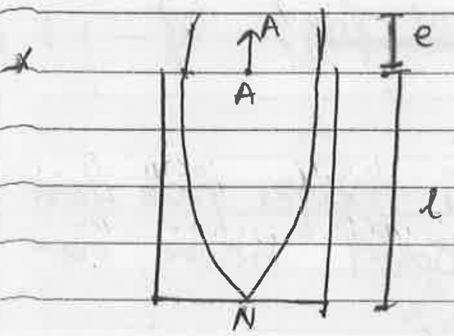
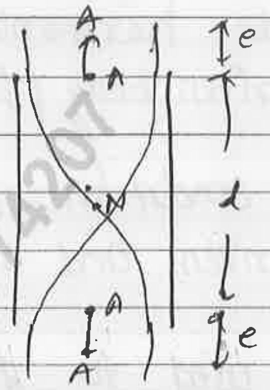
End correction (e): Due to inertia position of Antinode is slightly shifted from above from open end of an organ pipe. It is k/n as end correction of pipe.





$\frac{\lambda}{4} = l+e$

$\boxed{\lambda = 4(l+e)}$ ↑

$\boxed{n = \frac{V}{\lambda} = \frac{V}{4(l+e)}}$ ↓

$\frac{\lambda}{2} = l+2e$

$\boxed{\lambda = 2(l+2e)}$ ↑

$\boxed{n = \frac{V}{\lambda} = \frac{V}{2(l+2e)}}$ ↓

$e \propto r$

⇓

$e = 0.6r \Rightarrow e = 0.3D$

where r = radius of organ pipe
D = dia. of organ pipe

$n_{narrow} > n_{wider}$

If radius is given then consider (e)

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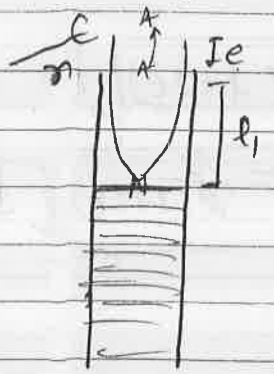
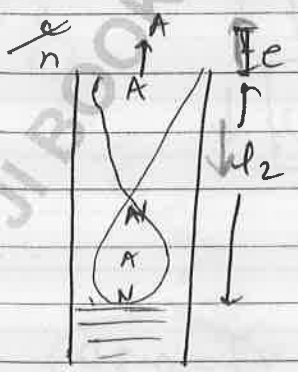
Resonance tube exp. →

→ It is based on Resonance condition. It is a special condition of forced vibration in which the freq. of driver (tuning fork) is equal to the one of the harmonic frequ (natural freq) of driven body (Resonance tube).

→ In this condition max^m energy is transfer from driver to driven and so intensity of ~~prod~~ sound become max^m.

→ It is used to find speed, and λ of sound wave in a gaseous medium and end correction of organ pipe.





resonan = 50

$$\frac{\lambda}{4} = l_1 + e \quad -① \qquad\qquad \frac{3\lambda}{4} = l_2 + e \quad -②$$

from ② − ①

$$\frac{3\lambda}{4} - \frac{\lambda}{4} = l_2 - l_1$$

$$\frac{\lambda}{2} = l_2 - l_1$$

$$\boxed{\lambda = 2(l_2 - l_1)}$$

$$\boxed{v = n\lambda = 2n(l_2 - l_1)}$$

from ② / ①

$$\frac{\frac{3\lambda}{4}}{\frac{\lambda}{4}} = \frac{l_2 + e}{l_1 + e}$$

$$\frac{3}{1} = \frac{l_2 + e}{l_1 + e}$$

$$e = \frac{l_2 - 3l_1}{2}$$

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If $e = 0 \Rightarrow l_2 = 3l_1$
$e \neq 0 \Rightarrow l_2 > 3l_1$

Conceptual Questions

Q. A tunning fork of freq 340 Hz is allowed to vibrate just above from opend end of resonance tube of length 120 cm if speed of sound is 340 m/s.

(i) find max^m no. of resonance condition.
(ii) max^m and min^m liquid level at resonance condition.

Sol^n $\lambda = \frac{v}{n} = \frac{340}{340} = 1\,m = 100\,cm$

(1) I resonance at $\frac{\lambda}{4} = \frac{100}{4} = 25\,cm$

(2) II " " $\frac{3\lambda}{4} = 75\,cm$

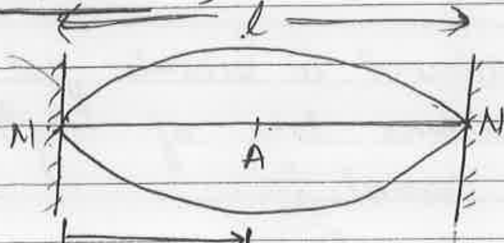
(3) III " " $\frac{5\lambda}{4} = 125\,cm > 120\,cm$ ✗

Ans (1) ⇒ 2 Resonance Condition (25, 75 cm)

An (2) max^m level $= 120 - 25 = 95\,cm$
min level $= 120 - 75\,cm = 45\,cm$

* Transverse stationary waves →

★ Sonometer →



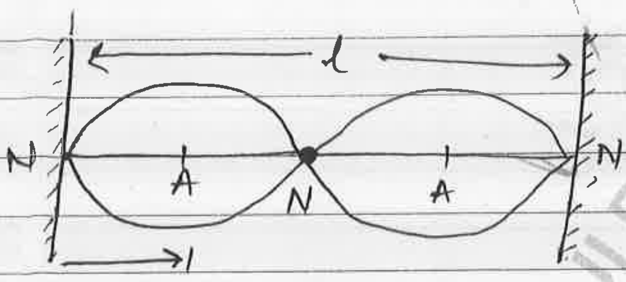
$x_p = \frac{l}{2} = \frac{l}{2\times 1}$

same size

$\frac{\lambda_1}{2} = l \Rightarrow \lambda_1 = 2l$

$$\boxed{n_1 = \frac{V}{\lambda_1} = \frac{V}{2l} = \frac{1}{2l}\sqrt{\frac{T}{\mu}}}$$

fundament freq. or I Harmonic

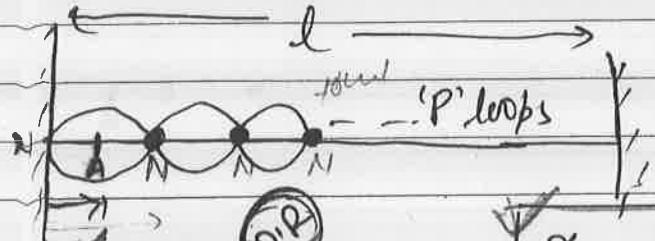


$x_p = \frac{l}{4} = \frac{l}{2\times 2}$

$2\frac{\lambda_2}{2} = l \Rightarrow \lambda_2 = \frac{2l}{2}$

$$\boxed{n_2 = \frac{V}{\lambda_2} = \frac{2V}{2l} = \frac{2}{2l}\sqrt{\frac{T}{\mu}}}$$

II Harmonic or I OT



$\boxed{x_p = \frac{l}{2P}}$ DIR

Pluck

$$\boxed{x_f = 2x_p = \frac{l}{P}}$$

Plucking → Antinode formation

Touch → Node formation

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$$p\frac{\lambda_p}{2} = l \Rightarrow \lambda_p = \frac{2l}{p}$$

$p =$ no of loops

$$\boxed{\nu_p = \frac{V}{\lambda_p} = \frac{pV}{2l} = \frac{p}{2l}\sqrt{\frac{T}{\mu}}}$$

- p^{th} Hormonic
- $(p-1)$ O.T
- p Antinode
- $(p+1)$ Nodes

D'R

ν_p $\boxed{\lambda = \frac{2l}{p}}$

Q. Standing waves are produced in a 10cm long streched wire. wire vibrate in 5 segments. if speed of transverse wave in wire is 20 cm/s then find. (1) plucking distance, (2) touching dist, (3) λ, (4) ν, (5) harmonics, (6) OT, (7) N & A (8)

Ans. $l = 10$ cm $\quad p = 5 \quad V = 20$ cm/s

(1) $x_p = \frac{l}{2p} = \frac{10}{2\times5} = 1$ cm

(2) $x_T = \frac{l}{p} = \frac{10}{5} = 2$cm

(3) $\lambda = \frac{2l}{p} = \frac{2\times10}{5} = 4$cm

(4) $\nu = \frac{V}{\lambda} = \frac{20}{4} = 5$ Hz

(5) Harmonics $= 5$

(6) OT $= 4$

(8) Antinode $= 5$

(7) Node $= 6$

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* Results

① $n = \frac{P}{2l}\sqrt{\frac{T}{A\rho}} = \frac{P}{2l}\sqrt{\frac{T}{AS}} = \frac{P}{2l}\sqrt{\frac{T}{(\pi r^2)\rho}} = \frac{P}{2l}\sqrt{\frac{\text{stress}}{\rho}} = \frac{P}{2l}\sqrt{\frac{Y \times \text{strain}}{\rho}}$

② $n \propto \sqrt{T} \Rightarrow \boxed{\frac{\Delta n}{n} \times 100 = \frac{1}{2}\frac{\Delta P}{P} \times 100}$

③ $n \propto \frac{1}{l} \Rightarrow \boxed{\frac{n_{more}}{n_{less}} = \frac{l_{max}}{l_{less}}}$

④ $n = \frac{P}{2}\sqrt{\frac{T}{l^2 \times \frac{M_{wire}}{l}}} = \boxed{\frac{P}{2}\sqrt{\frac{T}{M_{wire} \times l}}}$

⑤ $\frac{n_1}{n_2} = \frac{P_1}{P_2} \times \frac{l_2}{l_1}\sqrt{\frac{T_1}{T_2} \times \frac{r_2^2}{r_1^2} \times \frac{\rho_2}{\rho_1}}$

Pg-220
Q-94

Pg-194
Q-2

Pg-221
Q-102

Pg-210
Q-63

Pg-212
Q-75

Pg-195
Q-13

AIEEE RPMT

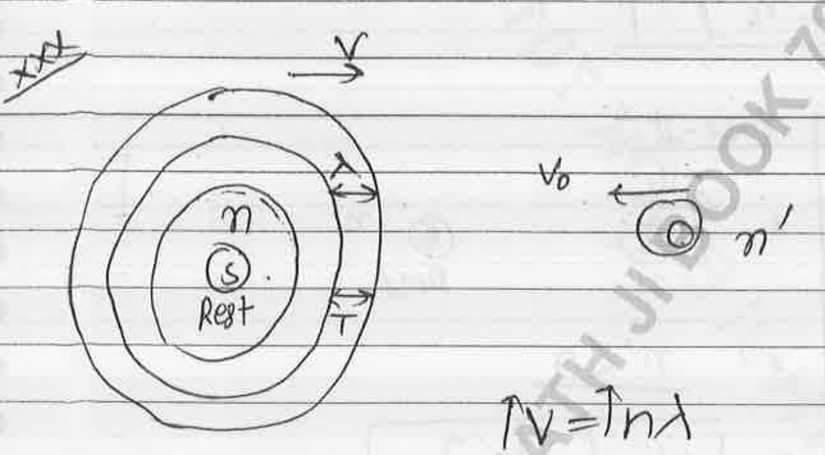
Q. A wire having linear mass density 5×10^{-3} kg/m is streched b/w 2 rigid support under tension 450N. This wire resosanat at freq. of 420 Hz. If the next higher resosant freq. of this wire is 490 Hz. then find min. possibly freq. (fundamental freq) of wire.

P-124
Q-13

* Doppler Effect

⇒ Due to relative motion b/w source & observer, observer observes diff. freq of sound from the freq. produced by the source, the change in freq due to relative motion b/w source & observer or change in pitch phenomenon is called as doppler effect.

* Doppler effect for sound waves

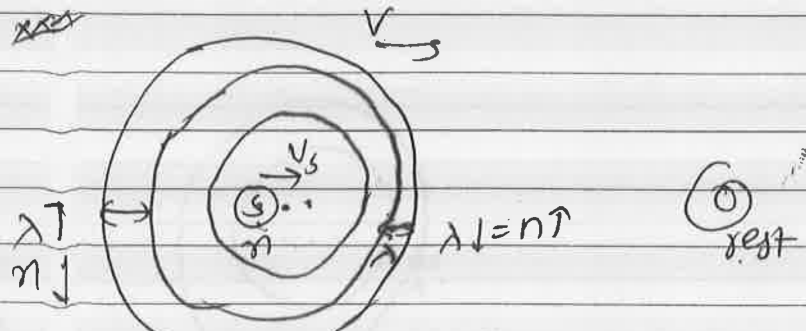


$$\uparrow V = \uparrow n\lambda$$
$$\lambda = const$$

$V = \frac{\lambda}{T}$

$\lambda = VT$

$$n' = \frac{V'}{\lambda} = \left(\frac{V+V_0}{VT}\right) = \left(\frac{V \pm V_0}{V}\right)n$$



$$V = \uparrow n\,\lambda\downarrow$$
$$V = const.$$

$$n' = \frac{V}{\lambda'} = \frac{V}{VT \mp V_sT} = \left(\frac{V}{V \mp V_s}\right)n$$

* General formula

$$\boxed{n' = \left[\frac{V \pm v_o}{V \mp v_s}\right] n}$$ ★★

*If V_w = speed of wind then –

$$\boxed{n' = \left[\frac{(V \pm V_w) \pm V_o}{(V \pm V_w) \mp V_s}\right] n}$$

* Case-I

* (S) $\xrightarrow{V_s}$, n (O) n' Rest

* $n' > n$

* $$\boxed{n' = \left[\frac{V}{V-V_s}\right] n}$$

* $$\frac{\cancel{V}}{\lambda'} = \left[\frac{V}{V-V_s}\right]\frac{\cancel{V}}{\lambda}$$

$$\boxed{\lambda' = \left[\frac{V-V_s}{V}\right]\lambda}$$

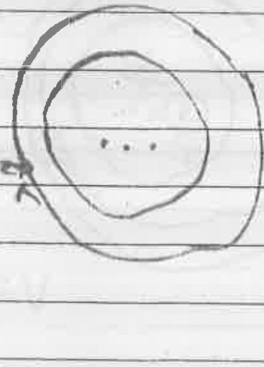
Case-II

* $\xleftarrow{V_s}$ (S) , n (O) n' Rest

* $n' < n$

* $$\boxed{n' = \left[\frac{V}{V+V_s}\right] n}$$

* $$\boxed{\lambda' = \left(\frac{V+V_s}{V}\right)\lambda}$$



↓ ↑

$f(n\lambda) = V$

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Case - III

n $\overleftarrow{V_0}$

(S) rest (O) n'

* $n' > n$
* $n' = \left[\dfrac{V+V_0}{V}\right] n$
* $\lambda' = \lambda = \text{const}$
* $V' = V + V_0$

Case - IV

n $\overrightarrow{V_0}$

(S) Rest (O) n'

* $n' < n$
* $n' = \left(\dfrac{V-V_0}{V}\right) n$
* $\lambda' = \lambda = \text{const}$
* $V' = V - V_0$

When both are in motion

Case - V

(S) $\rightarrow V_s$ (O) $\rightarrow V_0$

* $n' = \left[\dfrac{V-V_0}{V-V_s}\right] n$
* $V' = V - V_0$
* $\lambda' = \dfrac{V-V_s}{n} = \left(\dfrac{V-V_s}{V}\right)\lambda$
* $n' = \dfrac{V'}{\lambda'} = \left(\dfrac{V-V_0}{V-V_s}\right) n$

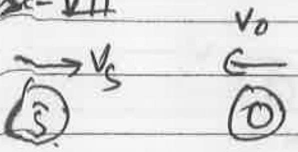
Case - VI

(S) $\overleftarrow{V_s}$ (O) $\overleftarrow{V_0}$

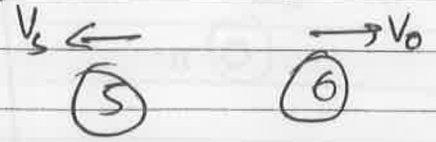
$n' = \left[\dfrac{V+V_0}{V+V_s}\right] n$ ✓

* $V' = V + V_0$
* $\lambda' = \left(\dfrac{V+V_s}{V}\right)\lambda$ ✓

Case-VII



Case-VIII

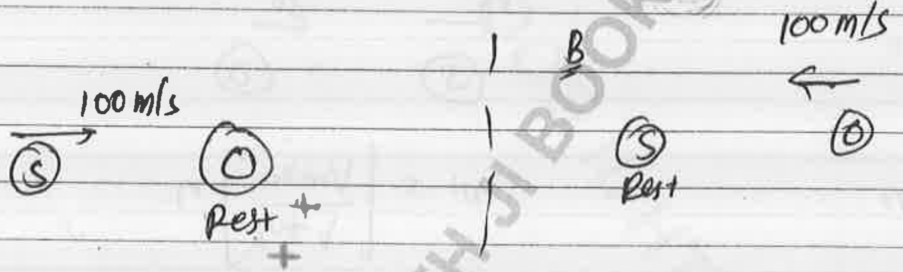


$\times$ $n' = \left[\dfrac{V+V_0}{V-V_s}\right] n$ $\qquad$ $n' = \left[\dfrac{V-V_0}{V+V_s}\right] n$

$\times$ $V' = V+V_0$ $\qquad$ $\times$ $V' = V-V_0$

$= \lambda' = \left(\dfrac{V-V_s}{V}\right)\lambda$ $\qquad$ $\times$ $\lambda' = \left(\dfrac{V+V_s}{V}\right)\lambda$

Q



If $n = 300\,Hz$, $V = 300\,m/s$ then find n'_A & n'_B

Sol

$$n'_A = \left(\frac{V}{V-V_s}\right) n$$

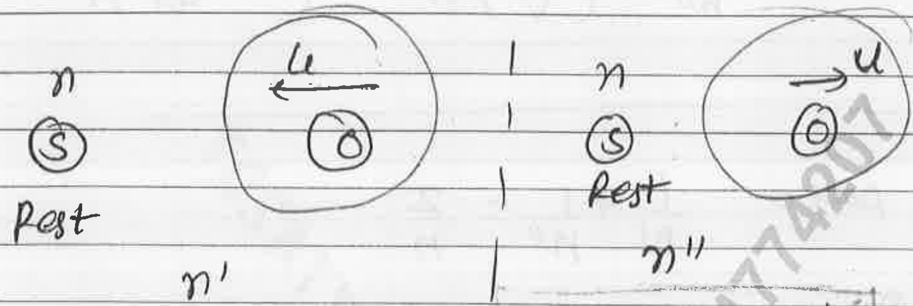
$$= \left[\frac{300}{300-100}\right] n = \frac{300}{200} \times 300$$

$$= \underline{\underline{450\,Hz}}$$

$$n'_B = \left[\frac{V+V_0}{V}\right] = \left(\frac{300+100}{300}\right) \times 300 = \underline{\underline{400\,Hz}}$$

Doppler effect is ~~asymmetric~~ for sound waves is an asymmetric phenomenon because of different freq. change factor (V & t)

Q



find relation b/w n', n'' & n

soln $n' = \left(\frac{V+u}{V}\right)n = \left(1 + \frac{u}{V}\right)n$ — (i)

$n'' = \left(\frac{V-u}{V}\right)n = \left(1 - \frac{u}{V}\right)n$ — (ii)

Add $n' + n'' = 2n$

D.P. $\boxed{n = \frac{n' + n''}{2}}$ → Arithmetic mean

Q

find relation b/w n' n'' & n

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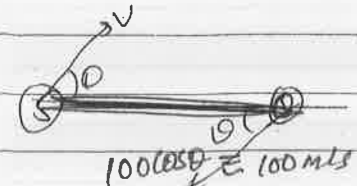
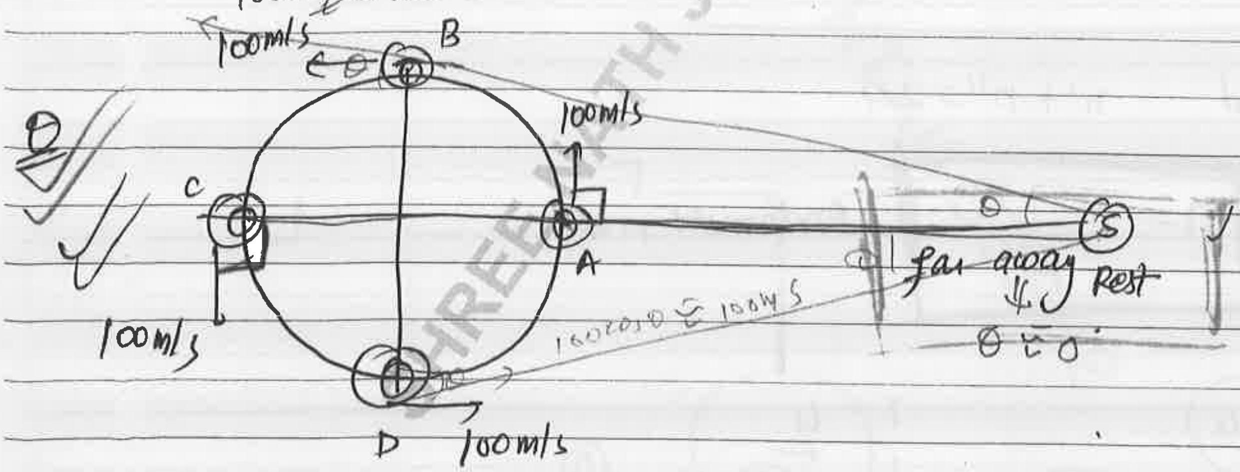
$$n' = \left(\frac{v}{v-u}\right)n \Rightarrow \frac{1}{n'} = \left(\frac{v-u}{v}\right)\frac{1}{n} = \left(1-\frac{u}{v}\right)\frac{1}{n} \quad (1)$$

$$n'' = \left(\frac{v}{v+u}\right)n \Rightarrow \frac{1}{n''} = \left(\frac{v+u}{v}\right)\frac{1}{n} = \left(1+\frac{v}{u}\right)\frac{1}{n} \quad (2)$$

Add = $\frac{1}{n'} + \frac{1}{n''} = \frac{2}{n}$

$$\boxed{n = \frac{2n'n''}{n'+n''}}$$ — H. Mean.

* 



If $n = 300$ Hz, $V = 300$ m/s, then find n'_A, n'_B, n'_C and n'_D

Sol^n * $n'_A = n = 300$ Hz $\Rightarrow \Delta n = 0$ (No doppler's effect) (No Relative motion)

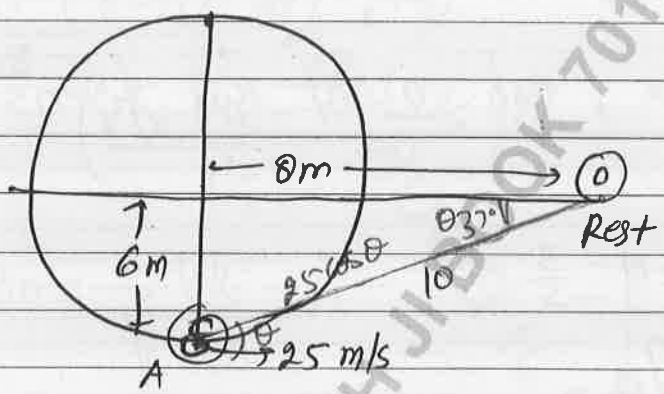
* $n'_C = n = 300$ Hz $\Rightarrow \Delta n = 0$

$$* n'_B = \left(\frac{V - V_0\cos\theta}{V}\right) n$$

$$= \left(\frac{300-100}{300}\right) \times 300 = 200 Hz$$

$$n'_D = \left(\frac{V + V_0\cos\theta}{V}\right) n = \left(\frac{300+100}{300}\right) \times 300 = 400 Hz$$

Q.



$$\cos\theta = \frac{8}{10} = \frac{4}{5}$$

if $n = 240$ Hz

$v = 100$ m/s then find n'_A

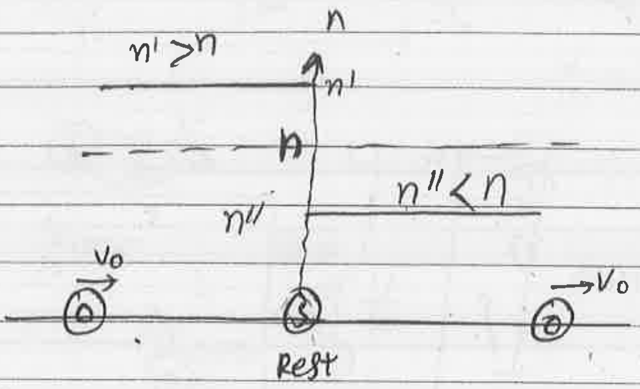
$$n'_A = \left(\frac{V}{V - V_s\cos\theta}\right) n$$

$$= \frac{100}{\left(100 - \cancel{25}^{5} \times \frac{4}{\cancel{5}}\right)} \times 240$$

$$= \frac{100}{80} \times 240$$

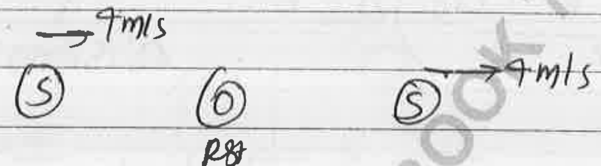
$$= 300 Hz$$

Q



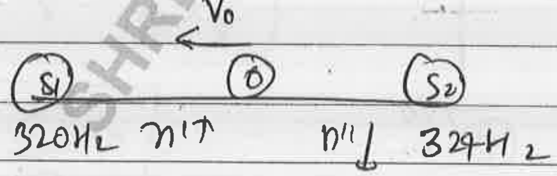
Doppler's effect depends on relative motion b/w source & observer, it does not depend on distance b/w b/w source & observe.

Q.NO 97
Pg 213



$$b = \Delta n = \frac{2V_s}{V} n = \frac{2 \times 4}{320} \times 240 = 6 \text{ beats/s}$$

Q.11
Pg NO. 214



for hearing zero beats –

$$n' = n''$$

$$\left(\frac{344 + V_0}{\cancel{344}}\right) 320 = \left(\frac{344 - V_0}{\cancel{344}}\right) \times 324$$

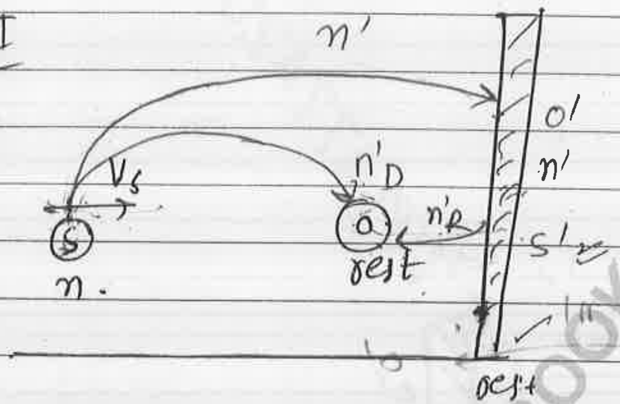
$$399 \times 320 + 320\, V_0 = 344 \times 324 - 324\, V_0$$

$$644\, V_0 = 344(4)$$

$$V_0 = \frac{4 \times 344}{644}$$

$$\simeq 2.14 \text{ m/s}$$

case-III



* $n'_D = \left(\frac{V}{V-V_s}\right) n$

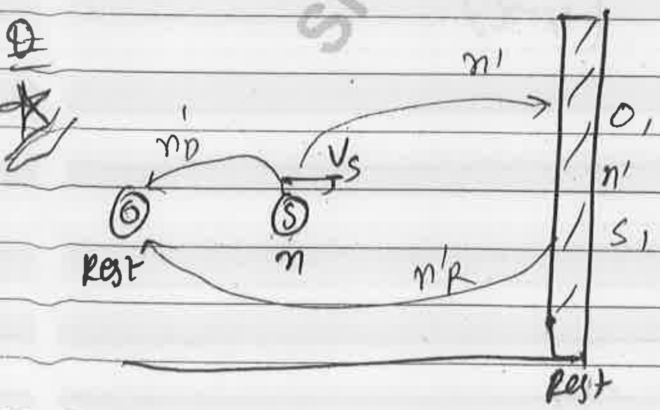
* $n' = \left(\frac{V}{V-V_s}\right) n$

* $\boxed{n'_R = n'}$ (no doppler effect)

* $b = \Delta n = n'_R - n'_D = 0$

No beat Phenomena.

Q



Calculate beat freq. observed by observer.

$n'_D = \left(\frac{V}{V+V_s}\right) n$

* $n' = \left(\frac{V}{V-V_s}\right) n$

* $n'_R = n'$

$b = \Delta n = n'_R - n'_D = \left(\frac{V}{V-V_s}\right) n - \left(\frac{V}{V+V_s}\right) n$

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$$\boxed{b = \Delta n \approx \frac{2V_s}{V} n}$$ Same as before

Trick →

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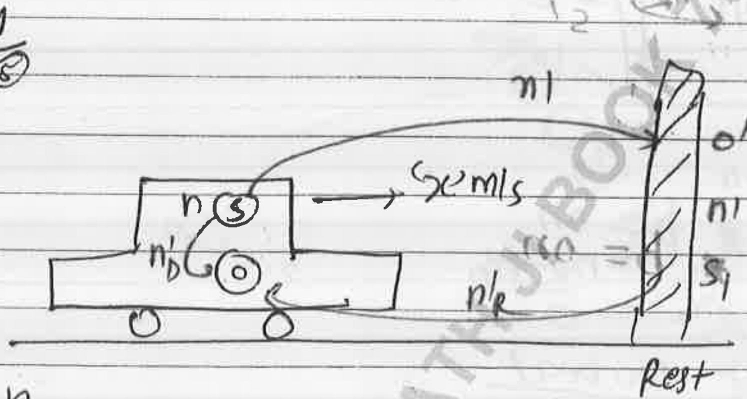
Pg-No-213
observer position not defined (obj)

case-IV
in moving (S)



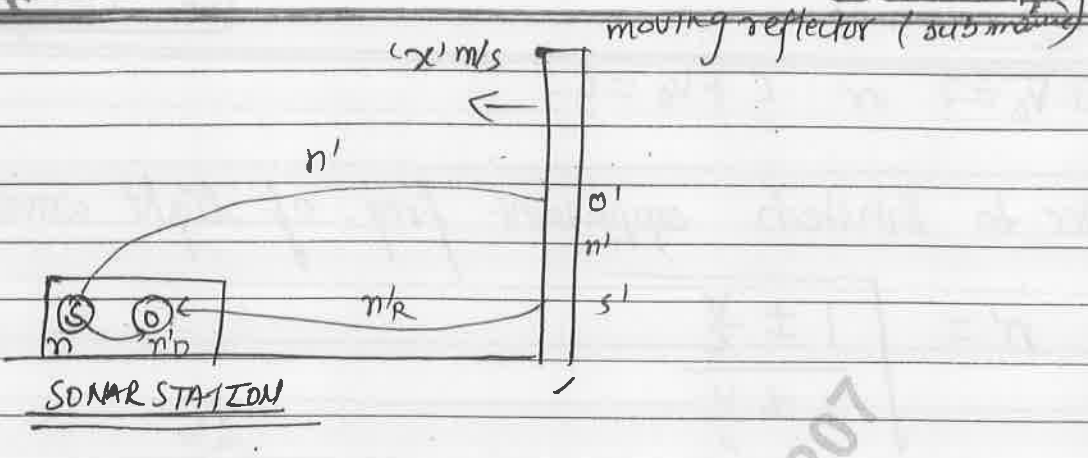
* $n'_D = n$
* $n' = \left(\frac{V+x}{V-x}\right) n$

amb
* $n'_R = \left(\frac{V+x}{V}\right) n' = \left(\frac{V+x}{V} \times \frac{V}{V-x}\right) n = \left(\frac{V+x}{V-x}\right) n$

* $b = \Delta n = n'_R - n'_D$
$= \left(\frac{V+x}{V-x}\right) n - n$

$$\boxed{b = \Delta n = \left(\frac{2x}{V-x}\right) n}$$

(case-1)



* $n'_D = n$
* $n' = \left(\dfrac{V+x}{V}\right) n$
* $n'_R = \left(\dfrac{V}{V-x}\right) n' = \left(\dfrac{V+x}{V-x}\right) n$

if $n'_R > n \Rightarrow$ coming towards.

if $n'_R < n \Rightarrow$ moving away!

: Light doppler's effect :

Doppler's effect for light waves is a symmetric phenomena but for sound waves it is an asymmetric phenomena.

becoz in light waves ——

① $$c = n\lambda$$
$$c = const$$
$$\Rightarrow n \propto \frac{1}{\lambda}$$

② it is based on Einstein's theory of relativity acc. to Einstein speed of light 'c' behaves as ∞ speed hence, means

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$C \pm V_0 = C$ or $C \mp V_s = C$.

So acc. to Einstein apparant freq. of light waves is

$$n' = \sqrt{\frac{1 \pm \frac{v}{c}}{1 \mp \frac{v}{c}}}$$

where, v = relative speed b/w source & observer

Case - I

$\overset{\rightarrow v}{S}$ $\quad$ O rest

$$n' = \sqrt{\frac{1+\frac{v}{c}}{1-\frac{v}{c}}}\, n$$

S rest $\quad$ $\overset{v \leftarrow}{O}$

$$n' = \left(1+\frac{v}{c}\right)^{\frac{1}{2}}\left(1-\frac{v}{c}\right)^{-\frac{1}{2}} n$$

$\overset{\rightarrow v/2}{S}$ $\quad$ $\overset{v/2 \leftarrow}{O}$

$$n' = \left(1+\frac{v}{2c}\right)^{2} n$$

$$\boxed{n' = \left(1+\frac{v}{c}\right) n}$$

$$\boxed{\Delta n = \frac{v}{c}\, n}$$

$$\boxed{\lambda' = \left(1-\frac{v}{c}\right)\lambda}$$

$$\Delta\lambda = -\frac{v}{c}\lambda$$ Ans

$$\lambda' - \lambda = \left(-\frac{v}{c}\right)\lambda$$

Case-II

$v \leftarrow$ (S) (O) rest

$$\boxed{n' = \left(1 - \frac{v}{c}\right) n}$$

$$\boxed{\Delta n = -\frac{v}{c} n}$$

(S) rest (O) $\rightarrow v$

$v/2 \leftarrow$ (S) (O) $\rightarrow v/2$

$$\boxed{\lambda' = \left(1 + \frac{v}{c}\right) \lambda}$$

$$\boxed{\Delta \lambda = + \frac{v}{c} \lambda}$$

Result

Always

$$\boxed{\frac{\Delta n}{n} = \pm \frac{v}{c}}$$

$$\boxed{\frac{\Delta \lambda}{\lambda} = \mp \frac{v}{c}}$$

$$\frac{\Delta n}{n} \times 100 = \pm \frac{v}{c} \times 100$$

$$\frac{\Delta \lambda}{\lambda} \times 100 = \mp \frac{v}{c} \times 100$$

Ques Wavelength of light received from a distant galaxy is 0.5 % more than then the wavelength received from an identical sources on earth. find relative speed of galaxy w.r.t earth.

$$\Delta \lambda = \frac{v}{c} \lambda$$

$$\frac{\Delta \lambda}{\lambda} \times 100 = \frac{v}{c} \times 100$$

$$0.5 = \frac{v}{3 \times 10^8} \times 100$$

$$\boxed{v = 1.5 \times 10^6 \ m/s}$$

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Spectrum

Violet end $\leftarrow$ λ ν Red end $\rightarrow$

4000Å $\leftarrow$ V I B G Y O R $\rightarrow$ 7800Å

Violet shift $\leftarrow$ $\rightarrow$ Red shift

$\lambda\downarrow \Rightarrow n\uparrow$ $\leftarrow$ $\rightarrow$ $\lambda\uparrow \Rightarrow n\downarrow$

Coming towards $\leftarrow$ $\rightarrow$ moving away

Use

Expansions of universe can be explained by using light doppler effect. (Due to red shift)

Sound characters (3)

Pitch $\xRightarrow{dep}$ ν
Quality $\xRightarrow{dep}$ no. of harmonic
Loudness $\Rightarrow$

* **Loudness [L]** $\rightarrow$ * It is measured in decibal 'db'
* It depends on shape and size of source and intensity of sound

$$L \propto \log_e I$$

$$\boxed{L = 10 \log_{10} \frac{I}{I_0}}$$

Where I_0 = threshold intensity of sound.
$= 10^{-12}$ watt/m² (Human)

* if $I = I_0$ then $L = 10 \log_{10} \frac{I_0}{I_0} = (0)\ db$

* if $I = I_1$ then $L_1 = 10 \log_{10} \frac{I_1}{I_0}$

$I = I_2$ then $L_2 = 10 \log_{10} \frac{I_2}{I_0}$

then, ~~*~~**

$$\boxed{\Delta L = L_1 - L_2 = 10 \log_{10} \frac{I_1}{I_2}}$$

Not imp.

Que-5

Pg No-172

385 Q

(5)

$L_1 = 70\ db \longrightarrow r_1 = 4m$

$L_2 = 10\ db \longrightarrow r_2 = ?$

$L_1 - L_2 = 10 \log_{10} \frac{I_1}{I_2}$

$70 - 10 = 10 \log_{10} \frac{r_2^2}{r_1^2}$ $\qquad \left(I \propto \frac{1}{r^2}\right)$

$6\cancel{0} = 1\cancel{0} \log_{10} \frac{r_2^2}{16}$

$10^6 = \frac{r_2^2}{16}$

$r_2^2 = 10^6 \times 16$

$r_2 = 10^3 \times 4$

$r_2 = 4000m$